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SEMI-PHYSICAL MODELLING OF NONLINEAR PROCESSES BY MEANS OF LOCAL MODEL APPROACHES

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Abstract: For many practical applications, a combination of theoretical and experimental modelling appears feasible. Qualitative knowledge about the most significant effects are often known or easily accessible. This contribution suggests a semi-physical modelling approach based on the special architecture of local models. Due to their inherent transparency, these models are very well suited for the incorporation of mainly qualitative process knowledge. The integration of prior knowledge is realised by an adaptation of the model structure to that one of the process. As a result, the final process-specific models are characterised by high generalisation performance also in situations with only few measurement data. *Copyright © 2002 IFAC*

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1. INTRODUCTION

The solution of many engineering problems requires accurate mathematical models of all kinds of processes. In control engineering, these static or dynamic models are the basis for e.g. process simulation, controller design, model-based control, process optimisation and model-based fault-diagnosis.

There exist mainly two ways to develop mathematical process models. *Theoretical modelling* on one side is based on the formulation of the underlying physical, biological or chemical laws in equations. This approach is usually called *white-box* modelling, since the model development implies a deep understanding of the physical background. As a result, the final models are transparent and interpretable, that means their parameters have a specific physical meaning. Difficulties arise if the considered processes are poorly understood or too complex. Then, the collection of a suitable degree of process knowledge demands high effort and the generated models are often not as accurate as desired.

On the other hand, *experimental modelling* approaches rely on the assumption of general *black-box* model structures with adaptable parameters. These parameters are estimated by an identification method based on measured data. The aim of parameter estimation is the adaptation of the model input/output behaviour to that one of the process. Generally, the structures of the model and that of the real process are not related. Consequently, internal model parameters have no direct physical interpretation. Since no process knowledge is required, experimental modelling approaches represent universally applicable methods. However, the achievable model quality is strongly restricted by quality and quantity of available measurement data. Classical polynomials as well as neural networks with a huge number of different existing structures can be applied for experimental modelling.

In real engineering applications at least to a particular degree process information is almost always available. Although insufficient for a fully physically parameterised model, this prior knowledge should be exploited during model development. There are numerous methods from pure physical to experimental modelling (Fischer et al., 1997). These so-called *grey-box* approaches attempt to combine the advantages of both white- and black-box models.

In this paper, a *semi-physical modelling* approach is pursued where process knowledge and experience of engineers are utilised for a knowledge-based prestructuring of originally very flexible model structures. Whereas in conventional neural networks the introduction of such knowledge is usually difficult or impossible, in this paper a special class of *local model architectures* is considered. These models are featured by their transparency and interpretability.

The paper is organised as follows: Section 2 describes the basics of local model approaches. The introduction of prior knowledge is pointed out in Section 3. In Sections 4 and 5, guidelines for semi-physical modelling are presented and applied to an example. Finally, conclusions are given in Section 6.

2. LOCAL MODEL APPROACHES

Local model architectures represent a promising alternative to many other nonlinear model structures with a pure black-box character like many neural networks. They have appeared in several scientific branches. Well known architectures, such as neuro-fuzzy systems and fuzzy models, belong to this class. Local models are based on the decomposition of the considered input space into different operating regimes. Within each operating regime a simple submodel is valid. The global model output $\hat{y}$ is given by the combination of all locally active submodels

$$\hat{y} = \sum_{i=1}^{M} \Phi_i(\mathbf{u}) g_i(\mathbf{u}) \,. \qquad (1)$$

The validity of each local submodel g_i is defined by its corresponding weighting, activation or membership function Φ_i. These functions describe the partitioning of the input space and determine the type of transition between neighbouring submodels.

Different local model architectures can be distinguished with respect to the following three properties:

- *Partitioning principle*: The membership functions introduce one of the following decomposition strategy: grid structure, recursive partitioning or partitioning into operating regimes of arbitrary form. Figure 1 shows examples of the mentioned partitioning principles. The determination of a problem-specific decomposition as well as the number of submodels is called structure identification.

- *Local model structure*: Local submodels can possess any structure. However, linear models are mostly applied. The optimisation of the corresponding local submodel parameters is called parameter identification. Figure 2 illustrates three differently complex local submodels.

- *Transition between submodels*: In principle, one can distinguish two forms of transition. Firstly, hard transitions perform a sudden switching between the local models. As in most cases, sudden changes are not natural, soft partitions provide smooth transitions between the submodels (compare Fig.3).

Murray-Smith and Johanson (1997) give a comprehensive overview of local model architectures. Most of them utilise linear submodels and an axis-orthogonal partitioning either by a grid structure or by a recursive decomposition. Examples are the ANFIS fuzzy model (Jang, 1993) and the LOLIMOT neuro-fuzzy model (Nelles, 2001). Local models with arbitrary operating regimes are usually based on clustering or product-space clustering algorithms (Babuska and Verbruggen, 1995).

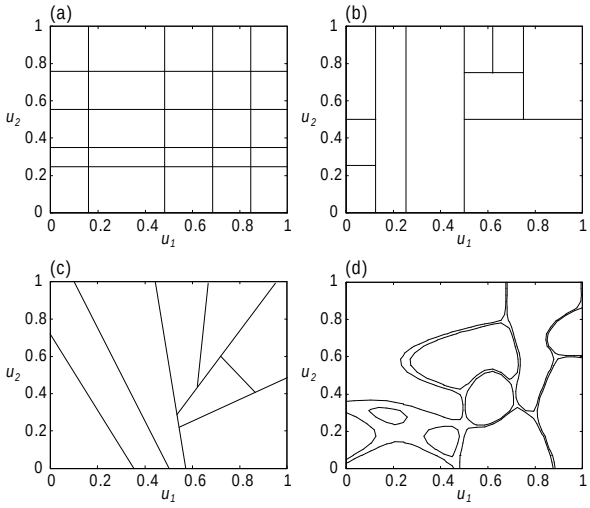

Figure 1: Partitioning strategies of local model approaches: (a) grid structure, (b) recursive partitioning by axis-orthogonal cuts, (c) recursive partitioning by axis-oblique cuts, (d) partitioning in operating regimes of arbitrary form

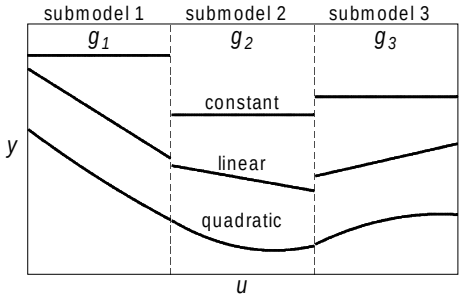

Figure 2: Different submodel structures

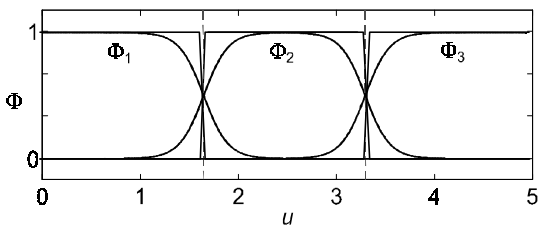

Figure 3: Membership functions with hard and soft transition between submodels

3. INTEGRATION OF PRIOR KNOWLEDGE

Local model architectures represent general and very flexible model structures. On one hand, this flexibility is an immense advantage since a wide class of nonlinear systems can be identified. On the other hand, they may need many variable parameters. A problem in practical applications often arises from the available amount of data. According to the "curse of dimensionality", high-dimensional input spaces cannot be filled equally dense with data. Because of these reasons, certain nonlinear effects may not be captured automatically by tuning available model parameters.

Due to their transparent structure, local models offer the possibility of adjusting the model structure to the process structure in terms of physical law based relationships. Such an incorporation of physical insight improves the training and the generalisation behaviour considerably and reduces the required model complexity in many cases.

In the following, two ways of integrating prior knowledge will be investigated.

3.1 *Distinction of Input Spaces for Local Submodels and Membership Functions*

So far, according to (1) identical input spaces for the local submodels $g_i(\mathbf{u})$ and the membership functions $\Phi_i(\mathbf{u})$ have been assumed. However, local models allow the realisation of distinct input spaces (Figure 4) (Murray-Smith and Johanson, 1997; Nelles, 2000) with

$$\hat{y} = \sum_{i=1}^{M} \Phi_i(\mathbf{z}) g_i(\mathbf{x}) \ . \tag{2}$$

The input vector $\mathbf{z}$ of the weighting functions comprises merely those inputs of the vector $\mathbf{u}$ having significant nonlinear effects which cannot be explained by the local submodels. Only those directions require a subdivision into different parts. The decisive advantage of this proceeding is the considerable reduction of the number of inputs in $\mathbf{z}$. Thus, the difficult task of structure identification can be simplified.

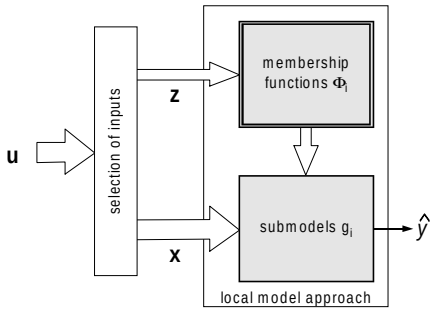

Figure 4: Structure of local model approaches with distinct inputs spaces for local submodels and membership functions

The use of separate input spaces for the local models (vector $\mathbf{x}$) and the membership functions (vector $\mathbf{z}$) becomes more precisely by considering another representation of the structure in (2). As normally local model approaches are assumed being linear with reference to their parameters according to

$$g_i(\mathbf{x}) = w_{i0} + w_{i1} x_1 + \ldots + w_{in_x} x_{n_x}, \tag{3}$$

(2) can be rearranged to

$$\hat{y} = w_0(\mathbf{z}) + w_1(\mathbf{z}) \cdot x_1 + \ldots + w_p(\mathbf{z}) \cdot x_{n_x} \quad \text{with}$$
$$w_j(\mathbf{z}) = \sum_{i=1}^{M} w_{ij} \cdot \Phi_i(\mathbf{z}) \ . \tag{4}$$

Thus, the specified local model approaches can be interpreted as linear-in-the-parameter relationships with operating point dependent parameters $w_j(\mathbf{z})$, whereupon these parameters depend on the input values in vector $\mathbf{z}$. Consequently, the process coefficients $w_j(\mathbf{z})$ still have a physical meaning.

3.2 *Structure of Submodels*

The choice of appropriate submodel structures always requires a compromise between submodel complexity and the number of submodels. The most often applied linear submodels have the advantage of being a direct extension of the well known linear models. However, under certain conditions more complex submodels may be reasonable. If the main nonlinear influence of input variables can be described qualitatively by a nonlinear transformation of the input variables (e.g. $f_i(\boldsymbol{x}) = x_1^2, x_1 x_2$), then the incorporation of that knowledge into the submodels leads to a considerable reduction of the required number of submodels. Generally, this approach can be realised by a preprocessing of the input variables $\mathbf{x}$ to the nonlinearly transformed variables

$$\mathbf{x}^* = F(\mathbf{x}) = \left[f_1(\mathbf{x}) \ f_2(\mathbf{x}) \ldots f_p(\mathbf{x}) \right]^T . \tag{5}$$

Besides those heuristically determined model structures, local model approaches also enable the incorporation of fully physically determined models. Furthermore, local models allow the employment of inhomogeneous models. Consequently, different local submodel structures are valid within the different operating regimes.

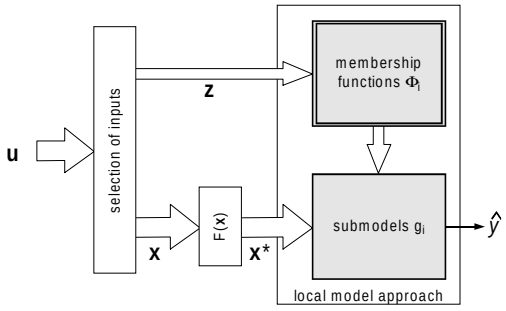

Figure 5: Pre-processing of input variables $\mathbf{x}$ for incorporation of prior knowledge into the submodel structure

4. GUIDELINES FOR SEMI-PHYSICAL MODELLING

The data-based modelling with local model architectures is an attractive alternative to theoretical approaches. Since in accordance with (4) these approaches can be interpreted as linear-in-the-parameter models with operating point dependent parameters, the application of experimental modelling methods and the physical interpretability of the results need not exclude each other. Thus, the identification of nonlinear processes can be performed systematically by regarding the following guidelines:

(i) *Physical analysis of the process*: Consideration of the physical relationships, derivation of the system order, discretisation of the time continuous differential equations. Thus, an initial modelling approach with the input vector **x** is obtained. Furthermore, non-modelled physical phenomena are considered, which contribute to the operating point dependency of the physical parameters. With this knowledge, a preselection of possible signals in the input vector **z** of the membership functions $\Phi_i(\mathbf{z})$ can be accomplished. Then, vector **z** is a subset of **u**.

(ii) *Performance of initial experiments on the process*: For the selection of the sampling rate and the excitation signal, process knowledge has to be gathered. This can be done by recording step responses at different operating points. Based on these measurements, the time constants can be roughly estimated. Furthermore, the dead times of the output with reference to the input signals are determined. In some cases, comparable expert knowledge is available by considering similar systems.

(iii) *Determination of the sampling time*: The sampling time T_s has to be selected small enough that the modelled smallest time constants are embodied adequately in the sampled signal. On the other hand, for implementation on cost-efficient micro computers, T_s should be chosen as large as possible. Unfortunately, no universally valid guidelines for the selection of the sampling period are known. Based on the knowledge of the smallest interesting time constant T_{min}, the following rule of thumb can be used

$$\frac{T_s}{T_{min}} = \frac{1}{5} \cdots \frac{3}{5} , \tag{6}$$

which is the generalised form of a guideline for linear systems (Isermann, 1992). From now on, in order to suppress higher order dynamics, the process signals should be filtered by analogue lowpass filters with an appropriate characteristic.

(iv) *Selection of the excitation signal*: Usually, input signals are chosen in a manner such that wide ranges of the input space are excited. As the dynamic model depends on past values of the input and output signals, not all of these variables can be excited independently. Hence, the design of adequate excitation signals is of great importance. In case of linear systems so-called pseudo random binary signals (PRBS) realised by feed back shift registers yield good results. However, in case of nonlinear systems, the PRBS is not suitable, as only small input domains are excited. An extension of these signals are so-called amplitude modulated pseudo random binary signals (APRBS), whereupon both, the impulse width and the amplitude are varied independently (Nelles and Isermann, 1995). In order to excite the process uniformly in different operating points, the amplitudes are equally distributed over the considered input regimes. For the selection of the minimum impulse length (cycle time) λ, the following rule of thumb can be specified

$$\lambda \leq \frac{T_{min}}{5} , \tag{7}$$

regarding the minimum process time constant T_{min}. The clock time of the shift register should be selected as large as possible since the resolution of the data grid is the finer the larger the length is chosen.

On the other hand the output signal is also desired to be equally distributed in order to achieve better identification results. If prior knowledge concerning input domains is available, in which the process characteristics (process gain and time constants) are nearly constant, an APRBS for each of those domains can be designed. The overall excitation signal arises by appending the individual signals (Fischer et al., 1997).

(v) *Measurement of datasets*: For the sake of model identification, different datasets can be measured. The dataset which arises from the excitation signal of step (iv) is utilised for training, whereas the other data are used for validation purposes.

(vi) *Model identification*: An appropriate local model architecture is identified and the validation quality is tested. If the result does not satisfy the requirements, the physical correctness of the approach should be proved and adapted. The considered dead times of output signals are also a potential source of error. In addition, further input variables can be added to the vectors **x** and **z**, respectively.

(vii) *Model reduction*: If the trained model yields satisfying results, it can be further investigated if simplifications are possible without affecting the approximation quality. Thus, some input signals x_v of the local models are negligible. On the other hand, the operating point dependency of the model parameters on different input signals z_v can be so small that no partition is performed along these directions and these variables thus can be neglected. For the sake of semi-physical modelling, special characteristics

in terms of nonlinear transformations $f_v(x_v)$ can be directly taken into account (Section 3.2). Such approaches often lead to considerable simplifications and therefore the number of local models can be reduced substantially.

The proposed guidelines to model identification are now utilised for modelling of a real world process.

5. PRACTICAL EXAMPLE

5.1 *Process Description*

The process under investigation is a centrifugal pump system as depicted in Figure 6. Measured signals are the pressures $p_I(t)$, $p_O(t)$ at the inlet and outlet, the flow rate $Q(t)$ and the angular speed $\omega(t)$.

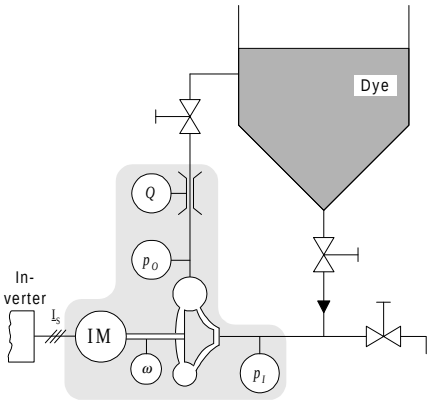

Figure 6: Pump system with measuring points

The pump is driven by an inverter-fed, speed variable induction motor (IM) which is controlled with a field oriented closed loop control. Here, the motor torque M_{el} can be easily calculated by means of

$$M_{el} = k_T \cdot \Psi_R I_{Sq} \ , \qquad (8)$$

with the constant k_T (depends on the electrical motor parameters), the rotor flux Ψ_R and the quadrature current in the field oriented reference frame I_{Sq}. The investigated motor is a three-phase, 50Hz, 2-pole, 1.5 kW, squirrel cage induction machine, rated at 400V, 3.4A and 2900rpmin. The pipe is here assumed to be constant and will not be changed during operation, e.g. by applying valves (Wolfram et al., 2001).

5.2 *Identification of the mechanical subsystem*

For the identification, the special local model structure LOLIMOT is applied (Nelles, 2000). This Local Linear Model Tree is a neuro-fuzzy system with linear submodels and an axis-orthogonal recursive decomposition of the input space. The guidelines described in section 4 are employed for the identification of appropriate models:

(i) In order to describe the mechanical subsystem, the torque balance must be taken into account:

$$M_{el}(t) = J_P \cdot \dot{\omega}(t) + M_{Fr}(t) + M_H(t) \ . \qquad (9)$$

The torque $M_{el}(t)$ generated by the motor is spent for the acceleration of the over-all inertia J_P, the friction losses summarised in term $M_{Fr}(t)$ and the production of the delivery head in term $M_H(t)$. The mechanical friction can be modelled by

$$M_{Fr}(t) = M_c \cdot sign(\omega) + M_v \cdot \omega \ , \qquad (10)$$

regarding Coulomb M_c and viscous M_v friction terms, whereupon the signum-function can be neglected as the pump is only driven in positive direction. The torque component for the generation of delivery head $M_H(t)$ can be specified by

$$M_H(t) = M_{TH1} \cdot \omega(t) \cdot Q(t) - M_{TH2} \cdot Q^2(t) \ . \qquad (11)$$

The continuous equation is discretised by replacing $\dot{\omega}(t)$ by $\left(\omega[k] - \omega[k-1]\right)/T_s$. Thus the initial difference equation approach is

$$M_{el}[k] = w_0 + w_1 \cdot \omega[k] + w_2 \cdot \omega[k-1] + w_3 \cdot \omega[k] \cdot Q[k] - w_4 \cdot Q^2[k] \ . \qquad (12)$$

Due to saturation effects, (8) yields only accurate results at low speeds. Thus, the parameters w_v of (12) may depend on the speed ω which is therefore considered as the input variable z_v of the membership functions (compare (4)).

(ii) Assuming that the relationship between the speed and the torque can be modelled by means of a first order lag and the additional terms are negligible, the minimum time constant is determined by evaluating step responses. Thus, a minimum time constant of T_{min}=50ms is obtained. The dead time between speed and torque can be neglected, whereas the dead time between speed and flow rate is approximately 0.5s.

(iii) In accordance with (6) the sample time is chosen to T_s=10ms. The signals are filtered with 4th order analogue lowpass filters with butterworth characteristic. The 3dB cut-off frequency is 40Hz.

(iv) The only variable which can be excited in the specified plant is the rotor speed $n=\omega/2\pi$. As described in Section 4, an APRBS is chosen for the excitation of the drive. According to (7) the minimum impulse width is λ=10msec. As the slew rate of the speed is restricted to 2000rpmin/sec it makes no sense to select λ too small. Thus, the minimum impulse width is here λ=0.3s.

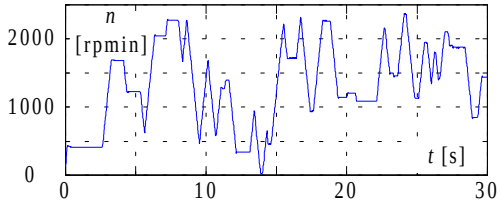

Figure 7: Measured excitation signal (speed)

(v) Several datasets are measured. Both, dynamic datasets as described in (iv) and static data are recorded for the sake of training and validation.

A short section of the measured excitation signal (APRBS with λ=0.3s) is depicted in Figure 7.

(vi) Based on the measured datasets, an initial neuro-fuzzy model with the input vector of the local models $\mathbf{x}^T=[\omega[k]\ \omega[k\text{-}1]\ \omega[k]\cdot Q[k]\ Q^2[k]\ 1]$ and the membership functions $\mathbf{z}^T=[\omega[k]]$ is identified.

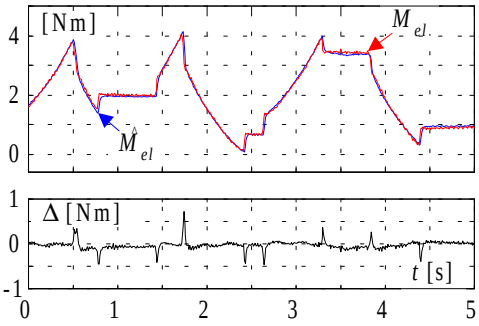

Figure 8: Initial approximation result

The achieved approximation result is illustrated in Figure 8 (5 local models). Hence, the model reproduces the static behavior well, whereas major deviations during dynamic operation occur. Further improvements can be achieved by additionally considering the past value of the torque $M_{el}[k\text{-}1]$ in the input vector $\mathbf{x}$.

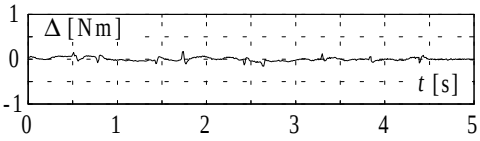

Figure 9: Model error after regarding $M_{el}[k\text{-}1]$

The resulting model error Δ (5 local models) considering the same excitation signal and time interval as in Figure 8 is depicted in Figure 9.

(vii) The model obtained in step (vi) provides already satisfactory approximation results. Unfortunately, many variables in $\mathbf{x}$ have to be taken into account. Thus, further considerations aiming at the reduction of the model without decreasing the quality are performed.
The flow rate is measured by an inductive sensor with a low cut-off frequency. Thus the contribution of the signal to the dynamic behaviour of the model is negligible. As the speed is the only driving input variable, the flow rate depends crucially on this signal. Thus, it should be possible to economise this input in the derived model. The new input vector is therefore $\mathbf{x}^T=[\omega[k]\ \omega[k\text{-}1]\ M_{el}[k\text{-}1]]$. The corresponding model error (5 local linear models) is illustrated in Figure 10.
Furthermore, due to the stationary proportionality between the flow rate and the speed, a quadratic term of the speed can be added supplementary to the input vector $\mathbf{x}^T=[\omega[k]\ \omega[k\text{-}1]\ \omega^2[k]\ M_{el}[k\text{-}1]]$, which can be interpreted as semi-physical modelling. With this, similar approximation results however with fewer local models (3 local models) are achieved.

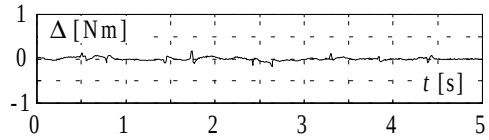

Figure 10: Model error without input $Q[k]$

Similar procedures are performed to identify the two remaining hydraulic subsystems of the process for fault detection purposes (Wolfram et al., 2001).

6. CONCLUSION

In this contribution, data-based modelling approaches which rely on local model architectures are proposed. For that purpose, the interesting operating range is partitioned in several regimes and thus the process is approximated by superposing local models, which are individually weighted by operating point dependent membership functions. Therefore, the input spaces of the local models and the membership functions need not to be necessarily identical.
As in technical systems always some physical insight is available, this knowledge can be utilised to improve the model structure and simplify the training process, which is referred to as semi-physical modelling. In order to enable a systematical modelling by including physical knowledge, a guideline in terms of several design steps is proposed. Finally, the application of the guidelines to a practical process is addressed.

REFERENCES

Babuska, R. and H. B. Verbruggen (1995). Identification of composite linear models via fuzzy clustering. *European Control Conference*. Rome, Italy, p. 1207-1212.

Fischer, M.; O. Nelles and R. Isermann (1997). Exploiting Prior Knowledge in Fuzzy Model Identification of a Heat Exchanger. *IFAC Symposium on Artificial Intelligence in Real Time Control*, Kuala Lumpur, Malaysia, p. 463-468.

Isermann, R. (1992). *Identifikation dynamischer Systeme, 1&2*. Springer-Verlag, Berlin, Germany.

Jang, J. S. R. (1993). ANFIS: Adaptive-Network-Based Fuzzs Inference Systems. *IEEE Transaction on Systems, Man, and Cybernetics 23*, p. 665-685.

Murray-Smith, R. and T. A. Johanson (1997). *Multiple Model Approaches to Modelling and Control*. Tayler & Francis, London, UK.

Nelles, O. (2001). *Nonlinear System Identification*. Springer Verlag, Germany.

Nelles, O. and R. Isermann (1995). Identification of Nonlinear Dynamic Systems - Classical Methods versus Radial Basis Function Networks. *American Control Conference*, 5, Seattle, USA, p. 3786-3790.

Wolfram, A.; D. Füssel; T. Brune and R. Isermann (2001). Component-Based Multi-Model Approach For Fault Detection and Diagnosis of a Centrifugal Pump. *ACC '2001*, Arlington, VA, USA.