

ALGEBRAIC DESIGN OF MULTIRATE CONTROLLERS

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Abstract: There are robotic and industrial control applications where the control action must be updated faster than the output measurement. This fact leads to multirate controllers. In this paper, it is exposed an algebraic design which is used to design minimum and finite time controllers based on the resolution of a coupled diophantine equations for different sampling rates. Also, it is explained the way in which these equations are solved. Finally, a cancellation controller is designed to control an interesting robotic environment case. Promising results are obtained.

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Keywords: Process control, Industrial control, Multirate, Robot control.

1. INTRODUCTION

It is called multirate sampling systems those sampled systems in which two or more variables are updated at different frequencies.

Multirate digital control or multiperiodic sampling control is a kind of digital control in which samplings at different frequencies are followed.

There are two particular cases in which multirate digital control can be considered: cyclic sampling and sampling-action delay. In the first case, variables are sampled at irregular intervals, but with the existence of a global period T_0 with cyclic repetition. The latter case involves the existence of a delay in the same control loop between the process output sampling and the input signal updating. It is normal to consider a synchronous and periodic sampling. When samplers are not synchronised it is referred to asynchronous multirate systems.

Multirate digital control is an important research area. Its application can be presented in a wide range of situations. For instance:

- Time sharing computer by means of several detection services (Jury and Mullin, 1959).
- Aerospace applications (Halevi and Ray, 1988), robotic applications (Tsao and Hutchinson, 1994), chemical process control (Morant, *et al.*, 1986), computer hard disc control (Baek and Lee 1999).
- Missing and scarced data (Albertos, *et al.*, 1999).
- Distributed and multiprocessors control systems (Hovestädt, 1991).
- Real-time control systems (Salt, *et al.*, 2000a).
- Multivariable control systems (Velez, 2000).

Under these settings, the main difficulty is the design of a proper controller. In this contribution it is proposed an algebraic procedure inspired in classical methodology, and moreover, different conditions will be experimented in order to overcome this restricted kind of controllers for industrial environments and apply another type of controllers whose structure will be based on the introduced here. A LTI SISO process

is assumed and it must be considered that there is not different time scales in the plant as it was introduced by (Litkouhi and Khalil, 1985).

2. NOTATION

The basic multirate control scheme is shown in fig. 1, where the plant is represented by an n -th order single-input-single-output LTI continuous system (CT), with transfer function G_p . The controller output is updated at a period T through the fast hold device, H . The output is measured at period NT and compared to the reference $R(t)$, which is available at any time, the error being elaborated by the multirate controller $G_R^{T,NT}$. To simplify the computation, and without lose of generality, N is assumed to be integer.

The multirate controller can be implemented as a periodic controller. In this paper, three parts, as depicted in figure 1, compose the multirate controller as it was proved in (Salt and Albertos, 2000b). The low rate computed error is first processed by the subcontroller G_1^{NT} and passed through a rate converter $H^{T,NT}$. The second and faster subcontroller, G_2^T , finally provides the control input to the plant.

First, there is a single rate model for each one of the sampling periods (T and NT). The fast sampling DT model (FSDT) is defined by:

$$G^T(\bar{z}) = \frac{B^T(\bar{z})}{A^T(\bar{z})} = \frac{\sum_{i=1}^n b_{i,T} \bar{z}^{-i}}{1 + \sum_{i=1}^n a_{i,T} \bar{z}^{-i}} = \frac{Y^T(\bar{z})}{U^T(\bar{z})} \quad (1)$$

where \bar{z} is the T unit-delay shift operator. For future developments, the notation $A(\bar{z}) \stackrel{\Delta}{=} A^T$ is also used, denoting a polynomial on \bar{z} (or, as before, a T -spaced sequence). The DT transfer function poles are denoted by $\mathbf{a}_{i,T}$. That is:

$$A^T(\bar{z}) = \prod_{i=1}^n (\bar{z} - \mathbf{a}_{i,T}) \quad (2)$$

The slow sampling DT model (SSDT) is given by:

$$G^{NT}(z) = \frac{B^{NT}(z)}{A^{NT}(z)} = \frac{\sum_{i=1}^n b_{i,NT} z^{-i}}{1 + \sum_{i=1}^n a_{i,NT} z^{-i}} = \frac{Y^{NT}(z)}{U^{NT}(z)} \quad (3)$$

Again, a simpler notation will be $A^{NT}(z) \stackrel{\Delta}{=} A^{NT}$, denoting a polynomial on z (or an NT -spaced sequence). In this case the poles are denoted by $\mathbf{a}_{i,NT}$. That is:

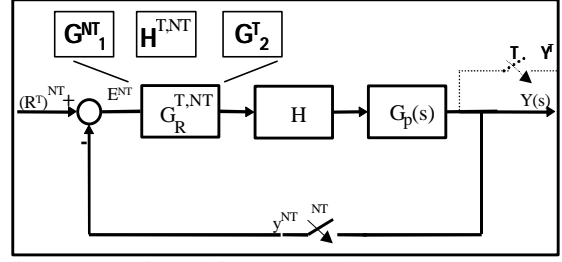


Figure 1. Basic multirate control loop.

$$A^{NT}(z) = \prod_{i=1}^n (z - \mathbf{a}_{i,NT}) \quad (4)$$

Note that, dealing with the same CT system, $\mathbf{a}_{i,NT} = \mathbf{a}_{i,T}^N; \forall i = 1, \dots, n$. Thus, the following useful relationship is derived:

$$W_A(\bar{z}) = \frac{\prod_{i=1}^n (\bar{z}^N - \mathbf{a}_{i,NT})}{\prod_{i=1}^n (\bar{z} - \mathbf{a}_{i,T})} \quad (5)$$

$$W_A(\bar{z}) = \prod_{i=1}^n (\bar{z}^{N-1} + \mathbf{a}_{i,T} \bar{z}^{N-2} + \dots + \mathbf{a}_{i,T}^{N-1}) \stackrel{\Delta}{=} \frac{A^{NT}}{A^T} \quad (6)$$

Similar polynomials $W(\cdot)$ can be obtained for any polynomial, other than A .

It is also possible to introduce dual-rate operators (DRDT) able to express the relationship between two differently time-spaced sequences. First, for input updating faster than output measurement sampling, the operator is:

$$G^{NT,T}(\bar{z}) = \frac{\bar{B}^T(\bar{z})}{A^{NT}(\bar{z}^N)} = \frac{\sum_{i=1}^{nN} \bar{b}_{i,T} \bar{z}^{-i}}{1 + \sum_{i=1}^n a_{i,NT} \bar{z}^{-iN}} \stackrel{\Delta}{=} \quad (7a)$$

$$\stackrel{\Delta}{=} \frac{\bar{B}^T}{A^{NT}} = \frac{Y^{NT}(\bar{z}^N)}{U^T(\bar{z})}$$

Based in the definition of $W(\cdot)$ it is possible to express:

$$G^T(\bar{z}) = \frac{Y^T(\bar{z})}{U^T(\bar{z})} = \frac{B^T(\bar{z})}{A^T(\bar{z})} = \frac{B^T(\bar{z})}{A^T(\bar{z})} \frac{W_A(\bar{z})}{W_A(\bar{z})} = \frac{\bar{B}^T(\bar{z})}{(A^{NT}(\bar{z}^N))^T} \quad (7b)$$

A special feature of this DRDT operator is the following: the numerator parameters of \bar{B}^T are distributed into n groups of N coefficients in such a way that the sum of each of these groups leads to the slow single-rate numerator coefficient. So, it is possible to obtain the SSDT model from the FSDT model (or from the DRDT model) (Albertos, *et al.*, 1996).

3. ALGEBRAIC DESIGN

3.1 Minimum time controllers.

The error signal at fast frequency can be expressed as

$$E^T = R^T - \bar{Y}^T \quad (8)$$

where \bar{Y}^T is the fast sequence of multirate-controlled process, that is $\bar{Y}^T = G_p^T U^T$, being $U^T = G_R^{T,NT} E^{NT}$ and the slow error sequence

$$E^{NT} = \frac{1}{1 + (G_p^T G_R^{T,NT})^{NT}} (R^T)^{NT} \quad (9)$$

For this reason and assuming the multirate loop of figure 1, it is obtained (Salt and Albertos, 2000b)

$$\bar{Y}^T = \frac{G_p^T G_R^{T,NT}}{1 + [(G_p^T G_R^{T,NT})^{NT}]^T} (R^T)^{NT} = \bar{M}^{T,NT} (R^T)^{NT} \quad (10)$$

where $\bar{M}^{T,NT}$ is a dual-rate operator, providing the output sampled at faster rate, $1/T$, and the input being updated at lower rate, $1/NT$.

Taking into account (Sklansky and Ragazzini, 1955)

$$\begin{aligned} (R^T)^{NT} &= Z[R(\bar{z})] = \\ &= \frac{1}{N} \left[R(\bar{z}) + R\left(\bar{z}e^{\frac{j2p}{N}}\right) + \dots + R\left(\bar{z}e^{\frac{j2p(N-1)}{N}}\right) \right] \end{aligned} \quad (11)$$

Substituting and reordering

$$\begin{aligned} E(\bar{z}) &= R(\bar{z}) \left[1 - \frac{1}{N} \tilde{M}(\bar{z}) \right] - \\ &- \frac{1}{N} \tilde{M}(\bar{z}) \left[R\left(\bar{z}e^{\frac{j2p}{N}}\right) + \dots + R\left(\bar{z}e^{\frac{j2p(N-1)}{N}}\right) \right] \end{aligned} \quad (12)$$

Minimum and finite time controllers are FIR (Finite Impulsional Response) controllers. For this reason, it can be expressed $\bar{M}^{T,NT}$ as \tilde{M}^T , which is denoted in the \bar{z} plane $\tilde{M}^T(\bar{z}) \stackrel{\Delta}{=} \tilde{M}(\bar{z})$.

Following (Kranc, 1957) if it is considered a reference of the way $R(s) = \frac{1}{s^R}$. Then $R(\bar{z})$ can be expressed as $\frac{P(\bar{z})}{(1 - \bar{z}^{-1})^R}$, where $P(\bar{z})$ is a

\bar{z}^{-1} polynomial, and $P(1) \neq 0$. Therefore (12) can be reformulated

$$\begin{aligned} E(\bar{z}) &= \frac{P(\bar{z})}{(1 - \bar{z}^{-1})^R} \left[1 - \frac{1}{N} \tilde{M}(\bar{z}) \right] - \\ &- \frac{1}{N} \tilde{M}(\bar{z}) \left[\frac{P\left(\bar{z}e^{\frac{j2p}{N}}\right)}{\left(1 - \bar{z}^{-1}e^{\frac{-j2p}{N}}\right)^R} + \dots + \frac{P\left(\bar{z}e^{\frac{j2p(N-1)}{N}}\right)}{\left(1 - \bar{z}^{-1}e^{\frac{-j2p(N-1)}{N}}\right)^R} \right] \end{aligned} \quad (13)$$

In order to make $E(\bar{z})$ of finite degree, it is necessary its expression has exact division (poles in the origin of \bar{z} plane). It is possible, if

- $\left[1 - \frac{1}{N} \tilde{M}(\bar{z}) \right]$ contains $(1 - \bar{z}^{-1})^R$ as a factor,

- $\tilde{M}(\bar{z})$ contains also as a factor

$$\left[\left(1 - \bar{z}^{-1}e^{\frac{-j2p}{N}}\right) \left(1 - \bar{z}^{-1}e^{\frac{-j4p}{N}}\right) \dots \left(1 - \bar{z}^{-1}e^{\frac{-j2p(N-1)}{N}}\right) \right]^R$$

This last expression is equal to

$$\left[\frac{1 - \bar{z}^{-N}}{1 - \bar{z}^{-1}} \right]^R = [1 + \bar{z}^{-1} + \dots + \bar{z}^{-(N-1)}]^R = W_R(\bar{z}^{-1}) \quad (14)$$

Therefore, it can be formulated the following diophantine equations to obtain a minimum time behaviour:

$$\tilde{M}(\bar{z}^{-1}) = W_R(\bar{z}^{-1}) \bar{z}^{-d} \Omega(\bar{z}^{-1}) \quad (15)$$

$$1 - M(z^{-1}) = (1 - z^{-1})^R \prod_j (1 - \mathbf{a}_{j,T}^N z^{-1}) \Lambda(z^{-1})$$

where

- $W_R(\bar{z}^{-1})$: polynomial which depends on the used reference, and has the following way:

$$W_R(\bar{z}^{-1}) = (1 + \dots + \bar{z}^{-(N-1)})^R$$

where N is the multiplicity and R the reference order. It is included to do the permanent error zero.

- d : difference between poles and zeros number of the process. It permits assuring the realizability.
- $(1 - \bar{z}^{-1})^R$: polynomial which depends on the used reference. It ensures that the polynomial $1 - M(z^{-1})$ has finite degree.

- $\prod_j (1 - \mathbf{a}_{j,T}^N z^{-1})$: poles of $G(z)$ process with module bigger or equal than 1. It permits assuring the stability.

3.2 Finite time controllers.

Two approaches appear in this case:

First approach:

$$\begin{aligned} \tilde{M}(\bar{z}^{-1}) &= W_R(\bar{z}^{-1})W_A^*(\bar{z}^{-1})\prod_i (1 - \mathbf{b}_{i,T}\bar{z}^{-1})\bar{z}^{-d}\Omega(\bar{z}^{-1}) \\ 1 - M(z^{-1}) &= (1 - z^{-1})^R \prod_j (1 - \mathbf{a}_{j,T}^N z^{-1})\Lambda(z^{-1}) \end{aligned} \quad (16)$$

It is practically equal than minimum time controller. The unique difference is, that now, it is beared in mind in $\tilde{M}(\bar{z}^{-1})$ outside zeros of unity circle, $\prod_i (1 - \mathbf{b}_{i,T}\bar{z}^{-1})$, to assure the stability. And also, $W_A^*(\bar{z}^{-1})$ takes into account only outside poles of unity circle; it is named as $W_A^*(\bar{z}^{-1})$ and not as $W_A(\bar{z}^{-1})$, because it is not considered all poles, but its construction is similar than $W_A(\bar{z}^{-1})$ (it is remembered in the following approach).

Second approach:

$$\begin{aligned} \tilde{M}(\bar{z}^{-1}) &= W_R(\bar{z}^{-1})\bar{B}(\bar{z}^{-1})\bar{z}^{-d}\Omega(\bar{z}^{-1}) \\ 1 - M(z^{-1}) &= (1 - z^{-1})^R \prod_j (1 - \mathbf{a}_{j,T}^N z^{-1})\Lambda(z^{-1}) \end{aligned} \quad (17)$$

This is a more complete approach, where:

- $W_R(\bar{z}^{-1})$: equal than in the first approach.
- $\bar{B}(\bar{z}^{-1})$: numerator of dual-rate operator. This term eliminates intersampling oscillations. As it is known, (see (7b)), $\bar{B}(\bar{z}^{-1}) = B^T W_A^T$, where B^T is the numerator of $G(\bar{z})$ process and W_A^T is constructed as $W_R(\bar{z}^{-1})$, that is

$$W_A(\bar{z}^{-1}) = \prod_{i=1}^n (1 + \dots + \mathbf{a}_{i,T}^{(N-1)} \bar{z}^{-(N-1)})^k$$

where N is the multiplicity, n the process poles number and k the multiplicity of $\mathbf{a}_{i,T}$ pole.

- d : difference between poles and zeros number of the process. It permits assuring the realizability.

- $(1 - z^{-1})^R$: polynomial which depends on the used reference. It ensures that the polynomial $1 - M(z^{-1})$ has finite degree.

- $\prod_j (1 - \mathbf{a}_{j,T}^N z^{-1})$: all process poles which are not considered in the reference term.

3.3 Resolution methodology.

In order to design a minimum or a finite time controller, it has to apply last equations and to study the order of the polynomials $\Omega(\bar{z}^{-1})$ and $\Lambda(z^{-1})$. Concretely, the order of $\Lambda(z^{-1})$ has to be as low as possible, but taking into account the following:

$$\tilde{m} = N \cdot m \quad (18)$$

where

$$\tilde{m} = \text{order}(\tilde{M}(\bar{z}^{-1})) \quad \text{and} \quad m = \text{order}(M(\bar{z}^{-1})).$$

Then it will be proposed the $\Lambda(z^{-1})$ polynomial, of λ degree, and later it will be obtained the degree q of $\Omega(\bar{z}^{-1})$.

Once diophantine equations are formulated, it is achieved

$$\begin{aligned} \tilde{M}(\bar{z}^{-1}) &= \tilde{m}_1 \bar{z}^{-1} + \dots + \tilde{m}_i \bar{z}^{-i} \\ 1 - M(z^{-1}) &= m_0 + m_1 z^{-1} + \dots + m_j z^{-j} \end{aligned} \quad (19)$$

where i and j depend on degrees of equations. Now an equations system can be constructed. It is obtained \tilde{m} equations from $\tilde{M}(\bar{z}^{-1})$, $m+1$ from $M(z^{-1})$ and m to do the link between both frequencies. This link establishes dual-rate closed loop and slow closed loop have to coincide in the kNT instants, where k is an integer. The following equation demonstrates it:

$$\left(\bar{Y}^T\right)^{NT} = \left(\bar{M}_N^T\right)^{NT} \left(R^T\right)^{NT} = M^{NT} R^{NT} = Y^{NT} \quad (20)$$

If the reference is a step, then

$$\begin{aligned} \left[M_N^T R^T\right]^{NT} &= \left[\frac{w_1 \bar{z}^{-1} + w_2 \bar{z}^{-2} + w_3 \bar{z}^{-3} + w_4 \bar{z}^{-4} + \dots}{1 - \bar{z}^{-1}} \right]^{NT} = \\ &= \left[w_1 \bar{z}^{-1} + (w_1 + w_2) \bar{z}^{-2} + (w_1 + w_2 + w_3) \bar{z}^{-3} + \dots \right. \\ &\quad \left. + (w_1 + w_2 + w_3 + w_4) \bar{z}^{-4} + \dots \right]^{NT} = \\ &= (w_1 + w_2) \bar{z}^{-2} + (w_1 + w_2 + w_3 + w_4) \bar{z}^{-4} + \dots \end{aligned} \quad (21)$$

Later, it is made equal coefficients of analog powers achieving the referred link:

$$\begin{aligned} w_1 + w_2 &= m_1 \\ w_1 + w_2 + w_3 + w_4 &= m_1 + m_2 \end{aligned} \quad (22)$$

If the reference is a ramp, proceeding similarly it is obtained the following link equations:

$$\begin{aligned} 2w_1 + w_2 &= m_1 \\ 4w_1 + 3w_2 + 2w_3 + w_4 &= 2m_1 + m_2 \end{aligned} \quad (23)$$

Finally, in order to solve the equations system, the number of variables has to be equal to the number of equations. If there are more variables than equations, it is had degrees of freedom; it can fix “*number(variables) - number(equations)*” equations freely to achieve design goals as it is usual in this digital design methodology.

4. EXAMPLE

An interesting industrial case is exposed in this section.

Authors of this paper have worked with a robot of high dimensions placed at Joint Research Centre of European Union in Ispra (Milan). Due to technical reasons, in this robotic environment there are two different work frequencies. On the one hand, it is considered the actuators period, which works at 5 ms. On the other hand, the samplers period, which works at 10 ms.

As it results obvious, it is necessary to implement a dual-rate controller to control the close loop of this process.

For this reason, it is presented the following multirate system where control actions at fast frequency are applied to compensate the slow frequency of the measurement system.

It will work with the system which is shown in figure 2. Anyway, the development is dually equivalent when it is worked with a faster measurement frequency than the control actions application frequency.

By means of Salt methodology (Salt, 1992) and (Salt and Albertos, 2000b) it can be split the controller into two subcontrollers, one working at slow frequency $1/NT$ and another working at fast frequency $1/T$. This new scheme is shown in figure 3. To simplify the computation, and without loss of generality, N is assumed to be integer.

The first thing to implement the controller is to identify the robot. Concretely, it is identified the X axis of the robot at 10 ms, being the resulting transfer function:

$$G(s) = \frac{-7.06}{s} e^{-0.01s} \quad (24)$$

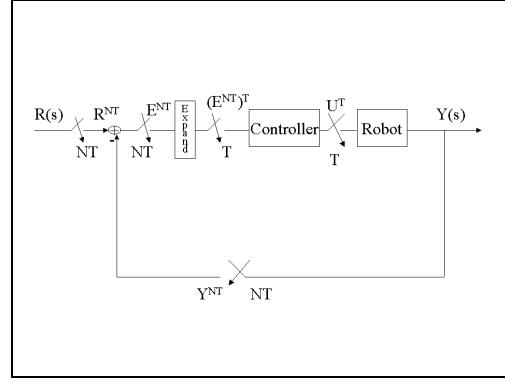


Figure 2. Basic multirate control scheme.

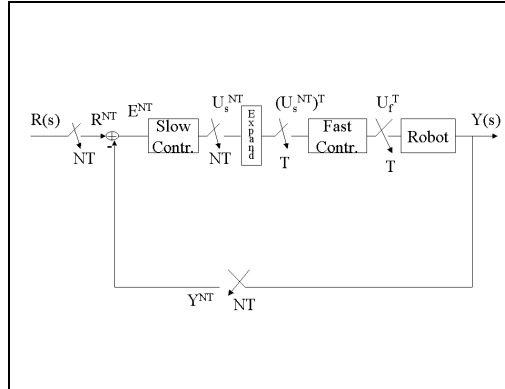


Figure 3. Slow/Fast multirate control scheme.

Later a finite time dual-rate controller has been designed. Results are presented next:

- Design parameters:

$$\begin{aligned} \text{Reference} &= \text{Ramp} \\ T &= 5 \text{ ms (fast period)} \\ N &= 2 \text{ (multiplicity)} \\ NT &= 10 \text{ ms (slow period)} \end{aligned}$$

- Transfer function of X axis discretized at fast period:

$$G(\bar{z}^{-1}) = \frac{-0.0353\bar{z}^{-3}}{1 - \bar{z}^{-1}} \quad (25)$$

- Diophantine equations to design the finite time dual-rate controller:

$$\begin{aligned} \tilde{M}(\bar{z}^{-1}) &= (1 + \bar{z}^{-1})^2 \cdot (-0.0353\bar{z}^{-3}) \cdot (1 + \bar{z}^{-1}) \cdot \\ &\cdot (\Omega_0 + \Omega_1\bar{z}^{-1} + \Omega_2\bar{z}^{-2}) \end{aligned} \quad (26)$$

$$1 - M(\bar{z}^{-1}) = (1 - \bar{z}^{-1})^2 \cdot (\Lambda_0 + \Lambda_1\bar{z}^{-1} + \Lambda_2\bar{z}^{-2})$$

- Fast and slow controllers have been obtained:

$$G_{SC} = \frac{1}{1 - 3\bar{z}^{-2} + 2\bar{z}^{-3}} \quad (27)$$

$$\begin{aligned} G_{FC} &= \frac{1.375 + 1.625\bar{z}^{-1} - 2.25\bar{z}^{-2} - \\ &- 2.75\bar{z}^{-3} + 0.875\bar{z}^{-4} + 1.125\bar{z}^{-5}}{-0.0353} \end{aligned} \quad (28)$$

- Graphical results of the simulation are shown in Figures 4 and 5:

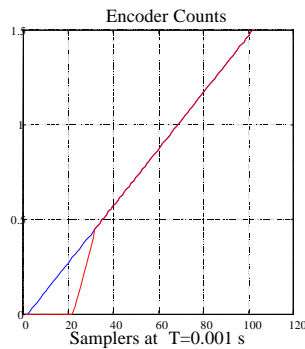


Figure 4. Reference vs. Position output.

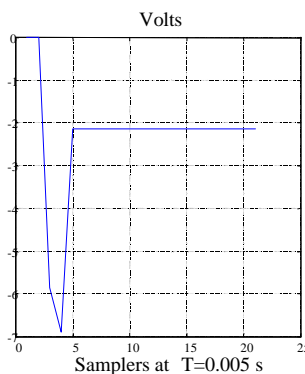


Figure 5. Control actions.

It can be observed that the position output follows accurately the ramp reference (figure 4). Only minimum errors are generated in the transitory (the biggest is approximately of 0.5 encoder counts). For this reason, it is achieved soft control actions, being the biggest approximately -7 Volts (figure 5).

More design examples can be found in (Albertos, *et al.*, 2000) and in (Salt, *et al.*, 2000a).

ACKNOWLEDGEMENTS

This work was supported by the Plan Nacional de I+D, Comisión Interministerial de Ciencia y Tecnología (CICYT), program TAP98-0252-C02-02.

REFERENCES

Albertos P., Salt J., Tornero J. (1996). Dual Rate Adaptive Control. *Automatica*, vol. AC-32, n. 7, pp. 1027-1030.

Albertos, P.; Sanchis, J; Sala, A. (1999). Output Prediction under scarce data operation: control applications. *Automatica*, vol. 35, n. 10, pp. 1671-1681.

Albertos P., Salt J., Cuenca A., Ibáñez A. (2000). Diseño Algebraico de Reguladores Bifrecuenciales. *XXI Jornadas de Automática*, Sevilla. (In Spanish)

Baek, S.;Lee, S. (1999). Design of Multi-Rate Estimator and its Application to a Disk Drive Servo System. *Proceedings of the American Control Conference*. San Diego.

Halevi, Y., Ray A. (1988). Integrated Communication and Control Systems. Part I - Analysis. *Journal of Dynamic Systems, Measurement and Control*, vol. 110, pp. 367-373.

Hovestädt, E. (1991). Multirate Control Algorithms for time-variable sampling periods. *IFAC Design Methods of Control Systems*, pp. 105-110. Zurich (Switzerland).

Jury, E.I.; Mullin, F.J. (1957). The analysis of sampled-data control systems with a periodically time varying sampling rate. *IRE Trans. on Automatic Control*, pp. 15-21.

Kranc G. (1957). Compensating Error-Sampled System by Multirate Controller. *Trans. AIEE*, vol. 6, M. 6, part II, pp.149-155.

Litkouhi, B.; Khalil, H. (1985). Multirate and Composite Control of Two-Time Scale Discrete-Time Systems. *IEEE Trans. on Automatic Control*, vol. AC-30, n. 7.

Morant, F., Albertos, P., Crespo, A. (1986). Oxide composition control in a raw material mill. *IFAC Symp. on Microprocessor in Control*. Istanbul.

Salt J. (1992). Control de Procesos Industriales con Muestreo No-convencional. *Tesis Doctoral Dpto. de Ingeniería de Sistemas y Automática Universidad Politécnica de Valencia*. (In Spanish)

Salt J., Valera A., Cuenca A., Ibáñez A. (2000a). Industrial Robot Multi-Rate Control with the VxWorks Real-Time Operating System. *IFAC AARTC'2000*, Palma Mallorca.

Salt J., Albertos P. (2000b). Multirate Controllers Design by Rate Decomposition, *Conference on Decision and Control*. Sydney.

Sklansky J., Ragazzini J.R. (1955). Analysis of errors in sampled-data feedback systems. *AIEE Trans.*, vol. 74, part II, pp. 65-71.

Tsao, T.; Hutchinson, S. (1994). Multi-rate Analysis and Design of Visual Feedback Digital Servo-Control. *Journal of Dynamic Systems Measurement and Control*, vol. 116, n. 1, pp. 45-55.

Vélez, C.M.; Salt, J. (2000). Simulation of Irregular Multirate Systems. *Proceedings of 8th Symposium on Computer Aided Control System Design*. Salford. United Kingdom.