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FAULT DIAGNOSIS USING SLIDING MODE OBSERVER FOR NONLINEAR SYSTEMS

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Abstract: In this paper, the fault diagnosis problem for a class of nonlinear multiple-input multiple-output (MIMO) systems with uncertainty is investigated. Under some geometric conditions, the system is transformed into two different subsystems. One is in the generalized observer canonical form and is not affected by actuator faults, so a nonlinear sliding mode observer for this subsystem is constructed. The other whose states can be measured is affected by the faults. The observation scheme is then used for actuator fault diagnosis with good accuracy. Extension to sensor fault diagnosis is also made. Finally, a numerical example is used to illustrate the efficiency of the proposed method.

Keywords: Fault diagnosis, nonlinear systems, sliding mode observer, uncertainty.

1. INTRODUCTION

There is an increasing demand for dynamic systems to become more safe and reliable. Fault diagnosis including fault detection and isolation (FDI) can help to improve plant efficiency, maintainability and reliability by early detection and accommodation of system failures. The design and analysis of FDI algorithms and their applications have received considerable attention during the past two decades. Fruitful results can be found in several survey papers (Isermann, 1984; Frank, 1990) and books (Gertler, 1998; Chen and Patton, 1999).

However, most research work on FDI has been concentrated on linear systems, and only limited results for nonlinear systems have been reported, for example, see an early work (Seliger and Frank, 1991) using unknown input observer approach, (Hammouri *et al.*, 1999; Polycarpou and Trunov, 2000; Jiang *et al.*, 2001) based on nonlinear observers, (Staroswiecki and Comtet-Varga, 2001; Zhang *et al.*, 1998) based on parity space approaches and (Cocquempot and Christophe, 2000) on the relationship between the two methods.

Sliding mode observers are a particular class of nonlinear observer, adapting ideas from the field of sliding mode control (Utkin, 1992). The main idea is to generate unbiased estimates of the system states despite the modelling errors and disturbance. This can be achieved by specifying extra conditions involving the sliding surfaces on the observer. Sliding mode observers have been used for fault detection: Sreedhar *et al* (Screedhar and Fernandez, 1993) considered the use of sliding mode observer under the assumption that full state measurements were available. Hermans and Zarrop (Hermans and Zarrop, 1996) designed an observer in such a way that in the presence of a fault the sliding motion was destroyed. In (Edwards *et al.*, 2000), Edwards *et al* dealt with observer design to maintain a sliding motion even in the presence of faults which are detected by analysing the so-called equivalent output injection. Recently, some research results have been obtained on fault diagnosis via a variable structure adaptive observer (Chen *et al.*, 2000) and FPRG based on sliding mode observer (Djemai *et al.*, 2000).

In this paper, we extend the results of fault diagnosis in (Jiang *et al.*, 1999) to a class of nonlinear systems with uncertainty via sliding mode observer. At first, under some geometric conditions, the system is transformed into two different subsystems. One is in the generalized observer canonical form which is not affected by actuator faults. As a generalization of the observer design approach in (Walcott and Zak, 1988), a sliding mode observer design is proposed for the first subsystem. The other whose states can be measured is affected by the actuator faults. By using the estimations of states, we can approximate the actuator faults with certain accuracy from the second subsystem. The proposed method can be extended to sensor fault diagnosis.

This paper is organized as follows. Section 2 gives geometric conditions under which the original nonlinear system with uncertainty and actuator fault can be transformed into a desired form. A sliding mode observer is designed for the transformed system in section 3. In section 4, actuator fault estimation is investigated. Extension to sensor fault diagnosis is discussed in section 5. A numerical example is included in section 6.

2. MIMO SYSTEM DESCRIPTION AND SOME PRELIMINARIES

Consider the following affine nonlinear system with uncertainty

$$\dot{x} = f(x) + \triangle f(x,\xi) + \sum_{i=1}^{m} q_i(x)u_i + \sum_{j=1}^{d} e_j(x) f_{aj} \quad (1)$$

$$y = [h_1(x), \cdots, h_l(x), h_{l+1}(x), \cdots, h_r(x)]^\tau \quad (2)$$

where the state is $x \in R^n$, the input is $u = [u_1, \cdots\ u_m]^\tau \in R^m$, and the output is $y \in R^r$. The actuator fault fault is represented by $f_a = [f_{a1}, \cdots, f_{ad}]^\tau \in R^d$ with $d \leq r < n$, and the disturbance or modelling uncertainty is denoted by $\triangle f(x,\xi)$, where ξ is a unknown parameter vector which is bounded. Furthermore, $f(\cdot)$, $q_i(\cdot)$ $(i = 1, \cdots, m)$ and $e_j(\cdot)(j = 1, \cdots, d)$ are smooth vector fields, and $h(\cdot)$ is a smooth vector function.

As a generalization of observer canonical form (OCF) in (Marino and Tomei, 1995), the following definition is given:

Definition 1: The generalized observer canonical form (GOCF) of system (1) and (2) is described as

$$\dot{z} = \begin{bmatrix} 0_{l\times l} & 0_{l\times(n-l)} \\ 0_{(n-l)\times l} & A \end{bmatrix} z + \gamma(y,u) + \begin{bmatrix} \triangle F_1(z,\xi) \\ \triangle F_2(z,\xi) \end{bmatrix} + \sum_{j=1}^{d} \begin{bmatrix} \psi_j(z) \\ 0_{(n-l)\times 1} \end{bmatrix} f_{aj} \quad (3)$$

$$y = \begin{bmatrix} I_{l\times l} & 0_{l\times(n-l)} \\ 0_{(r-l)\times l} & C \end{bmatrix} z \quad (4)$$

where

$$A = diag[A_1, \cdots, A_{r-l}]$$
$$C = diag[C_1, \cdots, C_{r-l}]$$

with (A_i, C_i) $(i = 1, \cdots, r-l)$ being in the observer canonical form.

Assumption 1: $\rho_1 = \cdots = \rho_l = 1$, $\sum_{i=1}^{r} \rho_i = n$, and the system is locally observable, i.e.

$$rank\{dh_i(x), \cdots, d(L_f^{\rho_i - 1} h_i) :\ 1 \leq i \leq r\} = n$$

where he relative degree ρ_i of h_i $(i = 1, \cdots, r)$ for the system described by (1) and (2) is defined in (Isidori, 1995) and denoted by:

$$\rho_i = min\{s \mid L_{q_j} L_f^{s-1} h_i(x) \neq 0, \quad j = 1, \cdots, m\}$$

Lemma 1: Under Assumption 1, there exists a global diffeomorphism $z = N(x)$ with $N(0) = 0$ and $z \in R^n$, transforming (1) and (2) into GOCF if and only if
(i) there exist r vector fields $g_1, \cdots, g_r$ satisfying

$$L_{g_s} L_f^{k-1} h_t = \delta_{s,t} \delta_{k,\rho_t},$$

for $1 \leq s \leq r,\ 1 \leq k \leq \rho_t,\ 1 \leq t \leq r$.
such that

$$[ad_f^i g_s,\ ad_f^j g_t] = 0,$$

for $1 \leq s,t \leq r,\ 0 \leq i \leq \rho_s - l,\ 0 \leq j \leq \rho_t - 1$.
where $\delta_{s,t} = 0$ for $s \neq t$, and $\delta_{s,s} = 1$.
(ii) the vector fields

$$ad_f^i g_s, \quad 1 \leq s \leq r,\ 0 \leq i \leq \rho_i - 1.$$

are complete.
(iii) $[q_i,\ ad_f^j g_s] = 0$, for $1 \leq i \leq m$, $0 \leq j \leq \rho_s - 1$, and $1 \leq s \leq r$.
(iv) $e_j = \sum_{i=1}^{l} \psi_{ji}(z) g_i$ for $j = 1, \cdots, d$.

Proof: According to (Marino and Tomei, 1995), conditions (i), (ii) and (iii) are necessary and sufficient for (1) and (2) (with $\triangle f(x) = 0$ and $f_a = 0$) to be transformable via a global diffeomorphism $z = N(x)$ into

$$\dot{z} = \begin{bmatrix} 0_{l\times l} & 0_{l\times(n-l)} \\ 0_{(n-l)\times l} & A \end{bmatrix} z + \gamma(y,u)$$

$$y = \begin{bmatrix} I_{l\times l} & 0_{(r-l)\times l} \\ 0_{(n-l)\times l} & C \end{bmatrix} z$$

where the change of coordinates is defined by

$$ad_f^i g_s = (-1)^i \frac{\partial}{\partial z_s^{i+1}}, \quad 0 \le i \le \rho_s - 1;\ 1 \le s \le r.$$

Condition (iv) is necessary and sufficient to transform e_j into $[\psi_j\ 0_{(n-l)\times 1}]^\tau$ whose last $n-l$ elements are zero in z-coordinates, while $\triangle F_1(z,\xi)$ and $\triangle F_2(z,\xi)$ in (3) can be described by

$$\triangle F_1(z,\xi) = \begin{bmatrix} L_{\triangle f} h_1 \\ \vdots \\ L_{\triangle f} h_l \end{bmatrix} \tag{5}$$

$$\triangle F_2(z,\xi) = \begin{bmatrix} L_{\triangle f} h_{(l+1)} \\ \vdots \\ L_{\triangle f}^{\rho_{(l+1)}} h_{(l+1)} \\ \vdots \\ L_{\triangle f} h_r \\ \vdots \\ L_{\triangle f}^{\rho_r} h_r \end{bmatrix} \tag{6}$$

This completes the proof. □

Denote

$$\bar{z}_1 \triangleq [z_1, \cdots, z_l]^\tau, \quad \bar{z}_2 \triangleq [z_{l+1}, \cdots, z_n]^\tau;$$

$$\bar{y}_1 \triangleq [y_1, \cdots, y_l]^\tau, \quad \bar{y}_2 \triangleq [y_{l+1}, \cdots, y_r]^\tau.$$

then the system (3) and (4) can be rewritten as

$$\dot{\bar{z}}_1 = \gamma_1(y,u) + \triangle F_1(z,\xi) + M(z) f_a \tag{7}$$
$$\bar{y}_1 = \bar{z}_1 \tag{8}$$
$$\dot{\bar{z}}_2 = A\bar{z}_2 + \gamma_2(y,u) + \triangle F_2(z,\xi) \tag{9}$$
$$\bar{y}_2 = C\bar{z}_2 \tag{10}$$

where

$$M(z) = \begin{bmatrix} L_{e_1} h_1 & \cdots & L_{e_d} h_1 \\ \vdots & \ldots & \vdots \\ L_{e_1} h_l & \cdots & L_{e_d} h_l \end{bmatrix} \tag{11}$$

Assumption 2: There exist vector function $B \in R^{(n-l)\times(r-l)}$, $\triangle \bar{F}_2(z,\xi) \in R^{(n-l)\times 1}$ and scalar functions $\alpha_i(y) (i = 1,2)$ such that

$$\| \triangle F_1(z,\xi) \| \le \alpha_1(y) \tag{12}$$
$$\triangle F_2 = B \triangle \bar{F}_2, \ \| \triangle \bar{F}_2(z,\xi) \| \le \alpha_2(y) \tag{13}$$

Further more, $C[sI - (A-KC)]^{-1}B$ is strictly positive real (SPR), where K is chosen such that $A - KC$ is stable.

Remark 1: The SPR requirement in the above assumption is equivalent to the following:
For a given positive definite matrix $Q > 0 \in R^{(n-l)\times(n-l)}$, there exists $P > 0 \in R^{(n-l)\times(n-l)}$ and $L \in R^{(r-l)\times(r-l)}$, such that

$$(A-KC)^\tau P + P(A-KC) = -Q \tag{14}$$
$$LC = B^\tau P \tag{15}$$

Remark 2: Assumption 2 is somewhat restrictive. However, it can be satisfied for a class of nonlinear systems with uncertainty, because of some degree of freedom in designing matrices K and Q (corresponding P) in Eq.(14).

3. SLIDING MODE OBSERVER DESIGN

In this section, we design a sliding mode observer for the subsystem which is not affected by any faults.

Theorem 1: Under Assumption 2 , there exists an exponentially convergent sliding mode observer for the subsystem described by (9) and (10), and the sliding mode observer is given by

$$\frac{d\hat{\bar{z}}_2}{dt} = A\hat{\bar{z}}_2 + \gamma_2(y,u) + K(\bar{y}_2 - \hat{\bar{y}}_2) + \alpha_2(y) B sign(L\bar{y}_2 - L\hat{\bar{y}}_2) \tag{16}$$
$$\hat{\bar{y}}_2 = C\hat{\bar{z}}_2 \tag{17}$$

where $sign(\cdot)$ denotes the usual sign vector function and L is given by (15).

Proof: Let $\tilde{\bar{z}}_2(t) \triangleq \bar{z}_2(t) - \hat{\bar{z}}_2$. From (9), (10),(16) and (17), the dynamic of observation error is given by

$$\frac{d\tilde{\bar{z}}_2}{dt} = (A-KC)\tilde{\bar{z}}_2(t) + B[\triangle \bar{F}_2 - \alpha_2(y) sign(L\bar{y}_2 - L\hat{\bar{y}}_2)] \tag{18}$$

Consider the following Lyapunov function

$$V(t) = (\tilde{\bar{z}}_2)^\tau(t) P \tilde{\bar{z}}_2(t) \tag{19}$$

Its time derivative with respect to (18) is

$$\dot{V}(t) = (\tilde{\bar{z}}_2)^\tau[(A-KC)^\tau P + P(A-KC)]\tilde{\bar{z}}_2 + 2(\tilde{\bar{z}}_2)^\tau PB \times [\triangle \bar{F}_2 - \alpha_2(y) sign(L\bar{y}_2 - L\hat{\bar{y}}_2)] \tag{20}$$

Substituting (13), (14) and (15) into (20) yields

$$\dot{V}(t) \le -[\lambda_{min}(Q)] \| \tilde{\bar{z}}_2 \|^2 + 2 \| LC\tilde{\bar{z}}_2 \| \| \triangle \bar{F}_2(z,\xi) \| - 2\alpha_2(y) \| LC\tilde{\bar{z}}_2 \| \le -[\lambda_{min}(Q)] \| \tilde{\bar{z}}_2 \|^2 \tag{21}$$

Noting that

$$\lambda_{min}(P) \| \tilde{\bar{z}}_2 \|^2 \le V \le \lambda_{max}(P) \| \tilde{\bar{z}}_2 \|^2 \tag{22}$$

one can further obtain

$$\dot{V}(t) \leq -\frac{\lambda_{min}(Q)}{\lambda_{max}(P)} V(t) \quad (23)$$

Therefore, $\tilde{\bar{z}}_2(t)$ converges to zero exponentially. This completes the proof.

Remark 3: Theorem 1 is a generalization of sliding mode observer design in (Walcott and Zak, 1988) to a class of nonlinear systems with uncertainties.

4. ACTUATOR FAULT DIAGNOSIS

For our result, we need to make the following assumption:

Assumption 3: $rank\ M(z) = d$, where $M(z)$ is defined in (11).

From the subsystem described by (7) and (8), one can obtain

$$\begin{aligned}\dot{\bar{y}}_1(t) = &\gamma_1(y(t), u(t)) + \triangle F_1(z, \xi)\\ &+M(z(t))f_a(t)\end{aligned} \quad (24)$$

Using the estimation $\hat{\bar{z}}_2$ for $\bar{z}_2$, we can estimate the actuator/component fault as

$$\begin{aligned}\hat{f}_a(t) = &(\hat{M}^\tau \hat{M})^{-1}\hat{M}^\tau(t)\\ &\times[\dot{\bar{y}}_1(t) - \gamma_1(y(t), u(t))]\end{aligned} \quad (25)$$

where $\hat{M}(t) \stackrel{\triangle}{=} M(\bar{y}_1(t), \hat{\bar{z}}_2(t))$.

Let $N(t) \stackrel{\triangle}{=} (M^\tau M)^{-1}M^\tau(\bar{y}_1(t), \bar{z}_2(t))$, $\hat{N}(t) \stackrel{\triangle}{=} (M^\tau M)^{-1}M^\tau(\bar{y}_1(t), \hat{\bar{z}}_2(t)$, one has

$$\begin{aligned}f_a(t) - \hat{f}_a(t) = &(N(t) - \hat{N}(t))\\ &\times[\dot{\bar{y}}_1(t) - \gamma_1(y(t), u(t)) - \triangle F_1(z, \xi)]\\ &-\hat{N}(t)\triangle F_1(z, \xi)\end{aligned} \quad (26)$$

Note that actuator faults are not involved in estimation of $\bar{z}_2$. Hence $\hat{N}(t) \to N(t)$ if $\hat{\bar{z}}_2(t) \to \bar{z}_2(t)$. If $f_a(t)$ is uniformly bounded, then for a arbitrarily given $\epsilon > 0$, there exists t_0 such that for $t > t_0$

$$\parallel f_a(t) - \hat{f}_a(t) \parallel \leq \epsilon + \parallel \hat{N}(t))\triangle F_1(z, \xi) \parallel (27)$$

On the other hand, from (12), one can get

$$\parallel \hat{N}(t)\triangle F_1(z, \xi) \parallel \leq \alpha_1 \sqrt{\lambda_{max}(\hat{N}^\tau \hat{N})} \quad (28)$$

Therefore

$$\parallel f_a(t) - \hat{f}_a(t) \parallel \leq \alpha_1 \sqrt{\lambda_{max}(\hat{N}^\tau \hat{N})} + \epsilon \ (29)$$

Remark 4: From (29), it can be seen that the estimation error of the fault is bounded, which can be used as the threshold for actuator fault detection. Furthermore, we can obtain the accurate estimation of the fault if $\alpha_1 = 0$ (it means that the original nonlinear system can be partially decoupled from the uncertainty).

Remark 5: The good feature of our method is that it not only enables the actuator fault detection, but also provides the shape (amplitudes) of actuator faults, which is very useful for fault accommodation such as application to aircraft flight control systems (Ochi and Kanai, 1991). Besides this, it is easy to implement as the design is based on the reduced-order observer.

Remark 6: Calculation of the output derivative is required to estimate the fault. Because of the presence of noise in practice, it is not easy to compute the signal derivative. Evaluating output derivative from noisy signals can be done using either observer or specific algorithms, which have been extensively investigated, for example in (Dierckx, 1993), and used for analytic redundancy based FDI of nonlinear systems in (Staroswiecki and Comtet-Varga, 2001).

Remark 7: In (Jiang *et al.*, 2001), adaptive observer-based fault diagnosis was investigated for a class of nonlinear systems with unknown constant parameters. Comparatively, the proposed approach in this paper works for nonlinear systems with time-varying uncertainties whose norm bounds are available.

5. DISCUSSION ON SENSOR FAULT DIAGNOSIS

In this section, we consider sensor fault diagnosis using similar methods as in section 3 and section 4.

Consider the following nonlinear system with uncertainty and sensor faults

$$\dot{x} = f(x) + \triangle f(x, \xi) + \sum_{i=1}^{m} q_i(x)u_i \quad (30)$$

$$y = h(x) + \sum_{j=1}^{d} e_j(x)f_{sj} \quad (31)$$

where the sensor fault vector is $f_s = [f_{s1}, \cdots, f_{sd}] \in R^d$ with $d \leq r$, other notations are the same as in section 2.

Similar to Definition 1, the robust observer canonical form (GOCF) for the system described by (30) and (31) is defined as follows:

Definition 2: GOCF of the system (30) and (31) is described as

$$\dot{z} = \begin{bmatrix} 0_{l\times l} & 0_{l\times(n-l)} \\ 0_{(n-l)\times l} & A \end{bmatrix} z + \gamma(y,u) + \begin{bmatrix} \triangle F_1(z,\xi) \\ \triangle F_2(z,\xi) \end{bmatrix} \quad (32)$$

$$y = \begin{bmatrix} I_{l\times l} & 0_{l\times(n-l)} \\ 0_{(r-l)\times l} & C \end{bmatrix} z + \sum_{j=1}^{d} \begin{bmatrix} \psi_j(z) \\ 0_{(r-l)\times 1} \end{bmatrix} f_{sj} \quad (33)$$

with (A,C) being an observable pair.

Under the conditions in Lemma 1, the nonlinear system in the sensor fault case can be transformed into GOCF described by (32) and (33), which can be rewritten as

$$\dot{\bar{z}}_1 = \gamma_1(y,u) + \triangle F_1(z,\xi) \quad (34)$$

$$\bar{y}_1 = \bar{z}_1 + D(z)f_s \quad (35)$$

$$\dot{\bar{z}}_2 = A\bar{z}_2 + \gamma_2(y,u) + \triangle F_2(z,\xi) \quad (36)$$

$$\bar{y}_2 = C\bar{z}_2 \quad (37)$$

It is assumed that $D(z)$ in Eq.(35) is of full column rank.

From (35), we estimate the sensor fault as

$$\hat{f}_s(t) = (\hat{D}^T\hat{D})^{-1}\hat{D}^T(t)[\bar{y}_1(t) - \hat{\bar{z}}_1(t)] \quad (38)$$

where $\hat{D}(t) = D(\hat{\bar{z}}_1(t), \hat{\bar{z}}_2(t))$, $\hat{\bar{z}}_1(t)$ is derived from (34) in which $\triangle F_1$ is replaced by some value chosen according to some specific problem statement (e.g. $\triangle F_1 = 0$), while $\hat{\bar{z}}_2(t)$ can be obtained from (36) and (37), using the same method as described in (14) and (15).

6. AN ILLUSTRATIVE EXAMPLE

Consider the following nonlinear system with uncertainty

$$\left.\begin{aligned} \dot{x}_1 &= -2x_1x_3 + 2x_2f_a \\ \dot{x}_2 &= x_1 + 2x_2^2x_3 + 3x_2^2cos(x_1)\xi(t) \\ \dot{x}_3 &= x_2 + u - f_a \\ &\qquad + (x_2 - 3x_2x_3)cos(x_1)\xi(t) \\ y_1 &= x_2 \\ y_2 &= x_3 \end{aligned}\right\} (39)$$

where $\xi(t) \in [0,\ 1]$ represents uncertain parameter, f_a stands for the actuator fault in the system.

It is easy to check that all the assumptions in Theorem 1 hold. In fact, the relative degree are $\rho_1 = 2,\ \rho_2 = 1$.

The transformation $z = N(x)$ is described as

$$\left.\begin{aligned} z_1 &= x_2 \\ z_2 &= x_1 + 2x_2x_3 \\ z_3 &= x_3 \end{aligned}\right\} \quad (40)$$

Under this transformation, the nonlinear system (39) is changed into the following GOCF

$$\left.\begin{aligned} \dot{z}_1 &= z_2 - 2y_1y_2 + 2y_1^2y_2 \\ &\qquad + 3z_1^2cos(z_2 - 2z_1z_3)\xi(t) \\ \dot{z}_2 &= 2y_1^2 + 4y_1^2y_2^2 + 2y_1u \\ &\qquad + 2z_1^2cos(z_2 - 2z_1z_3)\xi(t) \\ y_1 &= z_1 \end{aligned}\right\} (41)$$

$$\left.\begin{aligned} \dot{z}_3 &= y_1 + u - f_a \\ &\quad + (z_1 - 3z_1z_3)cos(z_2 - 2z_1z_3)\xi(t) \\ y_2 &= z_3 \end{aligned}\right\} (42)$$

Note that the subsystem (41) can be written as

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} -2y_1y_2 + 2y_1^2y_2 \\ 2y_1^2 + 4y_1^2y_2^2 + 2y_1u \end{bmatrix} + \begin{bmatrix} 3z_1^2 \\ 2z_1^2 \end{bmatrix} cos(z_2 - 2z_1z_3)\xi(t) \quad (43)$$

$$y_1 = [1\ 0] \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \quad (44)$$

Furthermore, one may choose

$$K = [-4\ \ -4]^\tau, \quad Q = \begin{bmatrix} 8 & 1 \\ 1 & 4 \end{bmatrix}$$

Then

$$\bar{A} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + [-4 - 4]^\tau[1\ 0] = \begin{bmatrix} -4 & 1 \\ -4 & 0 \end{bmatrix}$$

The positive definite matrix P can be solved from equations (14) and (15):

$$P = \begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix}, \qquad L = 5.$$

Thus, by simple calculation one can obtain

$$\alpha_1 =\mid y_1 - 3y_1y_2 \mid,\ B = [3\ \ 2]^\tau,$$
$$\triangle\bar{F}_2 = z_1^2cos(z_2 - 2z_1z_3)\xi(t),\ \alpha_2 = y_1^2$$

According to Theorem 1, there exists a sliding mode observer given by (16) and (17). In the simulation, $\xi(t) = rand$, the sampling period is 0.01s, the actuator fault considered is created as follows

$$f_a(t) = \begin{cases} 0 & \text{for } 0 \le t \le 2 \\ sin(\pi t) & \text{for } 2 < t \le 8 \end{cases} \quad (45)$$

Figure 1-2 show the response of the observer and estimation of the actuator fault as described by (45) when there is sensor noise corrupting the system. It can be seen that good estimation of the actuator fault can be achieved even in the presence of disturbance and noise. .

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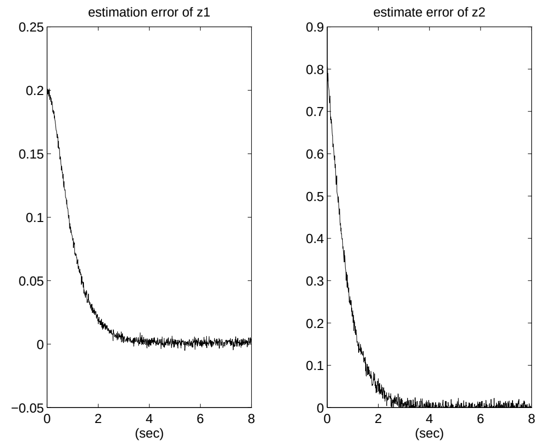

Fig. 1. State Estimation under Actuator Fault Occurrence

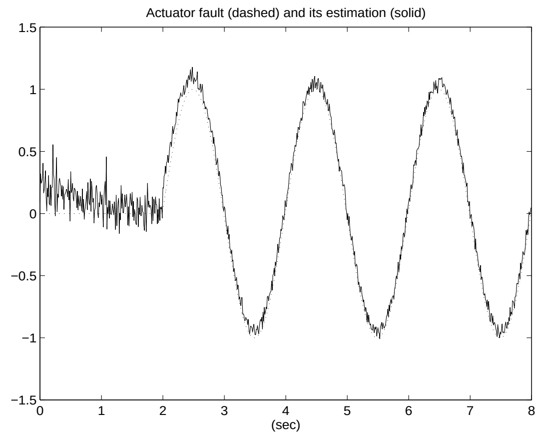

Fig. 2. Actuator Fault diagnosis

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