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SUBSPACE IDENTIFICATION OF CLOSED LOOP SYSTEMS BY STOCHASTIC REALIZATION

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Abstract: We develop a closed loop subspace identification method based on stochastic realization theory. Using the preliminary orthogonal decomposition of (Picci and Katayama, 1996*b*) we show that, under the assumption that the exogenous input is feedback-free and persistently exciting (PE), the identification of closed loop systems is divided into two subproblems: the deterministic identification of the plant and controller, and the stochastic identification of the noise filter. Subspace methods for identifying the deterministic and stochastic parts are derived by adapting the standard subspace methods to the deterministic and stochastic components. In both cases, a model reduction procedure should be applied in order to extract controllable and observable parts from non-minimal realizations. Some numerical results are included to show the applicability of the present technique.

Keywords: Closed loop systems, Subspace methods, Stochastic Realization, LQ decomposition.

1. INTRODUCTION

The identification of stochastic multivariable systems operating under feedback, without measurable exogenous inputs, has been considered in (Van der Klauw *et al.*, 1991); in this paper the joint input-output approach and stochastic realization theory followed by a balanced model reduction are used for deriving the state space models of the subsystems in the closed loop. Also, based on subspace methods, a related technique of identifying the state-space model of a plant operating in closed loop with exogenous inputs has been proposed by (Verhaegen, 1993), reformulating it as an equivalent open loop problem. In the framework of the indirect approach, the multivariable closed loop identification problem has been discussed in (Van den Hof and de Callafon, 1996), together with its relation to the dual Youla parametrization approach. Moreover, under the assumption that a finite number of Markov parameters of the controller are known, a closed loop subspace identification method (Van Overschee and Moor, 1997) has been derived by modifying the N4SID algorithm (Van Overschee and Moor, 1994). We have also presented a subspace-based method of identifying the deterministic part of a plant operating in closed loop based on the orthogonal decomposition of the joint input-output process (Katayama *et al.*, 2001).

In this paper, we develop a subspace method for identifying the plant, the controller and the noise filter operating in closed loop, by extending the orthogonal decomposition based technique (Katayama *et al.*, 2001). First we decompose the joint input-output process into stochastic and deterministic components by means of the orthogonal decomposition developed in (Picci and Katayama, 1996*b*). The basic assumption is that the exogenous inputs are feedback-free and persistently exciting (PE). It can be shown that, by the preliminary orthogonal decomposition, the identification of closed loop system is divided into two subproblems: the identification of the deterministic part, or the plant and controller, and the identification of the stochastic part, or the noise filter. Thus the plant and the controller can be obtained from the realization of deterministic component of the joint process, and the noise filter can be retrieved from the realization of stochastic component. In each case, a model reduction procedure (Zhou *et al.*, 1999), applicable to both stable and unstable systems, should be used for deriving low order models from higher order models, by canceling poles and zeros situated very closely.

2. PROBLEM STATEMENT

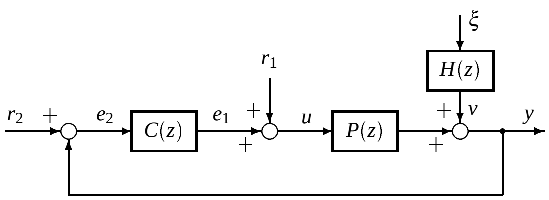

Fig. 1. Closed-loop system.

We consider the closed loop system shown in Fig. 1, where $y \in \mathbb{R}^p$ is the output vector of the plant, and $u \in \mathbb{R}^m$ is the input vector. An output disturbance $v \in \mathbb{R}^p$ is described by $v(t) = H(z)\xi(t)$, where $\xi \in \mathbb{R}^p$ is a white noise with mean zero and covariance matrix $\Lambda > 0$, and $H(z)$ is a minimum phase, square transfer matrix. It is also assumed that $H(\infty) = I_p$, where I_p is the $p \times p$ unit matrix. The signals $r_1 \in \mathbb{R}^m$ and $r_2 \in \mathbb{R}^p$ can be exogenous reference signals, or some probing inputs. We assume that $\{r_1(t), r_2(t), u(t), y(t), t = 0, \pm 1, \cdots\}$ are jointly stationary second-order processes with mean zero.

Let the plant be a finite dimensional, linear, time-invariant (FDLTI) system described by

$$y(t) = P(z)u(t) + H(z)\xi(t) \tag{1}$$

where $P(z)$ and $H(z)$ are the transfer matrices of the plant and the noise filter, respectively. The control input is generated by

$$u(t) = r_1(t) + C(z)[r_2(t) - y(t)] \tag{2}$$

where $C(z)$ denotes the transfer matrix of the FDLTI controller. We introduce the following assumptions on the closed loop system and exogenous inputs and noise.

A1: The closed loop system is well-posed in the sense that (u, y) are uniquely determined by the states of the plant, the controller, and by the exogenous inputs and noise. This generic condition is satisfied if $I_p + P(\infty)C(\infty)$ and $I_m + C(\infty)P(\infty)$ are non-singular. For the sake of simplicity, it is assumed throughout that the plant is strictly proper, namely, $P(\infty) = 0$.

A2: The controller internally stabilizes the closed loop system.

A3: The input vector $r = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} \in \mathbb{R}^l \quad (l = m + p)$ is PE, and r_1 and r_2 are mutually uncorrelated. Also, r is uncorrelated with the white noise ξ.

The objective of this paper is to obtain realizations of the plant $P(z)$, the controller $C(z)$ and of the noise filter $H(z)$, based on finite measurement data $\{r_1(t), r_2(t), u(t), y(t)\}$ by using a subspace identification method.

It may be observed that the identification of the controller is not needed, since often we know the dynamics of the controller. However, if the estimate of the controller agrees reasonably well with the actual controller implemented, this will be a confirmation of the validity of the identification procedure employed. Therefore, to identify all the components in the closed loop system is quite interesting from both practical and theoretical points of view.

The present problem is virtually the same as the one treated in (Verhaegen, 1993), but the approach is quite different. For, as shown below, we employ the approach based on orthogonal decomposition of (Picci and Katayama, 1996*b*) in order to decompose the joint input-output process into the deterministic and stochastic components; subspace methods are then applied to each component to get desired state space models. It may be also noted that further results in (Chiuso and Picci, 1999; Chiuso and Picci, 2000) show that in the case where the exogenous input is colored, the orthogonal decomposition-based subspace identification algorithms are more robust than the N4SID-like algorithms of (Van Overschee and Moor, 1994; Verhaegen, 1994).

3. THE JOINT INPUT-OUTPUT APPROACH

In order to obtain the state space models of the plant and controller in closed loop, we use the joint input-output approach. In this and next sections, we shall be concerned with the stationary case where infinite data are available.

Now define the joint input-output vector process as $w := \begin{bmatrix} u \\ y \end{bmatrix} \in \mathbb{R}^l$. From Fig. 1, the closed loop system is described by

$$\begin{aligned} w(t) &= \begin{bmatrix} T_{ur_1}(z) & T_{ur_2}(z) \\ T_{yr_1}(z) & T_{yr_2}(z) \end{bmatrix} r(t) + \begin{bmatrix} T_{u\xi}(z) \\ T_{y\xi}(z) \end{bmatrix} \xi(t) \\ &= T_{wr}(z)r(t) + T_{w\xi}(z)\xi(t) \end{aligned} \tag{3}$$

where $T_{ab}(z)$ denotes the transfer matrix from b to a. It is easy to see that

$$T_{wr}(z) = \begin{bmatrix} S_i(z) & S_i(z)C(z) \\ P(z)S_i(z) & P(z)S_i(z)C(z) \end{bmatrix} \tag{4a}$$

$$T_{w\xi}(z) = \begin{bmatrix} -C(z)S_o(z) \\ S_o(z) \end{bmatrix} H(z) \tag{4b}$$

where $S_i(z)$ and $S_o(z)$ are the input and output sensitivity matrices defined, respectively, by

$$S_i(z) = (I_m + C(z)P(z))^{-1}, \ S_o(z) = (I_p + P(z)C(z))^{-1}$$

Since there is no feedback from w to r, we can employ the stochastic realization method of (Picci and Katayama, 1996*b*) to get a state space model of the transfer matrix $T_{wr}(z)$ with input r and output w. Once the closed loop transfer matrices in $T_{wr}(z)$ are obtained, we can get the transfer matrices of the plant and the controller, say, from $P(z) = T_{yr_1}(z)T_{ur_1}^{-1}(z)$ and $C(z) = T_{ur_1}^{-1}(z)T_{ur_2}(z)$. There are also different combinations of transfer matrices in (4a) which may be used to recover the plant and the controller. Hence, unlike the indirect methods, no knowledge of the controller is employed and no auxiliary signals are generated.

Using the notations of (Picci and Katayama, 1996*b*), we shall write $\mathcal{R} = \overline{\text{span}}\{r(t) \mid t \in \mathbb{Z}\}$ and $\mathcal{W} = \overline{\text{span}}\{w(t) \mid t \in \mathbb{Z}\}$. We also define the linear spaces of second-order random variables spanned by the infinite past and future of the input and output vectors at the present time t as $\mathcal{R}_t^- := \overline{\text{span}}\{r(\tau) \mid \tau < t\}$, $\mathcal{W}_t^- := \overline{\text{span}}\{w(\tau) \mid \tau < t\}$ and $\mathcal{R}_t^+ := \overline{\text{span}}\{r(\tau) \mid \tau \geq t\}$, $\mathcal{W}_t^+ := \overline{\text{span}}\{w(\tau) \mid \tau \geq t\}$, respectively. These are regarded as subspaces of the ambient Hilbert space $\mathcal{H} := \mathcal{R} \vee \mathcal{W}$ containing all the linear functionals of the process (r, w).

Absence of feedback from w to r means that the future of r is conditionally uncorrelated with the past of w given the past of r itself. Hence we can write the feedback-free condition as $\mathcal{W}_t^- \perp \mathcal{R}_t^+ \mid \mathcal{R}_t^-$. It follows from the definition of conditional orthogonality that this condition can be rewritten as $\mathcal{W}_t^- \perp \mathcal{R} \mid \mathcal{R}_t^-$. Thus the absence of feedback is equivalent to the causality of the estimator, i.e.

$$\hat{E}\{w(t) \mid \mathcal{R}\} = \hat{E}\{w(t) \mid \mathcal{R}_{t+1}^-\}, \quad t \in \mathbb{Z}$$

It therefore follows that

$$\begin{aligned} w_s(t) &:= w(t) - \hat{E}\{w(t) \mid \mathcal{R}_{t+1}^-\} \\ &= \hat{E}\{w(t) \mid \mathcal{R}^\perp\} \end{aligned} \tag{5}$$

where $\mathcal{R}^\perp$ denotes the orthogonal complement of $\mathcal{R}$ with respect to $\mathcal{H}$. This implies that $w_s(t) \perp \mathcal{R}$ for all $t \in \mathbb{Z}$, i.e. the causal estimation error is uncorrelated with the whole history of the exogenous input r.

The process w_s defined by (5) is called the stochastic component of w. In the same way, the process defined by the orthogonal projection

$$w_d(t) := \hat{E}\{w(t) \mid \mathcal{R}\} \tag{6}$$

is called the deterministic component, the part of w linearly related to the exogenous reference input.

From above, we have the following orthogonal decomposition of the joint process:

$$w(t) = w_d(t) + w_s(t) \tag{7}$$

or equivalently

$$\begin{bmatrix} u(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} u_d(t) \\ y_d(t) \end{bmatrix} + \begin{bmatrix} u_s(t) \\ y_s(t) \end{bmatrix} \tag{8}$$

It should be noted that the deterministic and stochastic components are uncorrelated, i.e., $E\{w_s(t)w_d(\tau)^T\} = 0$ for all $t, \tau \in \mathbb{Z}$ (Picci and Katayama, 1996*b*). From this orthogonal decomposition, the basic relations satisfied by the deterministic and stochastic components in the closed loop system of Fig. 1, easily follow.

Proposition 1. The deterministic and stochastic components respectively satisfy

$$y_d(t) = P(z)u_d(t) \tag{9a}$$

$$u_d(t) = r_1(t) + C(z)[r_2(t) - y_d(t)] \tag{9b}$$

and

$$y_s(t) = P(z)u_s(t) + H(z)\xi(t) \tag{10a}$$

$$u_s(t) = -C(z)y_s(t) \tag{10b}$$

Proof: We see from (1), (2) and (8) that

$$y_d(t) + y_s(t) = P(z)[u_d(t) + u_s(t)] + H(z)\xi(t)$$

$$u_d(t) + u_s(t) = r_1(t) + C(z)[r_2(t) - y_d(t) - y_s(t)]$$

Taking the orthogonal projections of the above equations onto the spaces $\mathcal{R}$ and $\mathcal{R}^\perp$, we have (9) and (10) immediately. □

Solving (9) yields

$$\begin{bmatrix} u_d(t) \\ y_d(t) \end{bmatrix} = \begin{bmatrix} S_i(z) & S_i(z)C(z) \\ P(z)S_i(z) & P(z)S_i(z)C(z) \end{bmatrix} \begin{bmatrix} r_1(t) \\ r_2(t) \end{bmatrix}$$

Since the transfer matrices in the right hand side of the above equation are the same as those in (4a), they are completely determined by the realization of deterministic component. Moreover, the noise filter $H(z)$ is obtained from (10).

4. REALIZATION OF CLOSED LOOP SYSTEM

In this section, we derive the state space representations for the deterministic and stochastic components based on the theory of (Picci and Katayama, 1996*b*), and then compute the state space models of plant, controller and noise filter.

4.1 *Deterministic Component*

Suppose that for each t the input space $\mathcal{R}$ admits the direct sum decomposition $\mathcal{R} = \mathcal{R}_t^+ + \mathcal{R}_t^-$, that is, $\mathcal{R}_t^+ \cap \mathcal{R}_t^- = 0$. An analogous condition is that the spectral density matrix of r is strictly positive definite on the unit circle.

Let $\hat{\mathcal{W}}$ be spanned by the deterministic component w_d. Also, let $\hat{\mathcal{W}}_t^+$ denote the subspace generated by the future $w_d(\tau)$, $\tau \geq t$. According to Section 4 of

(Picci and Katayama, 1996*b*), we define the oblique predictor subspace as

$$\mathcal{X}_t^{+/-} := \hat{E}_{\|\mathcal{R}_t^+}\{\hat{\mathcal{W}}_t^+ \mid \mathcal{R}_t^-\} \tag{11}$$

This is the oblique projection of $\hat{\mathcal{W}}_t^+$ on the past $\mathcal{R}_t^-$ along the future $\mathcal{R}_t^+$, and $\mathcal{X}_t^{+/-}$ is the state space for the deterministic component w_d. Clearly, if r is a white noise process, (11) reduces to the ordinary orthogonal projection onto $\mathcal{R}_t^-$.

Let the dimension of the state space $\mathcal{X}_t^{+/-}$ be n. In general n equals the sum of the orders of the plant and of the controller. Any basis vector $x_d(t) \in \mathcal{X}_t^{+/-}$ yields a (joint) state space representation of w_d

$$x_d(t+1) = A_1 x_d(t) + [B_{11} \;\; B_{12}] r(t) \tag{12a}$$

$$w_d(t) = \begin{bmatrix} C_{11} \\ C_{21} \end{bmatrix} x_d(t) + \begin{bmatrix} D_{11} & D_{12} \\ 0 & 0 \end{bmatrix} r(t) \tag{12b}$$

where $D_{21} = 0$, $D_{22} = 0$ due to the fact that the plant $P(z)$ is strictly proper. Furthermore we have $D_{11} = I_m$ and $D_{12} = D_c$.

From (12) we derive state-space models for the plant and controller. First note that the closed loop transfer matrices are expressed as

$$\begin{bmatrix} T_{ur_1}(z) & T_{ur_2}(z) \\ T_{yr_1}(z) & T_{yr_2}(z) \end{bmatrix} = \left[\begin{array}{c|cc} A_1 & B_{11} & B_{12} \\ \hline C_{11} & D_{11} & D_{12} \\ C_{21} & 0 & 0 \end{array}\right]$$

We see from (4) that $T_{yr_1}(z) = P(z) T_{ur_1}(z)$ and $T_{ur_2}(z) = T_{ur_1}(z) C(z)$, so that the plant and controller are computed by

$$P(z) = T_{yr_1}(z) T_{ur_1}^{-1}(z), \quad C(z) = T_{ur_1}^{-1}(z) T_{ur_2}(z)$$

The state-space models are described in the following proposition.

Proposition 2. Suppose that $\det D_{11} \neq 0$. Then (non-minimal) state space representations of the plant and the controller are given by

$$P(z) = \left[\begin{array}{c|c} A_1 - B_{11} D_{11}^{-1} C_{11} & B_{11} D_{11}^{-1} \\ \hline C_{21} & 0 \end{array}\right] \tag{13a}$$

$$C(z) = \left[\begin{array}{c|c} A_1 - B_{11} D_{11}^{-1} C_{11} & B_{12} - B_{11} D_{11}^{-1} D_{12} \\ \hline D_{11}^{-1} C_{11} & D_{11}^{-1} D_{12} \end{array}\right] \tag{13b}$$

The models in (13) are not minimal (Verhaegen, 1993). Hence the observable and controllable part of each non-minimal state space model should be extracted from (13) to recover the state space models of the original plant and controller. This is performed by using a model reduction procedure due to (Zhou *et al.*, 1999; Varga, 2001).

4.2 *Stochastic Component*

Let the subspace $\tilde{\mathcal{W}}$ be spanned by the stochastic component w_s. Let $\tilde{\mathcal{W}}_t^+$ and $\tilde{\mathcal{W}}_t^-$ denote the linear spaces generated by the future and past of the stochastic component $w_s(t)$, respectively. According to Section 3 of (Picci and Katayama, 1996*b*), the predictor space $\tilde{\mathcal{X}}_t^{+/-}$ for the stochastic component is defined by

$$\tilde{\mathcal{X}}_t^{+/-} := \hat{E}\{\tilde{\mathcal{W}}_t^+ \mid \tilde{\mathcal{W}}_t^-\}$$

Suppose that $\dim \tilde{\mathcal{X}}_t^{+/-} = \tilde{n}$, which is the sum of the orders [1] of u_s and y_s. Then a particular realization of the stochastic system having the predictor space $\tilde{\mathcal{X}}_t^{+/-}$ as the state space is given by

$$x_s(t+1) = A_2 x_s(t) + [K_{21} \;\; K_{22}] \begin{bmatrix} \nu_1(t) \\ \nu_2(t) \end{bmatrix} \tag{14a}$$

$$w_s(t) = \begin{bmatrix} C_{12} \\ C_{22} \end{bmatrix} x_s(t) + \begin{bmatrix} \nu_1(t) \\ \nu_2(t) \end{bmatrix} \tag{14b}$$

where $x_s(t)$ is a basis in $\tilde{\mathcal{X}}_t^{+/-}$, and $\nu(t) := \begin{bmatrix} \nu_1(t) \\ \nu_2(t) \end{bmatrix}$ is the one-step prediction error of the stochastic component $w_s(t) = \begin{bmatrix} u_s(t) \\ y_s(t) \end{bmatrix}$ based on the past $\tilde{\mathcal{W}}_t^-$, i.e. the forward innovation process of $w_s(t)$ (Picci and Katayama, 1996*b*). It can be shown that the innovation of $w_s(t)$ is the conditional innovation of $w(t)$ given observations of $r(\tau)$, $\tau \leq t$, namely, $\nu(t) = w(t) - \hat{E}\{w(t) \mid \mathcal{R}_{t+1}^- \vee \mathcal{W}_t^-\}$.

Proposition 3. The realizations of $P(z)$ and $H(z)$ in (10a) are given by

$$P(z) = \left[\begin{array}{c|c} A_2 - K_{21} C_{12} & K_{21} \\ \hline C_{22} & 0 \end{array}\right] \tag{15a}$$

$$H(z) = \left[\begin{array}{c|c} A_2 - K_{21} C_{12} & K_{22} \\ \hline C_{22} & I_p \end{array}\right] \tag{15b}$$

Proof: Substituting $\nu_1(t)$ of (14b) into (14a) yields

$$x_s(t+1) = (A_2 - K_{21} C_{12}) x_s(t) + K_{21} u_s(t) + K_{22} \nu_2(t)$$
$$y_s(t) = C_{22} x_s(t) + \nu_2(t)$$

where $\nu_2(t)$ is a white noise. Thus eliminating x_s,

$$y_s(t) = C_{22}(zI - A_2 + K_{21} C_{12})^{-1} K_{21} u_s(t) + (I_p + C_{22}(zI - A_2 + K_{21} C_{12})^{-1} K_{22}) \nu_2(t)$$

Comparing this with (10a), we get (15a) and (15b). □

It should be noted that we can also include a regulator noise that acts on the output of the controller $C(z)$ in Fig. 1. If so, and if the controller is not strictly proper, the realization of stochastic component should be modified accordingly (Anderson and Gevers, 1982; Van der Klauw *et al.*, 1991).

As in the deterministic case, we also need a model reduction procedure to get reduced order models from non-minimal models.

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[1] The *order* of a process with a rational spectrum is meant to be the order of a minimal realization.

5. IMPLEMENTATION OF ORTHOGONAL DECOMPOSITION

Suppose that finite input-output data $r_1(t)$, $r_2(t)$, $u(t)$, $y(t)$ for $t = 0, 1, \cdots, N+2k-2$ are given with $k > 0$ and N sufficiently large. As usual, we fix a present time $t = k$ and define the block Hankel matrix $R_{0|k-1}$ of dimension $kl \times N$ formed by the exogenous input as

$$R_{0|k-1} = \begin{bmatrix} r(0) & r(1) & \cdots & r(N-1) \\ r(1) & r(2) & \cdots & r(N) \\ \vdots & \vdots & \ddots & \vdots \\ r(k-1) & r(k) & \cdots & r(N+k-2) \end{bmatrix}$$

and similarly for $R_{0|2k-1}$, $W_{0|2k-1}$, $W_{k|2k-1}$.

Consider the standard LQ factorization (Verhaegen, 1994; Van Overschee and Moor, 1994; Katayama and Picci, 1999)

$$\begin{bmatrix} R_{0|2k-1} \\ W_{0|2k-1} \end{bmatrix} = \begin{bmatrix} L_{11} & 0 \\ L_{21} & L_{22} \end{bmatrix} \begin{bmatrix} Q_1^T \\ Q_2^T \end{bmatrix}$$

where $L_{11}, L_{22} \in \mathbb{R}^{2kl \times 2kl}$ are the lower triangular matrices with $Q_i^T Q_j = I_{2kl} \delta_{ij}$.

Then it can be shown (Picci and Katayama, 1996*a*) that the finite-history of deterministic component is obtained by

$$\hat{W}^d_{0|2k-1} := \hat{E}\{W_{0|2k-1} \mid \mathcal{R}_{0|2k-1}\} = L_{21} Q_1^T$$

so that the stochastic component is given by

$$\hat{W}^s_{0|2k-1} = W_{0|2k-1} - \hat{W}^d_{0|2k-1}$$

Due to space limitation, we skip all the algorithms needed in subspace identification and subsequent model reduction; see e.g. (Katayama *et al.*, 2001).

6. SIMULATION RESULTS

Consider the following model (Codrons *et al.*, 2000):

$$P(z) = \frac{0.1208z + 0.1812}{z^4 - 1.992z^3 + 2.203z^2 - 1.841z + 0.8941}$$

$$C(z) = \frac{0.5517z^4 - 1.765z^3 + 2.113z^2 - 1.296z + 0.4457}{z^3(z-1)}$$

$$H(z) = \frac{z^4}{z^4 - 1.992z^3 + 2.203z^2 - 1.841z + 0.8941}$$

From this model, we observe that all the poles of the plant are located near the unit circle (see Fig. 2), and a zero is at $z = -1.5$. Thus $P(z)$ is stable, but non-minimal phase. The controller has a pole at $z = 1$, and has zeros at $z = 1.2629 \pm 0.2j$. Thus $C(z)$ is marginally stable, and non-minimal phase. The denominator of noise filter is the same as that of the plant.

We assume that the reference inputs r_1, r_2 are Gaussian white noises with mean zero and variances $\sigma^2_{r_1} = 1$, $\sigma^2_{r_2} = 1$, and that the noise ξ is a Gaussian white noise with mean zero and variance σ^2_ξ. For simulation studies, we take $N = 3000$, and $k = 15$, and generated 30 data set, where in each case different Gaussian random numbers with specific variances are generated.

First we show the identification results of deterministic part. The estimated poles of $P(z)$ by (13a) for $\sigma^2_\xi = (0.3)^2$ and the corresponding Bode plot are displayed in Figs. 2 and 3, respectively. We see that the identification result is quite good; though not included if we decrease the noise variance to $\sigma^2_\xi = (0.05)^2$, then the better estimates are obtained due to the increase of the power of deterministic component.

Also, the stochastic part, namely, the noise filter $H(z)$ is estimated by (15b). Bode plot of the estimates of noise filter is depicted in Fig. 4, where $\sigma^2_\xi = (0.3)^2$. But if we decrease the variance to $\sigma^2_\xi = (0.05)^2$, we see that the performance of the estimates is degraded as shown in Fig. 5, though that of the deterministic part improves. This may be inferred from the fact that the relative power of stochastic component increases as the variance σ^2_ξ increases.

7. CONCLUSIONS

In this paper, we have developed a subspace method for identifying the plant, the controller and the noise filter operating in closed loop, based on stochastic realization with exogenous inputs. In particular, by preliminary orthogonal decomposition, we have shown that the identification of closed loop system can be divided into two subproblems: the identification of the deterministic part and the identification of the stochastic part. The plant and the controller can be obtained from the realization of deterministic component of the joint process, and the noise filter can be retrieved from the realization of stochastic component. In each case, a model reduction procedure which is applicable to both stable and unstable systems is used for canceling poles and zeros situated very closely. Numerical results show the applicability of the present technique.

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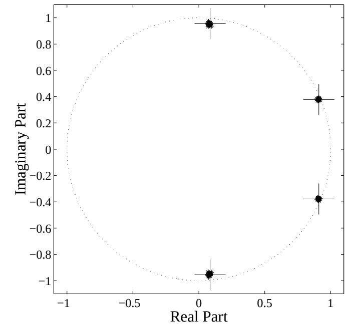

Fig. 2. True $(+)$ and estimated $(\times)$ poles of the plant $P(z)$ by (13a), where $\sigma_\xi^2 = (0.3)^2$.

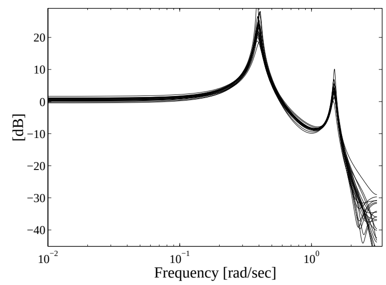

Fig. 3. Bode plot of the estimated plant $P(z)$ over 30 experiments, where $\sigma_\xi^2 = (0.3)^2$.

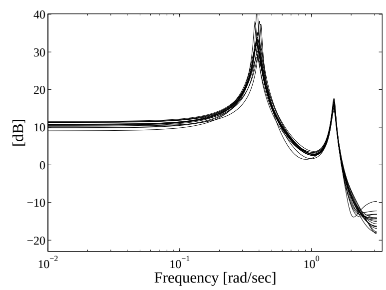

Fig. 4. Bode plot of the estimated noise filter $H(z)$ by (15b), where $\sigma_\xi^2 = (0.3)^2$.

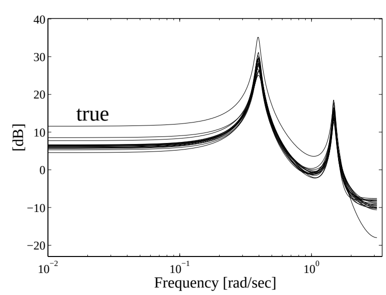

Fig. 5. Bode plot of the estimated noise filter $H(z)$ by (15b), where $\sigma_\xi^2 = (0.05)^2$.