

NONLINEAR DYNAMIC RESEARCH OF THE SPACECRAFT ROBUST FAULT TOLERANT CONTROL SYSTEMS*

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Abstract: New results on methods and software for nonlinear dynamic research and design of robust gyromoment fault tolerant control systems by the information spacecraft (remote sensing, communication, navigation and geodesic) developed by the leading Russian State Space Research and Production Associations (RPAs) — the *TsSKB-Progress* and the *NPO PM*, are presented. *Copyright © 2002 IFAC*

Keywords: spacecraft, attitude control system, fault tolerance, nonlinear dynamics

1. INTRODUCTION

Over the last two decades, the basic research on fault diagnosis, fault detection and isolation (FDI) at general control systems have received much attentions. The model-based approach is now recognized as an important and efficient method (Frank *et al.*, 2000), the trends in extending that methodology to *nonlinear* control systems today are practically realized at aerospace engineering by most modern motto in *Computer Science* (Kurakin, 1999; Somov, 2001b):

from *Artificial Intelligence* to *Natural Tricks*.

During the recent 30 years authors have been accumulated a substantial experience (*Natural Tricks*) in designing the spacecraft (SC) attitude control systems (ACSs) with high fault-tolerance, survivability and autonomy at the expense of functional *redundancy*. The dynamic requirements to the ACS for the communication, navigation and geodesic SCs are:

- continuous precision 3-axis orientation of the SC body by only minimum number of measurements under possible ACS on-board equipment failures, and also in executing a SC orbit correction;
- possibility of the SC body re-orientation for its orbit correction, as well autonomous control of the solar array panels (SAPs)

* This work was supported by the RKA and Russian Foundation for Basic Research (RFBR), grant 00-01-00293.

- and each high-gain receiving-transmitting antenna (RTA) with respect to the SC body;
- robustness to variations of the SC inertial and rigidity characteristics under minimum mass, size and power expenditures.

For the remote sensing SC these requirements are expressed by rapid angular manoeuvring and a spatial compensating motion with a variable vector of angular rate, there are the need:

- to orient the telescope line-of-sight to a pre-determined part of the Earth surface with the scan in designated direction;
- to compensate an image motion at the on-board optical telescope focal plane.

Increased requirements to such information satellites (lifetime up to 10 years, exactness of spatial rotation manoeuvres with the effective damping of the SC flexible construction oscillations, fault tolerance, reliability as well as to reasonable mass, size and energy characteristics) have motivated intensive development the moment gyrocomplex (MGC) based on excessive number of reaction wheels (RWs), gyrowhells (GWs) and gyrodines (GDs) — single-gimbal control moment gyros. Mathematical aspects of the spacecraft *nonlinear* gyromoment attitude control were represented in a number of research works (Junkins and Turner, 1986). The recent results (Schaub *et al.*, 1998) are based on methods of *Lyapunov functions*, whereas the *exact feedback linearization* (EFL) technique also was applied to this problem. This paper suggests original results on *nonlinear* methods and software for modeling, dynamic research, design and simulation of the SC gyromoment *robust fault tolerant* attitude control systems.

2. STATEMENT OF GENERAL PROBLEM

Let be given the nonlinear generalized controlled object \mathcal{O} for a time $t \in T_{t_0} \equiv [t_0, \infty)$

$$\begin{aligned} D^+x(t) &= \mathcal{F}(x(t), u, p(t, x), \gamma_\nu^f(t)), x(t_0) = x_0; (1) \\ y(t) &= \psi^o(x(t), \gamma_\nu^f(t)); \\ z^o(t) &= \phi^o(x(t), y(t), p(t, x)), \end{aligned} \quad (2)$$

where $x(t) \in \mathcal{H} \subset \mathbb{R}^{n_\nu}$; $x_0 \in \mathcal{H}_0 \subseteq \mathcal{H}$; $y(t) \in \mathbb{R}^{r_\nu^s}$ is an output vector for measurement and diagnosis of object's state, and $z^o(t) \in \mathbb{R}^{r_\nu^f}$ is a vector for description of its failure conditions; $u = \{u_j\} \in U \subset \mathbb{R}^{r_\nu^c}$ is a control vector, and $p(t, x) \in \mathcal{P}$ is the vector-function of disturbances in class \mathcal{P} ; D^+ is symbol of a right derivative with respect to time, and $\gamma_\nu^f(t) \in \mathcal{B}^m$ under $\mathcal{B}^m \equiv \mathcal{B} \times \mathcal{B} \cdots \times \mathcal{B}$, $\mathcal{B} = \{0, 1\}$ is vector of logic variables, which are outputs of a "fault's" asynchronous logic automaton (ALA) \mathcal{A}^f for its time $\nu \in \mathbb{N}_0 \equiv [0, 1, 2, \dots)$,

$$\gamma_\nu^f = \delta^f(\kappa_\nu^f, l_\nu^f); \kappa_{\nu+1}^f = \lambda^f(\kappa_\nu^f, l_\nu^f), \kappa_0^f = \kappa^f(0), \quad (3)$$

with memory, where logic vectors of object's state $\kappa_\nu^f = \kappa^f(\nu)$ and input $l_\nu^f = l^f(\nu) = g^f(z^o(t_\nu^f))$,

which are used for representing fault occurrences and damage development depending on the automaton time ν , bound up with the continuous time as $t = t_\nu^f + (\tau^f - t_\nu^f)$; $\tau^f \in \tau_\nu^f \equiv [t_\nu^f, t_{\nu+1}^f)$, $\nu \in \mathbb{N}_0$. Moreover, $l_\nu^f(t) = \text{const} \forall t \in \tau_\nu^f$ and change of the logic vector γ_ν^f in general case leads to variation of *dimensions* for vectors $x(t)$ and $y(t)$ under mappings in time moments $t = t_\nu^f$:

$$x(t_{\nu+}^f) = \mathcal{P}_\nu^x(x(t_{\nu-}^f)); \quad y(t_{\nu+}^f) = \mathcal{P}_\nu^y(y(t_{\nu-}^f)).$$

Let $T_u, T_q \leq T_u$ and $T_r \geq T_u$ are fixed sampling periods of control, state measurement and the control reconfiguration, moreover, multiplicity conditions must be satisfied for these periods, and

$$\begin{aligned} x_k &= x(t_k); \quad t_k = kT_u, \quad t_s = sT_q, \quad t_\mu = \mu T_r; \\ x_k^f &= \mathcal{F}_{T_u}(x_s); \quad x_\mu^f = \mathcal{F}_{T_r}(x_k), \end{aligned}$$

where x_k^f is the value of the variable x_s measured with the sampling period T_q , which is filtered out at the time $t = t_k$; $\mathcal{F}_{T_y}(\cdot)$ is the *digital* filtering operator with the sampling period T_y , $y = u, r$.

Let be also given subsystem of discrete measurement of the object state and digital filtering:

- for diagnostics of the object \mathcal{O}

$$y_s^d = \psi^d(y_s); \quad z_k^{df} = \mathcal{F}_{T_u}(y_s^d), \quad k, s \in \mathbb{N}_0; \quad (4)$$

- for forming the control and its reconfiguration

$$\begin{aligned} y_s^u &= \psi^u(y_s); \quad y_k^f = \mathcal{F}_{T_u}(y_s^u); \\ z_\mu^f &= \mathcal{F}_{T_r}(z_k^{df}), \quad \mu, k, s \in \mathbb{N}_0. \end{aligned} \quad (5)$$

Principal problems are contained in synthesis of:

- the synchronous logic automaton (SLA) \mathcal{A}^d with memory for the structural state diagnosis

$$\gamma_k^d = \delta^d(\kappa_k^d, l_k^d); \quad \kappa_{k+1}^d = \lambda^d(\kappa_k^d, l_k^d), \quad \kappa_0^d = \kappa^d(t_0), \quad (6)$$

with logic vectors of state κ_k^d , input $l_k^d = g^d(z_k^{df})$ and output γ_k^d ;

- the SLA \mathcal{A}^r , also with memory, for description of damage's block-keeping and reconfiguration

$$\gamma_\mu^r = \delta^r(\kappa_\mu^r, l_\mu^r); \quad \kappa_{\mu+1}^r = \lambda^r(\kappa_\mu^r, l_\mu^r), \quad \kappa_0^r = \kappa^r(t_0), \quad (7)$$

with logic vectors of state κ_μ^r , input $l_\mu^r = g^r(z_\mu^f, \gamma_\mu^{df})$, where $\gamma_\mu^{df} = \mathcal{F}_{T_r}(\gamma_k^d)$, and output γ_μ^r ;

- the nonlinear control law (NCL) with its reconfigurations due to the SLA \mathcal{A}^r routine

$$\begin{aligned} u_k &= \mathcal{U}(\hat{x}_{ek}, y_{ek}^f, y_{ok}, \gamma_\mu^r); \\ \hat{x}_{ek+1} &= \hat{\mathcal{F}}_e(\hat{x}_{ek}, y_{ek}^f, y_{ok}^o, u_k, \gamma_k^d, \gamma_\mu^r), \\ \hat{x}_{e0} &= \hat{x}_e(t_0); \quad k, \mu \in \mathbb{N}_0, \end{aligned} \quad (8)$$

where $y_{ek}^f = \mathcal{F}_{T_u}(\psi_e^u(y_{es}))$; $y_{es} = \psi_e^o(x_{es}, \gamma_k^d)$, and $x_{es} = x_e(t_s) \in \mathbb{R}^{n_\mu^e}$ is the state vector of a simplified discrete object's model

$$x_{es+1} = \mathcal{F}_e(x_{es}, u_k, \gamma_k^d, \gamma_\mu^r), \quad x_{e0} = x_e(t_0), \quad (9)$$

and $\hat{x}_{ek} = \hat{x}_e(t_k) \in \mathbb{R}^{n_\mu^e}$ is its estimation; $n_\mu^e \leq n = \max\{n_\nu\}$, and y_k^o is a programmed vector.

Feedback loops (4)–(9) are intended for fault-tolerant control of the object (1)–(3).

3. SYNTHESIS OF FEEDBACK CONTROL

Applied general approach to synthesis of *nonlinear* control systems (NCS) with a partial measurement of their state is presented when their structural state is known. Moreover, the method of *vector Lyapunov functions* (VLF), which has a strong mathematical basis for analysis of stability and other dynamical properties of various nonlinear interconnected systems with the *discontinuous right-hand side*, is used in cooperation with the EFL technique (Isidori, 1989).

Let there be given a nonlinear controlled object $D^+x(t) = \mathcal{F}(x(t), u, p(t, x))$; $x(t_0) = x_0$; $t \in T_{t_0}$, where $x(t) \in \mathcal{H} \subset \mathbb{R}^n$ is a state vector with an initial condition $x_0 \in \mathcal{H}_0 \subseteq \mathcal{H}$; $u = \{u_j\} \in U \subset \mathbb{R}^r$ is a control vector, and $p(t, x) \in \mathcal{P}$ is the vector-function of disturbances. Let some *vector norms* $\rho(x) \in \overline{\mathbb{R}}_+^l$ and $\rho^0(x_0) \in \overline{\mathbb{R}}_+^{l_0}$ also be *given*. For any NCL $u = \mathcal{U}(x)$ the closed-loop system has the form

$$D^+x(t) = \mathcal{X}(t, x); \quad x(t_0) = x_0, \quad (10)$$

where $\mathcal{X}(t, x) = \mathcal{F}(x, \mathcal{U}(x), p(\cdot))$, $\mathcal{X} : T_{t_0} \times \mathcal{H} \rightarrow \mathcal{H}$ is a *discontinuous* operator. Assuming the existence and the non-local continuability of the *right-sided* solution $x(t) \equiv x(t_0, x_0; t)$ of the system (10) for its *extended definition* in the aspect of physics, the most important dynamic property is obtained, that is $\rho\rho^0$ -*exponential invariance* of the solution $x(t) = 0$ under the *desired* $\gamma \in \overline{\mathbb{R}}_+^l$:

$$(\exists \alpha \in \mathbb{R}_+) (\exists \beta \in \overline{\mathbb{B}}_+^{l \times l_0}) (\exists \delta \in \mathbb{R}_+^{l_0}) (\forall \rho^0(x_0) < \delta) \\ \rho(x(t)) \leq \gamma + \beta \rho^0(x_0) \exp(-\alpha(t - t_0)) \quad \forall t \in T_{t_0}.$$

For the VLF $v : \mathcal{H} \rightarrow \overline{\mathbb{R}}_+^k$ with the components $v^s(x) \geq 0$, $v^s(0) = 0$, $s = 1 : k$ and the norm $\|v(x)\| = \max\{v^s(x), s = 1 : k\}$, defined are the scalar function $\bar{v}(x) = \max\{v^s(x), s = 1 : l_k, 1 \leq l_k \leq k\}$ and a *lower right derivative* with respect to (10):

$$\underline{v}'(x) = \lim_{\delta t \rightarrow 0+} \{(v(x + \delta t \mathcal{X}(t, x)) - v(x)) / \delta t\}.$$

Theorem. *Let there exist the VLF v , so that:*

- 1) $(\exists a \in \mathbb{R}_+^l) (\forall x \in \mathcal{H}) \quad \rho(x) \leq a \cdot \bar{v}(x)$;
- 2) $(\exists b \in \mathbb{R}_+^{l_0}) (\forall x_0 \in \mathcal{H}_0) \|v(x_0)\| \leq \langle b, \rho^0(x_0) \rangle$;
- 3) $\exists \gamma_c \in \overline{\mathbb{R}}_+^k$ and a function $\varphi_\gamma(\cdot)$ exists so that $\gamma_c \leq \varphi_\gamma(a, \gamma)$;
- 4) $\forall (t, x) \in (T_{t_0} \times \mathcal{H})$ the conditions are satisfied:
 - a) $\underline{v}'_\gamma(x) \leq \tilde{f}_c(t, v_\gamma(x)) \equiv P v_\gamma(x) + \tilde{f}_c(t, v_\gamma(x))$;
 - b) *Hurwitz condition for the positive matrix P* ;
 - c) *Ważewski condition on quasi-monotonicity for the function $\tilde{f}_c(t, y)$* ;
 - d) *Carateodory condition for the function $\tilde{f}_c(t, y)$, bounded in each domain $\Omega_c^r = (T_{t_0} \times \mathcal{S}_c^r)$, where $r > 0$ and $\mathcal{S}_c^r = \{y \in \mathbb{R}^k : \|y\|_E < r\}$* ;
 - e) $(\tilde{f}_c(t, y) / \|y\|) \xrightarrow{t \in T_{t_0}} 0$ for $y \rightarrow 0$ uniformly with respect to time $t \in T_{t_0}$,

where $v_\gamma = v - \gamma_c$. Then solution $x(t) = 0$ of the system (10) is $\rho\rho^0$ -*exponential invariant* and the matrix \mathcal{B} has the form $\mathcal{B} = c \cdot ab^t$ with $c \in \mathbb{R}_+$.

Proof see for example in Somov *et al.* (1999a): the basis of inequality for vector norm $\rho(x(t))$ is attained by the *comparison principle*, using the *maximum right-sided* solution $\bar{x}_c(t) \equiv \bar{x}_c(t_0, x_{c0}; t)$ of a *comparison system*.

There is such an important problem: by what approach is it possible to create *constructive* techniques for constructing the VLF $v(x)$ and *simultaneous* synthesis of a nonlinear control law $u = \mathcal{U}(x)$ for the close-loop system (10) with given vector norms $\rho(x)$ and $\rho^0(x_0)$? Recently, a pithy technique on constructing VLF at such synthesis has been elaborated. This method is based on a *nonlinear transformation* of the NCS model and solving the problem in two stages.

In stage 1, the right side $\mathcal{F}(\cdot)$ in (10) is transformed as $\mathcal{F}(\cdot) = f(x) + G(x)u + \tilde{\mathcal{F}}(t, x(t), u)$, some *principal variables* in a state vector $x \in \mathcal{H} \subset \mathbb{R}^n \subseteq \mathbb{R}^n$ with $\tilde{n} \leq n$, $x_0 \in \mathcal{H}_0 \subseteq \mathcal{H}$ are selected and a *simplified nonlinear model* of the object (10) is presented in the form of an affine *quite smooth* nonlinear control system

$$\dot{x} = F(x, u) \equiv f(x) + G(x)u \equiv f(x) + \sum g_j(x)u_j,$$

which is structurally synthesized by the EFL technique. In this aspect, based on the structural analysis of *given* vector norms $\rho(x)$ and $\rho^0(x)$, and also vector-functions $f(x)$ and $g_j(x)$, the *output vector-function* $h(x) = \{h_i(x)\}$ is carefully selected. Furthermore, the nonlinear invertible (one-to-one) coordinate transformation $z = \Phi(x) \forall x \in \mathcal{S}_h \subseteq \mathcal{H}$ with $\Phi(0) = 0$ is analytically obtained with *simultaneous* constructing the VLF.

Finally, bilateral component-wise inequalities for the vectors $x, z, v(x), \rho(x), \rho^0(x_0)$ are derived, it is most desirable to obtain the *explicit* form for the nonlinear transformation $x = \Psi(z)$, inverse with respect to $z = \Phi(x)$, and the VLF aggregation procedure is carried out with analysis of proximity for a singular directions in the *Jacobian* $[\partial F(x, \mathcal{U}(x)) / \partial x]$.

In stage 2, the problem of the NCL synthesis for the *complete model* of the NCS (10), taking rejected coordinates and nonlinearities, and also the restriction on control, into account is solved by the VLF-method.

If a forming control is digital and a measurement of coordinate's state is discrete and incomplete, then a simplified nonlinear discrete object's model is obtained by *Taylor-Lie* series (Kazantzis and Kravaris, 1997), a *nonlinear* discrete observer in the NCL is formed and its parametric synthesis is carried out with a simultaneously (Somov, 1997) construct a *discrete sub-vector* VLF.

4. SYNTHESIS OF LOGIC AUTOMATA

For the FDI a three-level logic-digital system is generally applied on-board Russian spacecraft for remote sensing, communication and navigation:

- on the lower level — the *integral local* SLAs \mathcal{A}_d^d with memory for automatic monitoring of the relevant device status by measurement of available physical variables (currents, movements, rates, etc.)
- on the middle level — the *local loop* SLAs \mathcal{A}_c^d with memory for automatic monitoring of the control loop status (the roll, yaw and pitch channels, the SAP control loop etc.);
- on the higher "system" level — a SLA \mathcal{A}^d , also with memory, for the *global* functional diagnostics of the main control loop by comparison of outputs for normal and emergency models of the ACS operation.

For high fail-safe operation of the ACS, maximum employment of *functional redundancy* has been provided by using the SLA to apply all the reverse complete sets of devices or their electric circuits.

At synthesis of the diagnosis SLAs \mathcal{A}_d^d , \mathcal{A}_c^d and \mathcal{A}^d (6) and also of a damage block-keeping & reconfiguration SLA \mathcal{A}^r (7), the *Natural Tricks* are used. They are based on both well-known physical invariant relations (for example, general momentum invariant for the "SC+MGC"-system) and engineering inventiveness, presented at a perfect logic-uncontradictory form.

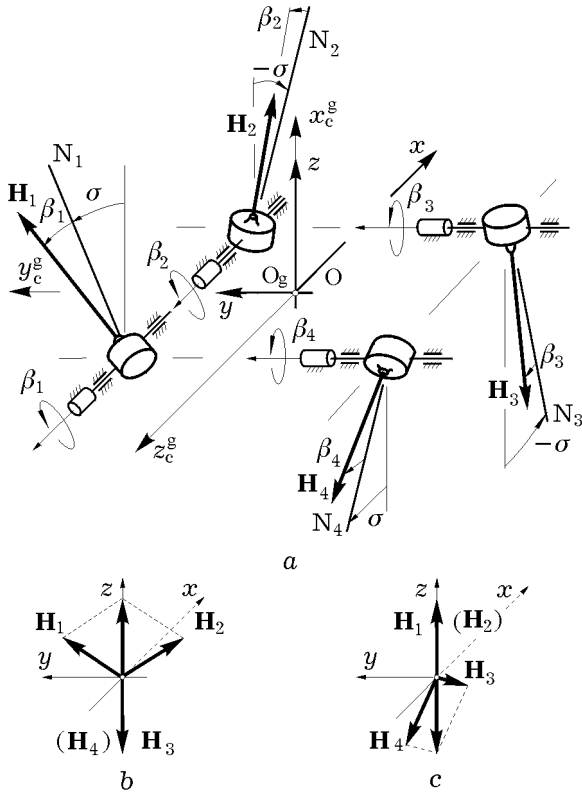


Fig. 1. The fault-tolerant "2-SPEED"-type minimal-redundant scheme of the MGC

5. A MOMENT GYROCOMPLEX

For information SC it is important to minimize the MGC mass and provide the possibility for reconfiguration of its structure and control algorithms for 2–3 possible faults in any executive device of the MGC. The authors had been executed multilateral analysis of schemes for constructing the small-mass MGC based on RWs, various types of single- and two-gimbal GWs, the GDs with both the gear stepping drives (GSDs) and the moment gearless drives (MGDs) on their precession axes, in combination with unloading loops of accumulated angular momentum (AM) by reaction trusters and/or a magnetic torquers (Somov *et al.*, 1999b).

For example, let the point O be the spacecraft mass center and $Oxyz$ is the body reference frame (BRF), see Fig. 1a. In the MGC canonical reference frame $O_g x_c^g y_c^g z_c^g$ the angular momentum projections of the first (GD-1 & GD-2) and the second (GD-3 & GD-4) pairs of gyrodines always are summed up along the axis $O_g x_c^g$. The GD's neutral positions $N_p, p = 1 : 4$ are directed at the angles $\pm\sigma$ with respect to positive (for the 1st GD's pair) and to negative (for the 2nd GD's pair) directions of the axis $O_g x_c^g$, see Fig. 1a. Under the MGC Z-arrangement on the spacecraft body, when the axis $O_g x_c^g$ is the same as the axis Oz of BRF, the following 4 efficient (for 3-axis spacecraft attitude control) GD configurations are possible on the basis of *only 3 active* gyrodines:

- the configurations Z-I, $I=1:4$ — the MGC without GD-I, represented at the nominal state in Fig. 1b (configurations Z-4 or Z-3) and in Fig. 1c, (configurations Z-2 or Z-1).

So, the "2-SPEED"-type (Crenshaw, 1973) gyrocomplex scheme in Fig. 1a is *fault-tolerant* under diagnostics of a faulted GD and the gyrocomplex reconfiguration by some *passages* between configurations Z-I on *specific logic conditions*.

6. MATHEMATICAL MODELS

The BRF attitude with respect to the inertial reference frame (IRF) is defined by quaternion $\mathbf{\Lambda} = (\lambda_0, \boldsymbol{\lambda}), \boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3)$. Assume that $\mathbf{\Lambda}^p(t)$ is a quaternion, and $\boldsymbol{\omega}^p(t) = \{\omega_i^p(t)\}$ and $\dot{\boldsymbol{\omega}}^p(t)$ are angular rate and acceleration vectors of the programmed SC body's motion in the IRF. The error quaternion is $\mathbf{E} = (e_0, \mathbf{e}) = \hat{\mathbf{\Lambda}}^p(t) \circ \mathbf{\Lambda}$, the *Euler's* parameters vector is $\mathcal{E} = \{e_0, \mathbf{e}\}$, the attitude error matrix is $\mathbf{C}_e \equiv \mathbf{C}(\mathcal{E}) = \mathbf{I}_3 - 2[\mathbf{e} \times] \mathbf{Q}_e$, where $\mathbf{Q}_e \equiv \mathbf{Q}(\mathcal{E}) = \mathbf{I}_3 e_0 + [\mathbf{e} \times]$ with $\det(\mathbf{Q}_e) = e_0$. Here symbols $\langle \cdot, \cdot \rangle, \times, \{ \cdot \}, [\cdot]$ for vectors and $[\mathbf{a} \times], (\cdot)^t$ for matrix are conventional denotations.

The BRF attitude with respect to the orbital reference frame (ORF) is defined by quaternion $\mathbf{\Lambda}^o = \tilde{\mathbf{\Lambda}}_o(t) \circ \mathbf{\Lambda}$, where $\mathbf{\Lambda}_o$ is known quaternion of the ORF attitude with respect to the IRF,

and also by vector of Euler's parameters \mathcal{E}^o , moreover the matrix $\mathbf{C}_e^o = \mathbf{C}(\mathcal{E}^o)$. For a fixed position of *flexible* structures on the SC body with some simplifying assumptions and for $t \in \mathbb{T}_{t_0} = [t_0, +\infty)$ a SC angular motion model appears as:

$$\dot{\Lambda} = \Lambda \circ \omega / 2; \mathbf{A}^o \{ \dot{\omega}, \ddot{\mathbf{q}}, \ddot{\beta}, \dot{\Omega} \} = \{ \mathbf{F}^\omega, \mathbf{F}^q, \mathbf{F}^\beta, \mathbf{F}^h \}, \quad (11)$$

$$\mathbf{F}^\omega = \mathbf{M}^g - \omega \times \mathbf{G} + \mathbf{M}_d^o + \mathbf{Q}^o; \mathbf{M}^g = -\dot{\mathcal{H}} = -\mathbf{A}_h \dot{\beta};$$

$$\mathbf{F}^q = \{ -a_{jj}^q ((\delta^q / \pi) \Omega_j^q \dot{q}_j + (\Omega_j^q)^2 q_j) + \mathbf{Q}_j^q(\omega, \dot{q}_j, q_j) \};$$

$$\mathbf{F}^\beta = \mathbf{A}_h \omega + \mathbf{M}_c^g + \mathbf{M}_d^g + \mathbf{M}_b^g + \mathbf{M}_f^g + \mathbf{Q}^g(\cdot);$$

$$\mathbf{F}^h = \mathbf{M}_c^h + \mathbf{M}_d^h + \mathbf{M}_f^h + \mathbf{Q}^h(\cdot); \mathbf{M}_c^h = \mathbf{M}^h + \mathbf{M}^{ha};$$

$$\mathbf{A}^o = \begin{bmatrix} \mathbf{J}^o & \mathbf{D}_q & \mathbf{D}_g & \mathbf{D}_h \\ \mathbf{D}_q^t & \mathbf{A}^q & \mathbf{0} & \mathbf{0} \\ \mathbf{D}_g^t & \mathbf{0} & \mathbf{A}^g & \mathbf{0} \\ \mathbf{D}_h^t & \mathbf{0} & \mathbf{0} & \mathbf{A}^h \end{bmatrix}; \begin{cases} \mathbf{M}_c^g = \mathbf{M}^g + \mathbf{M}^{gd} + \mathbf{M}^{ga}; \\ \mathbf{G} = \mathbf{G}^o + \mathbf{D}_q \dot{\mathbf{q}} + \mathbf{D}_g \dot{\beta}; \\ \mathbf{G}^o = \mathbf{J}^o \omega + \mathcal{H}(\beta); \\ \mathbf{A}_h = [\partial \mathcal{H}(\beta) / \partial \beta]; \end{cases}$$

$$\mathbf{H} = \{ \mathbf{H}_p \}; \Omega = \{ \Omega_p \}; \beta = \{ \beta_p \}; \omega = \{ \omega_i \};$$

$$\mathbf{q} = \{ q_j \}; \mathbf{H}_p(\beta_p) = \mathbf{H}_p \mathbf{h}_p; \mathcal{H}(\beta) = \sum \mathbf{H}_p(\beta_p),$$

where torques \mathbf{M}_d^g and \mathbf{M}_d^h of a physical damping, and also the electro-magnetic damper (EMD) torques $\mathbf{M}_{dp}^g(k_d^g, \dot{\beta}_p)$ with gain k_d^g are nonlinear continuous functions; vectors of the rolling friction torques in bearings on the gyro-rotor (GR) axes \mathbf{M}_f^h and on GD's precession axes \mathbf{M}_f^g , and also in general case the torque's vector \mathbf{M}_b^g describing the influence of limiting supports on GD's precession axes, are *nonlinear discontinuous* functions.

The components of the MGC control vectors \mathbf{M}_c^g and \mathbf{M}_c^h with regard for the possible *faults* in electric circuits of MGDs or GSDs as well as the EMDs on the GD precession axes, and also that of the electric drives on the GD's rotor axes and arresters (cages) are described by *hybrid functions*

$$\mathbf{M}_p^x = \sum_{l=1}^2 \gamma_p^{fxl}(\nu) \gamma_p^{rxl}(\mu) a_p^x i_p^{xl}, \quad (12)$$

where $x = g, gd, ga, h, ha$, coordinates $\gamma_p^{yxl}, y = f, r$ are *logic* variables $\gamma_p^{yxl} \in \{0, 1\}$; $\gamma_p^{yx1} \wedge \gamma_p^{yx2} = 0$; $\gamma_p^{yx1} \vee \gamma_p^{yx2} = 1, p = 1:4$; i_p^{xl} are the control currents and currents at the GD electro-magnetic arresters in main ($l = 1$) and in reserve ($l = 2$) circuits.

The functions $\gamma_p^{fxl}(\nu)$ are outputs of an ALA \mathcal{A}^f with memory used for representing fault occurrences and damage development depending on the automaton time $\nu \in \mathbb{N}_0$. Functions $\gamma_k^{rxl}(\mu)$ are outputs of a SLA \mathcal{A}^r , also with memory, for description of damage's or fault's block-keeping and the reconfiguration sequence depending on the automaton time $\mu \in \mathbb{N}_0$. The currents in GD's control circuits $i_p^{gl}(t)$ for $\gamma_p^{rgl} = 1$ and $i_p^{hl}(t)$ for $\gamma_p^{rhl} = 1$ are proportional to GD's digital control voltages $u_p^x(t) = \text{Zh}[\text{Sat}(\text{Qntr}(u_{pk}^x, b_u^x), B_u^x), T_u]$, where $u_{pk}^x, x = g, h$ are the outputs of NCLs on the GDs precession and GRs axes, and functions $\text{Sat}(x, a)$ and $\text{Qntr}(x, a)$ are general-usage ones, while the holder model with the period T_u is of the type: $y(t) = \text{Zh}[x_k, T_u] = x_k \quad \forall t \in [t_k, t_{k+1})$.

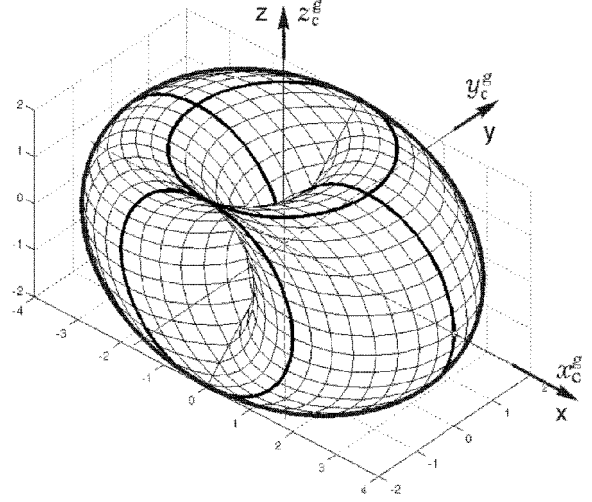


Fig. 2. Main singular sets of "2-SPEED" scheme

7. THE MGC ADJUSTMENT

Within *precession theory* of control moment gyros for $\mathbf{H}_p = h_g = \text{const}$ the MGC's AM vector is such:

$$\mathcal{H}(\beta) = h_g \mathbf{h}(\beta); \mathbf{h}(\beta) \equiv \sum \mathbf{h}_p(\beta_p) = \{x, y, z\},$$

$$x = x_{12} + x_{34}; y = y_1 + y_2; z = -(z_3 + z_4); x_p = c_{\beta_p}; y_p = s_{\beta_p}; z_p = s_{\beta_p}; x_{12} = x_1 + x_2; x_{34} = x_3 + x_4.$$

The MGC scheme have a lot of singular sets, main sets are presented in Fig. 2. The distribution law of normed AM $\mathbf{h}(\beta)$ between GD's pairs

$$f_\rho(\beta) = (\tilde{x}_1 - \tilde{x}_2) + \rho (\tilde{x}_1 \cdot \tilde{x}_2 - 1) = 0,$$

with $\tilde{x}_1 = x_{12}/q_y; \tilde{x}_2 = x_{34}/q_z; q_s = \sqrt{4 - s^2}, s = y, z$, at "right-sided differential relay-hysteresis" tuning due to $D^+ f_\rho(\beta) = \Phi_\rho(f_\rho(\beta), \mathbf{h}(\beta))$, see Somov (2001a), for the MGC's AM vector ensures its belonging to the *imaginary* singular sets only at *separate* time moments, and *bijectively* connects the vector \mathbf{M}^g with vectors β and $\tilde{\beta}$.

8. SPACECRAFT CONTROL LAW

If the error $\tilde{\omega}$ in the angular rate vector ω is defined as $\tilde{\omega} = \omega - \mathbf{C}_e \omega^p(t)$, and the MGC's required control torque vector \mathbf{M}^g is formed as $\mathbf{M}^g = \omega \times \mathbf{G}^o + \mathbf{J}^o (\mathbf{C}_e \dot{\omega}^p(t) - [\omega \times] \mathbf{C}_e \omega^p(t) + \tilde{\mathbf{m}})$, then the simplest normed nonlinear model of the SC attitude error dynamics is as follows

$$\dot{e}_0 = -\langle \mathbf{e}, \tilde{\omega} \rangle / 2; \dot{\mathbf{e}} = \mathbf{Q}_e \tilde{\omega} / 2; \dot{\tilde{\omega}} = \tilde{\mathbf{m}}, \quad (13)$$

for which the EFL synthesis results in the NCL

$$\tilde{\mathbf{m}}(\mathcal{E}, \tilde{\omega}) = -\mathbf{A}_0 \cdot \mathbf{e} \cdot \text{Sgn}(e_0) - \mathbf{A}_1 \cdot \tilde{\omega}, \quad (14)$$

where $\mathbf{A}_0 = ((2a_0^* - \tilde{\omega}^2 / 2) / e_0) \mathbf{I}_3; \mathbf{A}_1 = a_1^* \mathbf{I}_3 - \mathbf{R}_{e\omega}, \text{Sgn}(e_0) = (1, \text{if } e_0 \geq 0) \vee (-1, \text{if } e_0 < 0)$, and matrix $\mathbf{R}_{e\omega} = \langle \mathbf{e}, \tilde{\omega} \rangle \mathbf{Q}_e [\mathbf{e} \times] / (2e_0)$. Moreover, the closed-loop nonlinear system (13), (14) is transformed to a simplest *linear* system for the SC *spatial* attitude control. *Simultaneously* constructing the VLF's components using the *Vandermonde* matrix is analytically executed, the VLF's structure $\mathbf{v}(\mathbf{x}) = \mathbf{v}(\mathcal{E}, \tilde{\omega})$ has become known, and for obtaining the inverse transformation $\mathbf{x} = \Psi(\mathbf{z})$ in the explicit form it is sufficient to use the analytic inverse of matrix \mathbf{Q}_e (Somov *et al.*, 1999b).

As result of the SC ACS (11) synthesis with the MGC scheme in Fig. 1a for the command order vector \mathcal{E}_{ck}^o and the measured vector \mathcal{E}_k^{of} , the attitude error vector $\mathcal{E}_k^\varepsilon = \{\varepsilon_{0k}, \varepsilon_k\}$, matrix $\mathbf{A}_{hk}^f = \mathbf{A}_h(\mathbf{H}_k^f, \beta_{pk}^f)$ and control vector \mathbf{u}_k^g are computed by relations

$$\begin{aligned} \varepsilon_{0k} &= e_{0k}^{of} e_{c0k}^o + \langle \mathbf{e}_k^{of}, \mathbf{e}_{ck}^o \rangle; \quad \beta_{pk}^{of} = \beta_{pk}^f + (-1)^{p-1} \sigma; \\ \boldsymbol{\varepsilon}_k &= \mathbf{Q}(\mathcal{E}_{ck}^o) \mathbf{e}_k^{of} - e_{0k}^{of} \mathbf{e}_{ck}^o; \quad \boldsymbol{\varepsilon}_k = -2\boldsymbol{\varepsilon}_k; \\ S_{hpk}^{of} &= \mathbf{H}_{pk}^f \sin \beta_{pk}^{of}; \quad C_{hpk}^{of} = \mathbf{H}_{pk}^f \cos \beta_{pk}^{of}; \\ \mathbf{A}_{hk}^f &= \begin{bmatrix} -C_{h3k}^{of} & -C_{h4k}^{of} & 0 & 0 \\ 0 & 0 & C_{h1k}^{of} & C_{h2k}^{of} \\ -S_{h1k}^{of} & -S_{h2}^{of} & S_{h3k}^{of} & S_{h4k}^{of} \end{bmatrix}; \\ \mathbf{w}_k^g &= \mathbf{w}_{k-1}^g + k_g(\boldsymbol{\varepsilon}_k + a_g \boldsymbol{\varepsilon}_{k-1}); \quad \boldsymbol{\omega}_{ok}^{of} = \mathbf{C}(\mathcal{E}_k^{of}) \boldsymbol{\omega}_o; \\ \mathbf{u}_k^g &= -(\mathbf{A}_{hk}^f) (\mathbf{w}_k^g + \boldsymbol{\omega}_{ok}^{of}), \quad k \in \mathbb{N}_0, \end{aligned} \quad (15)$$

where k_g, a_g are the constant scalar parameters.

Digital NCL (15) is *universal* for given MGC type, it provides the precise 3-axis attitude control of the SC body at the MGC reconfiguration, it is robust with respect to an accumulated AM of the "SC+MGC"-system and to variations of the SC inertial and rigidity characteristics. Operability of this NCL for large torque disturbances arising at the SC orbit correction as well as for spatial control of the SC body, flexible SAPs and RTAs, has been investigated.

9. SOFTWARE AND SIMULATION

The developed methods for dynamic research and design of robust fault-tolerant ACS had been implemented in the software system DYNAMICS and in well-known MatlabTM. Some simulation results for the SC gyromoment ACS (11) see in Somov (2001a).

10. CONCLUSIONS

Contemporary nonlinear methods for design of robust fault-tolerant control systems applied at Russian information spacecraft (Kozlov *et al.*, 1999; Batrenev *et al.*, 1999), were presented. Designing such spacecraft ACSs have been conducted, including international projects: *Ikar* (Somov *et al.*, 2001a) and *Sesat* (Somov *et al.*, 2001b). Flight exploitation these ACSs have proved high efficiency of principles and methods employed for providing a robust fault-tolerant operation.

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