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DISCRETE-TIME LPV CONTROLLER FOR ROBUST MISSILE AUTOPILOT DESIGN

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Abstract: This paper proposes a method to obtain a discrete-time LPV (Linear Parameter Varying) controller for a non-linear system represented with the LFT (Linear Fractional Transformation) framework. This synthesis can be integrated into a loop-shaping/LPV approach to obtain a discrete gain-scheduled control law. A detailed application of this method is here performed with a classical example of non-linear missile pitch-axis control. *Copyright © 2002 IFAC*

Keywords: Aerospace control, Missile, Discrete-time systems, Robust Control, Time-varying systems.

1. INTRODUCTION

For a missile with a high level of maneuverability, linear control cannot complete the required performances for all flight conditions because of the very non-linear behaviour of the system. To compare the quality of different non-linear control laws, one classical missile pitch-axis model (Reichert, 1992) has been extensively studied (Nichols, *et al.* 1993; Biannic and Apkarian, 1999; Devaud, *et al.* 2001). Among the proposed methods, some are based on LPV control theory which has focused attention of many people in the control community during the past decade (Apkarian and Gahinet 1995; Apkarian and Adams, 1998), but almost all of these control laws have been performed in continuous-time domain.

The objective here is to design a digital controller for this missile pitch-axis control example via a discrete LPV method that could also be used in a very general case. The chosen strategy is based on the quasi-LPV/LFT continuous-time representation of the plant and requires a recent method of discretisation for continuous-time LFT models (Imbert, 2001). The discrete–time synthesis of the controller is then performed with a LPV/LFT loop-shaping method similar to the continuous-time case described in (Devaud, *et al.*, 1999).
The paper is organised as follows: the theoretical discrete LPV/LFT synthesis used in the loop-shaping design is developed in section 2. Section 3 is devoted to the control problem description and a discrete-time LFT model of the non-linear system is obtained in section 4. Section 5 details the chosen loop-shaping control strategy and the obtained discrete-time LPV-controller is finally analysed in time and frequency-domain in section 6.

2. DISCRETE LPV/LFT CONTROL

The main theoretical point of this paper is detailed in this section: for a discrete-time LPV plant with a LFT representation, it consists in the synthesis of a discrete-time LPV controller of the same structure (fig. 1).

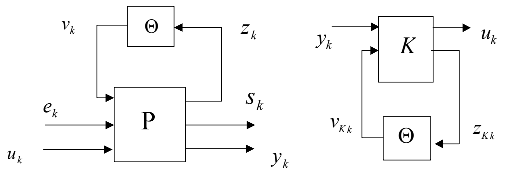

Fig. 1: discrete LFT representations of the plant and the controller

The LPV/LFT plant is described by:

$$P:\begin{cases} x_{k+1} = Ax_k + B_1 v_k + B_2 e_k + B_3 u_k \\ z_k = C_1 x_k + D_{11} v_k + D_{12} e_k + D_{13} u_k \\ s_k = C_2 x_k + D_{21} v_k + D_{22} e_k + D_{23} u_k \\ y_k = C_3 x_k + D_{31} v_k + D_{32} e_k + D_{33} u_k \end{cases}$$

with $v_k = \Theta z_k \quad \Theta = diag(\theta_1 I_{n_1},\ldots,\theta_n I_{n_n}), \quad \|\Theta\|_{L_2} < 1$

where:

- $\|\cdot\|_{L_2}$ denotes the L_2-gain of discrete time-varying systems.
- Θ represents the varying parameters block.
- z_k and v_k are the input and the output of the parametric block.
- e_k and s_k are the disturbance input and the output of the plant.
- u_k and y_k are the control input and the measurement of the plant.

All signals can be vector-valued. For the rest of the study, without loss of generality it is assumed that:

$$D_{33} = 0$$

The design problem considered is the following:

> Find a LPV/LFT controller $F_l(K,\Theta)$ such that the closed-loop system $F_l(F_u(P,\Theta),F_l(K,\Theta))$ is internally stable and has a L_2-gain less than a given number γ:
>
> $$\|F_l(F_u(P,\Theta),F_l(K,\Theta))\|_{L_2} < \gamma \qquad (1)$$

where $F_l(.)$ (resp. $F_u(.)$) denotes the lower (resp. upper) Linear Fractional Transformation.

Define the system P_a as follows:

$$\left[\begin{array}{c|ccccc} A & B_1 & 0 & B_2 & B_3 & 0 \\ \hline C_1 & D_{11} & 0 & D_{12} & D_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & I_{n_\theta} \\ C_2 & D_{21} & 0 & D_{22} & D_{23} & 0 \\ C_3 & D_{31} & 0 & D_{32} & 0 & 0 \\ 0 & 0 & I_{n_\theta} & 0 & 0 & 0 \end{array}\right]$$

where $n_\theta = size(v_k) = size(z_k)$, and consider the following sets of scaling:

$$\Delta = \{diag(\theta_1 I_{n_1},\ldots,\theta_n I_{n_n}),\ \theta_i \in [-1;1]\}$$

$$S_1 = \{S > 0,\ S\Theta = \Theta S,\ \forall \Theta \in \Delta\}$$

$$S_2 = \left\{\begin{bmatrix} S_A & S_B \\ S_B & S_C \end{bmatrix} > 0,\ (S_A, S_C) \in S_1^{\,2},\ S_B\Theta = \Theta S_B,\ \forall\Theta \in \Delta\right\}$$

<u>Proposition (Apkarian and Gahinet, 1995)</u>: if there exists any K and $L \in S_2$ that verify

$$\left\| \begin{pmatrix} L & 0 \\ 0 & I \end{pmatrix}^{\frac{1}{2}} F_l(P_a, K) \begin{pmatrix} L & 0 \\ 0 & I \end{pmatrix}^{-\frac{1}{2}} \right\|_\infty < \gamma \qquad (2)$$

then the closed-loop system satisfies (1).

Let define a state space representation of K:

$$\begin{cases} x_{K\,k+1} = A_K x_{K\,k} + B_{K1} y_k + B_{K2} v_{K\,k} \\ u_k = C_{K1} x_{K\,k} + D_{K11} y_k + D_{K12} v_{K\,k} \\ z_{K\,k} = C_{K2} x_{K\,k} + D_{K21} y_k + D_{K22} v_{K\,k} \end{cases}$$

It induces the following state-space representation of $F_l(P_a, K)$:

$$A_{F_l} = \begin{bmatrix} A + B_3 D_{K11} C_3 & B_3 C_{K1} \\ B_{K1} C_3 & A_K \end{bmatrix}$$

$$B_{F_l} = \begin{bmatrix} B_1 + B_3 D_{K11} D_{31} & B_3 D_{K12} & B_2 + B_3 D_{K11} D_{32} \\ B_{K1} D_{31} & B_{K2} & B_{K1} D_{32} \end{bmatrix}$$

$$C_{F_l} = \begin{bmatrix} C_1 + D_{13} D_{K11} C_3 & D_{13} C_{K1} \\ D_{K21} C_3 & C_{K2} \\ C_2 + D_{23} D_{K11} C_3 & D_{23} C_{K1} \end{bmatrix}$$

$$D_{F_l} = \begin{bmatrix} D_{11} + D_{13} D_{K11} D_{31} & D_{13} D_{K12} & D_{12} + D_{13} D_{K11} D_{32} \\ D_{K21} D_{31} & D_{K22} & D_{K21} D_{32} \\ D_{21} + D_{23} D_{K11} D_{31} & D_{23} D_{K12} & D_{22} + D_{23} D_{K11} D_{32} \end{bmatrix}$$

Using the discrete real bounded lemma, the following statements are then equivalent:

- There exist a controller K and a scaling $L \in S_2$ such that $F_l(F_u(P,\Theta),F_l(K,\Theta))$ is internally stable and such that K and L verify (2).
- There exist a controller K, a scaling $L \in S_2$ and $X > 0$ such that:

$$\begin{bmatrix} -X^{-1} & A_{F_l} & B_{F_l} & 0 \\ A_{F_l}^{\ T} & -X & 0 & C_{F_l}^{\ T} \\ B_{F_l}^{\ T} & 0 & \begin{bmatrix} -L & 0 \\ 0 & -\gamma I \end{bmatrix} & D_{F_l}^{\ T} \\ 0 & C_{F_l} & D_{F_l} & \begin{bmatrix} -L^{-1} & 0 \\ 0 & -\gamma I \end{bmatrix} \end{bmatrix} < 0 \qquad (3)$$

Furthermore each $X > 0$ and each $L \in S_2$ can be parameterised as follows:

$X = X_2 X_1^{-1},\ X_1 = \begin{pmatrix} R & I \\ M^T & 0 \end{pmatrix},\ X_2 = \begin{pmatrix} I & S \\ 0 & N^T \end{pmatrix}$, with

$$R > 0,\ S > 0, \begin{pmatrix} R & I \\ I & S \end{pmatrix} > 0,\ RS + MN^T = I$$

$$L = L_2 L_1^{-1},\ L_1 = \begin{pmatrix} \Pi & I \\ U^T & 0 \end{pmatrix},\ L_2 = \begin{pmatrix} I & \Sigma \\ 0 & V^T \end{pmatrix}, \text{ with}$$

$$\Pi > 0,\ \Sigma > 0,\ \begin{pmatrix} \Pi & I \\ I & \Sigma \end{pmatrix} > 0,\ \Pi\Sigma + UV^T = I$$

The proposed approach to solve problem (1) extends to LPV/LFT systems the results of H$_\infty$ control exposed in (Gahinet, 1996) or (Guo, *et al.*, 1999) in the stationary case. It can be proved that inequality (3) is equivalent to a LMI:

- *First step*: left and right-multiply inequality (3) with $diag(X_2{}^T, X_1{}^T, L_1{}^T, I_{n_e}, L_2{}^T, I_{n_s})$ and $diag(X_2, X_1, L_1, I_{n_e}, L_2, I_{n_s})$ respectively (where $n_s = size(s_k)$ and $n_e = size(e_k)$).

- *Second step*: introduce the following intermediate variables:

$$\begin{cases} F = SB_3 D_{K11} + NB_{K1} \\ G = D_{K11} C_3 R + C_{K1} M^T \\ H = SAR + SB_3 D_{K11} C_3 R + NB_{K1} C_3 R \\ \quad + SB_3 C_{K1} M^T + NA_K M^T \\ F_2 = SB_1 \Pi + FD_{31} \Pi + SB_3 D_{K12} U^T + NB_{K2} U^T \\ G_2 = \Sigma C_1 R + \Sigma D_{13} G + VD_{K21} SC_3 R + VC_{K2} M^T \\ W = \Sigma D_{13} D_{K11} + VD_{K21} \\ Y = D_{K12} U^T + D_{K11} D_{31} \Pi \\ Z = \Sigma D_{11} \Pi + \Sigma D_{13} D_{K11} D_{31} \Pi + VD_{K21} D_{31} \Pi \\ \quad + \Sigma D_{13} D_{K12} U^T + VD_{K22} U^T \end{cases} \tag{4}$$

Consequently, (3) is equivalent to:

$$\begin{bmatrix} \begin{bmatrix} -R & -I \\ -I & -S \end{bmatrix} & \Omega_1 & \Omega_2 & (0) \\ \Omega_1{}^T & \begin{bmatrix} -R & -I \\ -I & -S \end{bmatrix} & (0) & \Omega_3{}^T \\ \Omega_2{}^T & (0) & \begin{bmatrix} -\Pi & -I & 0 \\ -I & -\Sigma & 0 \\ 0 & 0 & -\gamma I \end{bmatrix} & \Omega_4{}^T \\ (0) & \Omega_3 & \Omega_4 & \begin{bmatrix} -\Pi & -I & 0 \\ -I & -\Sigma & 0 \\ 0 & 0 & -\gamma I \end{bmatrix} \end{bmatrix} < 0 \tag{5}$$

with

$$\Omega_1 = \begin{bmatrix} AR + B_3 G & A + B_3 D_{K11} C_3 \\ H & SA + FC_3 \end{bmatrix}$$

$$\Omega_2 = \begin{bmatrix} B_1 \Pi + B_3 Y & B_1 + B_3 D_{K_{11}} D_{31} & B_2 + B_3 D_{K_{11}} D_{32} \\ F_2 & SB_1 + FD_{31} & SB_2 + FD_{32} \end{bmatrix}$$

$$\Omega_3 = \begin{bmatrix} C_1 R + D_{13} G & C_1 + D_{13} D_{K11} C_3 \\ G_2 & \Sigma C_1 + WC_3 \\ C_2 R + D_{23} G & C_2 + D_{23} D_{K11} C_3 \end{bmatrix}$$

$$\Omega_4 = \begin{bmatrix} D_{11} \Pi + D_{13} Y & D_{11} + D_{13} D_{K11} D_{31} & D_{12} + D_{13} D_{K_{11}} D_{32} \\ Z & \Sigma D_{11} + WD_{31} & \Sigma D_{12} + WD_{32} \\ D_{21} \Pi + D_{23} Y & D_{21} + D_{23} D_{K11} D_{31} & D_{22} + D_{23} D_{K11} D_{32} \end{bmatrix}$$

The minimisation of γ solution to problem (1) has thus been turned into the following problem:

$$\min \gamma \ / \ \left\{ \begin{cases} R > 0,\ S > 0 \\ \begin{pmatrix} R & I \\ I & S \end{pmatrix} > 0 \end{cases} \text{and} \begin{cases} \Pi > 0,\ \Sigma > 0 \\ \begin{pmatrix} \Pi & I \\ I & \Sigma \end{pmatrix} > 0 \end{cases} \text{and (5)} \right\} \tag{6}$$

where D_{K11}, F, G, F_2, G_2, H, W, Y, Z, Π, Σ, R, S and γ are the optimisation variables.

As problem (6) is a minimisation problem under LMI constraints (i.e. a convex problem), the convergence to the global minimum is guaranteed.
Once the optimal values of the optimisation variables are recollected, the matrices M, N, U and V can be computed and a state-space representation of the LPV/LFT controller is easily obtained by inverting equations (4).

Remark: if Π, Σ, R and S are full-rank matrices, then the controller satisfying (3) is unique, involving the inversion of M and N (resp. U and V). If not, a controller of reduced-order (resp. reduced parametric dependence) is obtained using the pseudo-inverses.

3. DESCRIPTION OF THE MISSILE PITCH-AXIS CONTROL PROBLEM

The control problem discussed in this paper is a classical benchmark detailed in (Reichert, 1992): this study deals with the control of a missile pitch-axis under high variations of the angle of attack, which induces a very non-linear behaviour of the plant to control.

Consider the following equations of flight dynamics:

$$\begin{cases} \dot{\alpha} = K_\alpha(z) M C_n(\alpha, \delta, M) \cos(\alpha) + q \\ \dot{q} = K_q(z) M^2 C_m(\alpha, \delta, M) \\ \eta = K_\eta M^2 C_n(\alpha, \delta, M) \end{cases} \tag{7}$$

where:

- α, δ, q, M, z and η denotes respectively the angle of attack, the pitch fin deflection, the pitch rate, the number of Mach, the altitude and the normal acceleration.
- $C_n(\alpha,\delta,M)$ and $C_m(\alpha,\delta,M)$ are the normal force and moment aerodynamical coefficients which have the following expressions:

$$\begin{cases} C_n(\alpha,\delta,M) = a_n \alpha^3 + b_n \alpha|\alpha| + (2 - M/3) c_n \alpha + d_n \delta \\ C_m(\alpha,\delta,M) = a_m \alpha^3 + b_m \alpha|\alpha| + (-7 + 8M/3) c_m \alpha + d_m \delta \end{cases}$$

A particular flight point will be considered in this study: $M = 3$ and $z = 20000\,ft$, which induces the following numerical values:

$a_n = 1.0286*10^{-4}\ \deg^{-3}$ $\quad a_m = 2.1524*10^{-4}\ \deg^{-3}$
$b_n = -0.94457*10^{-2}\ \deg^{-2}$ $\quad b_m = -1.9546*10^{-2}\ \deg^{-2}$
$c_n = -0.1696\ \deg^{-1}$ $\quad c_m = 0.051\ \deg^{-1}$
$d_n = -0.034\ \deg^{-1}$ $\quad d_m = -0.206\ \deg^{-1}$

$$K_\alpha = 2.069*10^{-2} \quad K_q = 1.2320 \quad K_\eta = 21.4432$$

Furthermore actuators are modelled with a second-order low-pass filter (with natural frequency $\omega_a = 150\, rads^{-1}$ and damping $\xi = 0.7$).

The normal acceleration η is the variable that has to track a given reference but both η and the pitch rate q are available measurements. This missile control problem has then the following performance requirements:

- 95% time response: 0.35s.
- static error less than 1%.
- overshoot less than 15%.
- limitation on actuator rate : 25 deg.s^{-1} for 1g step.
- Sufficient stability margins for the equivalent open loop of the linearised models for all values of the angle of attack: 20ms for delay margin, 8 dB for gain margin, 40° for phase margin.

4. LPV DISCRETE-TIME MODEL OF THE MISSILE

The objective of this section is to build a convenient LPV/LFT discrete-time model of the missile in which the angle of attack α appears both as a state and as a varying parameter.

4.1. LFT continuous-time model

The range of variation for the angle of attack is [-20, +20] (in degrees). As the missile is supposed to be symmetric, only positive values for α will be taken into account. For this range of variation, $\cos(\alpha)$ can also be approximated by 1. With those assumptions, the quasi-LPV model can be expressed with a continuous-time LFT representation in which the parameter block is αI_2 (for a minimal standard representation).

4.2. Discretisation

An approximated discrete-time LFT model with the same parametric block as the one of the continuous-time LFT model is required for the synthesis of the discrete LPV/LFT controller. The chosen method to discretise the continuous-time LFT model consists in two points (Imbert, 2001):

- Add a first-order hold for the outputs of the parameter block (that are inputs for the system).
- Add a zero-order hold for the other inputs of the system.

The choice of the sample-time T_e depends on the closed-loop performance specifications. As the required time-response is 0.35s, a convenient value of T_e can be 0.01s. Figure 2 compares non-linear step responses of the discretised LFT model and of the continuous-time model (with zero-order hold).

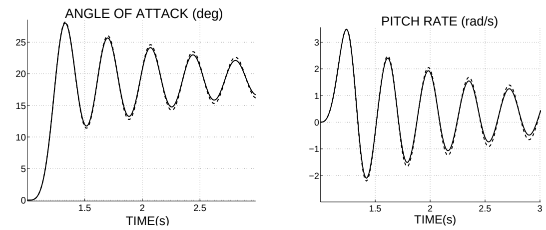

Figure 2: comparison of LFT models : continuous-time (solid) and discretised (dash)

Even if such a comparison is not sufficient to justify entirely the quality of the discretisation, it can although be considered that the discrete-time model reflects quite properly the system to be controlled.

Let now describe the chosen control strategy.

5. THE LOOP-SHAPING PROCEDURE

The pertinence of loop-shaping methods (MacFarlane and Glover, 1990) for missile autopilots design has already been shown for both linear (Friang, *et al.*, 1998; Iglesias and Urban 1999) and non-linear (Hiret, *et al.*, 1998) control laws. The main principles of this methodology are quickly described below:

- The first step is to choose post and pre-compensators W_2 and W_1 to shape the open loop (fig. 3):

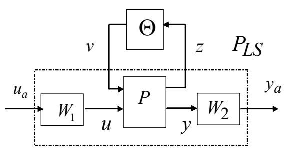

Fig. 3: Augmented Plant

Generally those compensators are tuned to provide, for given values of Θ, low-gain in high frequency domain to increase robustness to model uncertainties and high-gain in low-frequency domain for embedded precision (fig. 4):

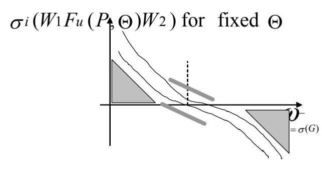

Fig. 4: Loop-shaping principle

- The second step consists in the controller synthesis: the principle of H_∞ robust stabilisation problem (one particular H_∞ problem) is extended to LPV discrete-time context (fig. 5), giving directly the following problem:

Find a stabilising controller $F_l(K,\Theta)$ that minimise the L_2-norm of the plant between (e_1,e_2) and (s_1,s_2).

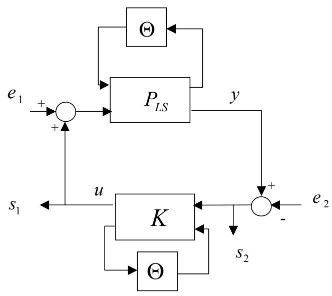

Fig. 5: LPV/LFT robust stabilisation problem

The synthesis method detailed in section 2 is then used to find a solution to this problem.

- Finally the implanted controller is the obtained LPV controller associated with the post and pre-compensators (fig. 6):

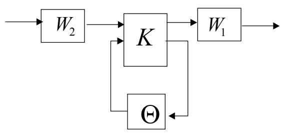

Fig. 6: Implanted controller

The application of this synthesis procedure to obtain a non-linear autopilot for the missile control problem is detailed in the next section.

6. SYNTHESIS AND ANALYSIS OF THE LPV/LFT LOOP-SHAPING CONTROLLER

For the synthesis of the missile autopilot, it is sufficient to choose linear invariant compensators W_2 and W_1: the non-linear behaviour will be taken into account by the LPV controller. W_1 is a first-order low-pass filter and W_2 has the same structure as one of the classical autopilot configuration: a static gain for each measure is combined with a PI compensator on the accelerometric error (fig. 7).

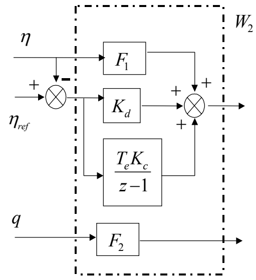

Fig. 7: Structure of the post-compensator W_2

Remark: This configuration guarantees the cancellation of any static error thanks to the PI compensator on the accelerometric channel.

Numerical data:

W_1 cut-off frequency: 58 s^{-1}

$$W_2 \text{ coefficients: } \begin{cases} F_1 = -0.0052 & F_2 = -0.2912 \\ K_c = 0.1258 & K_d = 0.0157 \end{cases}$$

The resulting implanted LPV controller (with the compensators) has a state-space representation of order 8 and a parameter block of order 2. The obtained performance level γ is 3.7.

6.1. Time-domain analysis

The behaviour of the non-linear closed-loop system is observed for a manoeuvre with different step responses of the normal acceleration, which induces large variations of the angle of attack (fig. 8).

The main result consists in the fact that all time-domain specifications are completed (Tab. 1):

Table 1: Time-domain performances

	Max. 95% time response	Max overshoot	Max normalised actuator rate
LPV/LFT Controller	0.343 s	5.9 %	16.1 deg/s

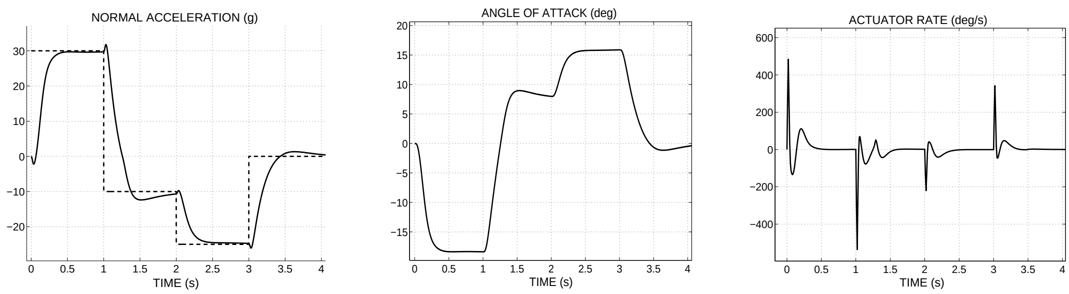

Fig. 8: Non-linear simulation

6.2. Frequency-domain analysis

For 10 values of α between 0 and +20 degrees, the complete non-linear model is linearised and exactly discretised with a zero-order hold (sample-time 0.01s). The non-linear LPV discrete controller is frozen for the same values of α, so that the discrete-time open loop becomes linear invariant. In this study, the expression "open-loop" denotes the classical equivalent loop for autopilot analysis: the closed-loop system is opened at the input of the actuator. Stability margins are deduced from the plots given in fig. 9. The results are recollected in Tab. 2.

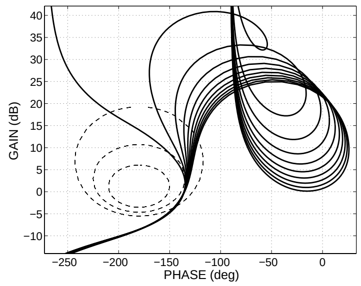

Fig. 9: Black diagram of equivalent open-loop

Remark: the worst configuration for stability is obtained for $\alpha = 0$, which is a classical result for missile control. Consequently the worst phase and gain margins are obtained for this flight. Furthermore the bandwidth increases with high angle of attack, so that the worst case for delay margin is achieved for $\alpha = \alpha_{\max} = 20\,\deg$.

Table 2: Frequency-domain performances

	Gain margin	Phase margin	Delay margin
LPV/LFT Controller	> 8.5 dB	> 44 deg	> 21.5 ms

Conclusion: The LPV/LFT discrete control law designed with the loop-shaping methodology achieves all the performance requirements of this missile autopilot benchmark: the good continuous-time performance of the loop-shaping LFT/LPV controllers (Devaud, *et al.*, 1999) is here confirmed in discrete-time. Furthermore the stability margins are satisfactory and very homogenous for the whole range of variation of the angle of attack.

7. CONCLUSION

An original method based on H_∞ loop-shaping principles has been proposed to design quasi-LPV or gain-scheduled discrete-time controllers and has been successfully tested on a missile control problem. Its main advantage consists in the resulting LPV controller expressed in discrete-time domain, which can simplify its implementation and validation process in case of use in real applications.

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