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STATE AND PARAMETER ESTIMATION FOR NONLINEAR SYSTEMS

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Abstract: A *constructive* method is proposed for the design of nonlinear *adaptive observers* with *global convergence* for recursive joint estimation of states and parameters. It extends an earlier result to systems with a more general parametrization. The considered nonlinear systems are those typically considered for the design of high gain observers with additional terms involving unknown parameters. A numerical example is presented for illustration.

Keywords: nonlinear system, adaptive observer, high gain observer, state and parameter estimation.

1. INTRODUCTION

Recursive *joint estimation of states and parameters* in dynamic systems has important applications in fault detection and isolation (FDI) and in adaptive control. For this purpose, we consider the design of *adaptive observers* for a class of *truly nonlinear* systems (which cannot be linearized by coordinate change and output injection). Based on a recent result on adaptive observers for linear time varying (LTV) systems (Zhang and Delyon, 2001; Zhang, 2001) and on the techniques of *high gain observers* (Gauthier *et al.*, 1992; Gauthier and Kupka, 1994), a global adaptive observer for nonlinear systems has been proposed in (Zhang and Xu, 2001*b*). That algorithm is implicit, *i.e.*, in the form of differential-algebraic equations (DAE). It has been improved by (Zhang and Xu, 2001*a*) with an algorithm in the form of ordinary differential equations (ODE). However, the latter assumes that the number of unknown parameters is exactly equal to the number of states. *The main contribution of this paper* is to extend the result of (Zhang and Xu, 2001*a*) to the case with a more general parametrization involving an arbitrary number of parameters. The proposed method is *constructive*. Roughly speaking, if a high gain observer can be designed for a given nonlinear system, then it is possible to design an adaptive observer for the system obtained by adding into the state equation additional terms with unknown coefficients.

Some early works on adaptive observers for linear systems can be found in (Lüders and Narendra, 1973; Kreisselmeier, 1977). Adaptive observers for nonlinear systems then have drawn more attentions of researchers. For instance, results on global state and parameter estimation have been reported in (Bastin and Gevers, 1988; Marino and Tomei, 1995*b*; Marino and Tomei, 1995*a*). However, these results are restricted to the nonlinear systems whose dynamics can be linearized by coordinate change and output injection. Their applicability is limited by the restrictive linearization condition. More recently, some more general results on nonlinear systems have been published (Rajamani and Hedrick, 1995; Cho and Rajamani, 1997; Besançon, 2000). These methods do not require the considered nonlinear system to be linearizable, instead, they assume the existence of some Lyapounov function satisfying particular conditions. They are not constructive methods in the sense that there is no systematic way for the design of the required Lyapounov function.

The paper is organized as follows. In Section 2 formulate the considered problem. Section 3 recalls an earlier result and suggests an improvement. Section 4 is devoted to the new method. A simulation example is given in Section 5. Finally, the paper is concluded by Section 6.

2. PROBLEM STATEMENT

In this paper, we consider the joint estimation of the state vector $x(t)$ and the parameter vector θ in the system

$$\dot{x}(t) = A_o x(t) + f(x(t)) + g(x(t))u(t) + \Psi(t)\theta \quad (1a)$$
$$y(t) = c_o x(t) \quad (1b)$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^l$ and $y(t) \in \mathbb{R}$ are system state, input and output,

$$A_o = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \end{bmatrix} \qquad c_o = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} \quad (2)$$

$f : \mathbb{R}^n \to \mathbb{R}^n$ and $g : \mathbb{R}^n \to \mathbb{R}^n \times \mathbb{R}^l$ are two nonlinear functions in the triangular form:

$$f(x)=\begin{bmatrix} f_1(x_1) \\ f_2(x_1, x_2) \\ \vdots \\ f_n(x_1, \dots, x_n) \end{bmatrix} g(x)=\begin{bmatrix} g_1(x_1) \\ g_2(x_1, x_2) \\ \vdots \\ g_n(x_1, \dots, x_n) \end{bmatrix} \quad (3)$$

$\Psi(t) \in \mathbb{R}^n \times \mathbb{R}^p$ is a matrix of known signals, possibly depending on u and y, $\theta \in \mathbb{R}^p$ is an unknown constant parameter. It is assumed that $u(t)$ and $\Psi(t)$ are both uniformly bounded.

We assume that system (1) cannot be linearized by coordinate change and output injection, otherwise classical methods could be applied to the linearized system.

Note that if the term $\Psi(t)\theta$ is deleted from system (1), then the high gain observer of (Gauthier *et al.*, 1992) can be designed for state estimation (without unknown parameter).

The case with $p = n$ and a square diagonal matrix $\Psi(t)$ has been considered in (Zhang and Xu, 2001*a*). In that paper, the attempt to design an adaptive observer directly for system (1) has failed. Then the key idea to solve the problem has been to construct an adaptive observer for the extended system obtained by delaying the system variables in time. It has been proved that, the adaptive observer for the extended system converges globally. Following the same idea, in this paper, for the case with a general matrix $\Psi(t)$, we need to consider the same extended system constructed in the following.

For any integer k and some real constant $\Delta > 0$, define

$$x^k(t) = x(t - k\Delta)$$
$$u^k(t) = u(t - k\Delta)$$
$$y^k(t) = y(t - k\Delta)$$
$$\Psi^k(t) = \Psi(t - k\Delta)$$

Then these variables satisfy

$$\dot{x}^k(t) = A_o x^k(t) + f(x^k(t)) + g(x^k(t))u^k(t) + \Psi^k(t)\theta \quad (4a)$$
$$y^k(t) = c_o x^k(t) \quad (4b)$$

Consider the extended system

$$\begin{bmatrix} \dot{x}^0(t) \\ \dot{x}^1(t) \\ \vdots \\ \dot{x}^{m-1}(t) \end{bmatrix} = \begin{bmatrix} A_o & 0 & \cdots & 0 \\ 0 & A_o & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & A_o \end{bmatrix} \begin{bmatrix} x^0(t) \\ x^1(t) \\ \vdots \\ x^{m-1}(t) \end{bmatrix} + \begin{bmatrix} f(x^0(t)) \\ f(x^1(t)) \\ \vdots \\ f(x^{m-1}(t)) \end{bmatrix} + \begin{bmatrix} g(x^0(t))u^0(t) \\ g(x^1(t))u^1(t) \\ \vdots \\ g(x^{m-1}(t))u^{m-1}(t) \end{bmatrix} + \begin{bmatrix} \Psi^0(t) \\ \Psi^1(t) \\ \vdots \\ \Psi^{m-1}(t) \end{bmatrix} \theta \quad (5a)$$

$$\begin{bmatrix} y^0(t) \\ y^1(t) \\ \vdots \\ y^{m-1}(t) \end{bmatrix} = \begin{bmatrix} c_o & 0 & \cdots & 0 \\ 0 & c_o & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & c_o \end{bmatrix} \begin{bmatrix} x^0(t) \\ x^1(t) \\ \vdots \\ x^{m-1}(t) \end{bmatrix} \quad (5b)$$

Throughout this paper, the following notations and assumptions will be used.

For any positive real number ρ, define

$$\Lambda = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & \rho^{-1} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \rho^{-(n-1)} \end{bmatrix} \quad (6)$$

Let S be the positive definite solution of the matrix equation

$$A_o^T S + S A_o + S = c_o^T c_o \quad (7)$$

and

$$k_o = \frac{1}{2} S^{-1} c_o^T \quad (8)$$

Assumption 1. The functions $f(x)$ and $g(x)$ are globally Lipschitz.

Assumption 2. The input $u(t)$ and the matrix of signals $\Psi(t)$ are both bounded.

3. FROM DIAGONAL TO NON DIAGONAL Ψ

We first shortly recall the adaptive observer of (Zhang and Xu, 2001*a*) for the case with a diagonal matrix $\Psi(t)$, then explain how it should be modified to deal with the more general case. The electronically available manuscript (Zhang and Xu, 2001*a*) provides explanations on how the adaptive observer has been obtained by combining the linear adaptive observer of (Zhang, 2001) and the high gain observer of (Gauthier *et al.*, 1992).

3.1 *Recall of the case with diagonal* $\Psi(t)$

In this subsection we consider the case with $p = n$ and a square diagonal $\Psi(t) \in \mathbb{R}^n \times \mathbb{R}^n$.

Assumption 3. Let $\Upsilon(t) \in \mathbb{R}^n \times \mathbb{R}^n$ be generated through the ODE

$$\dot{\Upsilon}(t) = \rho(A_o - k_o c_o)\Upsilon(t) + \rho\Psi(t) \qquad (9)$$

with some initial condition and some $\rho > 0$. Denote

$$\Upsilon^k(t) = \Upsilon(t - k\Delta) \qquad (10)$$

For a given integer $m \geq n$, assume that there exits a constant $\alpha > 0$ such that, for all t, the following inequality holds:

$$\sum_{k=0}^{m-1} \Upsilon^{k^T}(t) c_o^T c_o \Upsilon^k(t) \geq \alpha I \qquad (11)$$

Remark 1. Due to the stability of the matrix $\rho(A_o - k_o c_o)$, the boundedness of of $\Psi(t)$ implies the boundedness of $\Upsilon(t)$. If the time scale is changed with $s = \rho t$, equation (9) becomes

$$\frac{d\Upsilon}{s} = (A_o - k_o c_o)\Upsilon + \Psi$$

It then becomes clear that the upper bound of Υ is independent of ρ. □

Theorem 1. Consider the case with $p = n$ and $\Psi(t) \in \mathbb{R}^n \times \mathbb{R}^n$ being a square diagonal matrix. Let $\Gamma \in \mathbb{R}^n \times \mathbb{R}^n$ be any symmetric positive definite matrix. Under Assumptions 1, 2 and 3, for sufficiently large $\rho > 0$, the ODE system for $k = 0, \ldots, m-1$

$$\begin{aligned}\dot{\hat{x}}^k(t) &= A_o \hat{x}^k(t) + f(\hat{x}^k(t)) + g(\hat{x}^k(t))u^k(t) \\ &+ \Psi^k(t)\hat{\theta}(t) + \rho\Lambda^{-1}k_o[y^k(t) - c_o\hat{x}^k(t)] \\ &+ \Lambda^{-1}\Upsilon^k(t)\Gamma \sum_{i=0}^{m-1} \Upsilon^{i^T}(t) c_o^T [y^i(t) - c_o \hat{x}^i(t)] \qquad (12a)\end{aligned}$$

$$\dot{\hat{\theta}}(t) = \rho\Lambda^{-1}\Gamma \sum_{i=0}^{m-1} \Upsilon^{i^T}(t) c_o^T [y^i(t) - c_o\hat{x}^i(t)] \qquad (12b)$$

with k_o as in (8) and Λ as in (6), is a global adaptive observer for the extended system (5), *i.e.*, for any initial conditions $x(t_0), \hat{x}^k(t_0)$, $\hat{\theta}(t_0)$ and for all $\theta \in \mathbb{R}^n$, the errors $\hat{x}^k(t) - x^k(t)$ and $\hat{\theta}(t) - \theta$ tend to zero when $t \to \infty$. □

Note that there are m state estimation equations with $k = 0, 1, \ldots, m-1$, each being of dimension n. These ODEs are all coupled through the last term. See (Zhang and Xu, 2001*a*) for a proof of Theorem 1.

In order to provide some heuristics about the modification of the algorithm for the case with non diagonal $\Psi(t)$, let us shortly recall part of the proof of Theorem 1. As the purpose here is to get some heuristics, we only consider the simplest case with $m = 1$. Then the extended system (5) reduces to system (1), and the adaptive observer (12) becomes

$$\begin{aligned}\dot{\Upsilon}(t) &= \rho(A_o - k_o c_o)\Upsilon(t) + \rho\Psi(t) \qquad (13a)\\ \dot{\hat{x}}(t) &= A_o\hat{x}(t) + f(\hat{x}(t)) + g(\hat{x}(t))u + \Psi(t)\hat{\theta}(t) \\ &+\Lambda^{-1}\left[\rho k_o + \Upsilon(t)\Gamma\Upsilon^T(t)c_o^T\right][y(t) - c_o\hat{x}(t)] \qquad (13b)\\ \dot{\hat{\theta}}(t) &= \rho\Lambda^{-1}\Gamma\Upsilon^T(t)c_o^T[y(t) - c_o\hat{x}(t)] \qquad (13c)\end{aligned}$$

Combine (13b) and (13c) to obtain

$$\begin{aligned}\dot{\hat{x}} &= A_o\hat{x} + f(\hat{x}) + g(\hat{x})u + \Psi\hat{\theta} \\ &+ \rho\Lambda^{-1}k_o(y - c_o\hat{x}) + \Lambda^{-1}\Upsilon\rho^{-1}\Lambda\dot{\hat{\theta}}\end{aligned}$$

Let $\tilde{x} = \hat{x} - x$, $\tilde{\theta} = \hat{\theta} - \theta$ and notice that $\dot{\theta} = 0$, then

$$\begin{aligned}\dot{\tilde{x}} &= (A_o - \rho\Lambda^{-1}k_o c_o)\tilde{x} + f(\hat{x}) - f(x) \\ &+ g(\hat{x})u - g(x)u + \Psi\tilde{\theta} + \Lambda^{-1}\Upsilon\rho^{-1}\Lambda\dot{\tilde{\theta}}\end{aligned}$$

It is easy to check that $\Lambda A_o = \rho A_o \Lambda$ and $c_o\Lambda = c_o$. Define $\tilde{z} = \Lambda\tilde{x}$ and $\tilde{\vartheta} = \rho^{-1}\Lambda\tilde{\theta}$, then

$$\dot{\tilde{z}} = \rho(A_o - k_o c_o)\tilde{z} + \xi + \rho\Psi\tilde{\vartheta} + \Upsilon\dot{\tilde{\vartheta}} \qquad (14)$$

where

$$\begin{aligned}\xi &= \Lambda[f(\Lambda^{-1}\hat{z}) - f(\Lambda^{-1}z)] \\ &+ \Lambda[g(\Lambda^{-1}\hat{z}) - g(\Lambda^{-1}z)]u \qquad (15)\end{aligned}$$

Note that we have used the fact that Ψ is diagonal and thus $\Lambda\Psi = \Psi\Lambda$.

Now define

$$\eta(t) = \tilde{z}(t) - \Upsilon(t)\tilde{\vartheta}(t)$$

then we get

$$\begin{aligned}\dot{\eta} &= \rho(A_o - k_o c_o)(\eta + \Upsilon\tilde{\vartheta}) + \xi + \rho\Psi\tilde{\vartheta} - \dot{\Upsilon}\tilde{\vartheta} \\ &= \rho(A_o - k_o c_o)\eta + \xi \\ &+ [\rho(A_o - k_o c_o)\Upsilon + \rho\Psi - \dot{\Upsilon}]\tilde{\vartheta} \qquad (16)\end{aligned}$$

Because Υ is generated by (13a), we have simply

$$\dot{\eta} = \rho(A_o - k_o c_o)\eta + \xi \qquad (17)$$

This simplification is an important step in the proof of Theorem 1. The diagonality of $\Psi(t)$ has been used to to achieve this simplification.

3.2 *Towards non diagonal* $\Psi(t)$

Let us first consider the case with a square non diagonal $\Psi(t)$. Then, equation (14) becomes

$$\dot{\tilde{z}} = \rho(A_o - k_o c_o)\tilde{z} + \xi + \rho\Lambda\Psi\Lambda^{-1}\tilde{\vartheta} + \Upsilon\dot{\tilde{\vartheta}}$$

Consequently, equation (16) becomes

$$\begin{aligned}\dot{\eta} &= \rho(A_o - k_o c_o)\eta + \xi \\ &\quad + [\rho(A_o - k_o c_o)\Upsilon + \rho\Lambda\Psi\Lambda^{-1} - \dot{\Upsilon}]\tilde{\vartheta}\end{aligned}$$

In this case, in order to achieve the simplification (17), equation (13a) of the adaptive observer should be modified as

$$\dot{\Upsilon}(t) = \rho(A_o - k_o c_o)\Upsilon(t) + \rho\Lambda\Psi\Lambda^{-1} \qquad (18)$$

It is desired that the $\Upsilon(t)$ has an upper bound independent of ρ. It is the case when $\Psi(t)$ is diagonal, as explained in Remark 1. Unfortunately, this behavior of $\Upsilon(t)$ cannot be guaranteed by (18) in general. For this reason, we will replace the matrix Λ^{-1} in the last term of (18) by another matrix Ω which depends on ρ and is chosen so that the upper bound of $\Upsilon(t)$ is independent of ρ. Moreover, Ω does not necessarily has the same size as Λ, so that the case of non square $\Psi(t)$ can be handled. The detailed solution is presented in the next section.

4. THE NEW ADAPTIVE OBSERVER

Assumption 4. Let $\Upsilon(t) \in \mathbb{R}^n \times \mathbb{R}^p$ be generated through the ODE

$$\dot{\Upsilon}(t) = \rho(A_o - k_o c_o)\Upsilon(t) + \rho\Lambda\Psi(t)\Omega \qquad (19)$$

where $\rho > 0$ and $\Omega \in \mathbb{R}^p \times \mathbb{R}^p$ is an invertible square matrix depending on ρ such that, for large ρ, the upper bound of $\Upsilon(t)$ is independent of ρ. Denote

$$\Upsilon^k(t) = \Upsilon(t - k\Delta) \qquad (20)$$

For a given integer $m \geq p$, assume that there exits a constant $\alpha > 0$ such that, for all t, the following inequality holds:

$$\sum_{k=0}^{m-1} \Upsilon^{k^T}(t) c_o^T c_o \Upsilon^k(t) \geq \alpha I \qquad (21)$$

Remark 2. In order to ensure that, for large ρ, the upper bound of $\Upsilon(t)$ is independent of ρ, it is sufficient to consider a diagonal matrix

$$\Omega = \begin{bmatrix} \rho^{q_1} & 0 & \cdots & 0 \\ 0 & \rho^{q_2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \rho^{q_p} \end{bmatrix}$$

where $q_1, q_2, \ldots, q_p$ are integers to be chosen. Denote

$$\Psi = \begin{bmatrix} \psi_{1,1} & \psi_{1,2} & \cdots & \psi_{1,p} \\ \psi_{2,1} & \psi_{2,2} & \cdots & \psi_{2,p} \\ \vdots & \vdots & \vdots & \vdots \\ \psi_{n,1} & \psi_{n,2} & \cdots & \psi_{n,p} \end{bmatrix}$$

Then

$$\Lambda\Psi\Omega = \begin{bmatrix} \rho^{q_1}\psi_{1,1} & \rho^{q_2}\psi_{1,2} & \cdots & \rho^{q_p}\psi_{1,p} \\ \rho^{q_1-1}\psi_{2,1} & \rho^{q_2-1}\psi_{2,2} & \cdots & \rho^{q_p-1}\psi_{2,p} \\ \vdots & \vdots & \vdots & \vdots \\ \rho^{q_1-(n-1)}\psi_{n,1} & \rho^{q_2-(n-1)}\psi_{n,2} & \cdots & \rho^{q_p-(n-1)}\psi_{n,p} \end{bmatrix}$$

Consider each column of the matrix $\Lambda\Psi\Omega$. In the j-th column, some of the signals $\psi_{i,j}$ may be zero. Choose the value of q_j such that, for the entries with non zero $\psi_{i,j}$ in the j-th column, the highest exponent of ρ is zero. By choosing the values of q_j in this manner for all the columns, the upper bound of the matrix $\Lambda\Psi\Omega$ is independent of ρ for large value of ρ.

Change the time scale with $s = \rho t$, then equation (19) becomes

$$\frac{d\Upsilon}{s} = (A_o - k_o c_o)\Upsilon + \Lambda\Psi\Omega$$

It follows that the upper bound of Υ is independent of ρ for large value of ρ. □

Theorem 2. Let $\Gamma \in \mathbb{R}^n \times \mathbb{R}^n$ be any symmetric positive definite matrix. Under Assumptions 1, 2 and 4, for sufficiently large $\rho > 0$, the ODE system for $k = 0, \ldots, m-1$

$$\begin{aligned}\dot{\hat{x}}^k(t) &= A_o\hat{x}^k(t) + f(\hat{x}^k(t)) + g(\hat{x}^k(t))u^k(t) \\ &\quad + \Psi^k(t)\hat{\theta}(t) + \rho\Lambda^{-1}k_o[y^k(t) - c_o\hat{x}^k(t)] \\ &\quad + \Lambda^{-1}\Upsilon^k(t)\Gamma\sum_{i=0}^{m-1}\Upsilon^{i^T}(t)c_o^T[y^i(t) - c_o\hat{x}^i(t)] \qquad (22a)\end{aligned}$$

$$\dot{\hat{\theta}}(t) = \rho\Omega\Gamma\sum_{i=0}^{m-1}\Upsilon^{i^T}(t)c_o^T[y^i(t) - c_o\hat{x}^i(t)] \qquad (22b)$$

is a global adaptive observer for the extended system (5), *i.e.*, for any initial conditions $x(t_0)$, $\hat{x}^k(t_0)$, $\hat{\theta}(t_0)$ and for all $\theta \in \mathbb{R}^n$, the errors $\hat{x}^k(t) - x^k(t)$ and $\hat{\theta}(t) - \theta$ tend to zero when $t \to \infty$. □

Note that the only differences of Theorem 2 from Theorem 1 are that Assumption 3 is replaced by assumption 4 and, in the last equation, Λ^{-1} is replaced by Ω.

Proof of Theorem 2 Combine (22a) and (22b):

$$\begin{aligned}\dot{\hat{x}}^k(t) &= A_o\hat{x}^k(t) + f(\hat{x}^k(t)) + g(\hat{x}^k(t))u^k + \Psi^k(t)\hat{\theta}(t) \\ &\quad + \rho\Lambda^{-1}k_o[y^k(t) - c_o\hat{x}^k(t)] + \Lambda^{-1}\Upsilon^k(t)\rho^{-1}\Omega^{-1}\dot{\hat{\theta}}(t)\end{aligned}$$

Let $\tilde{x}^k(t) = \hat{x}^k(t) - x^k(t)$, $\tilde{\theta}(t) = \hat{\theta}(t) - \theta$ and notice that $\dot{\theta} = 0$, then

$$\dot{\tilde{x}}^k(t) = (A_o - \rho\Lambda^{-1}k_oc_o)\tilde{x}^k(t) + f(\hat{x}^k(t)) - f(x^k(t)) + g(\hat{x}^k(t))u^k - g(x^k(t))u^k + \Psi^k(t)\tilde{\theta}(t) + \Lambda^{-1}\Upsilon^k(t)\rho^{-1}\Omega^{-1}\dot{\tilde{\theta}}(t)$$

Define

$$z^k(t) = \Lambda x^k(t)$$
$$\hat{z}^k(t) = \Lambda\hat{x}^k(t)$$
$$\tilde{z}^k(t) = \Lambda\tilde{x}^k(t)$$
$$\tilde{\vartheta}(t) = \rho^{-1}\Omega^{-1}\tilde{\theta}(t)$$

Then,

$$\dot{\tilde{z}}^k(t) = \rho(A_o - k_oc_o)\tilde{z}^k(t) + \xi^k(t) + \rho\Lambda\Psi^k(t)\Omega\tilde{\vartheta}(t) + \Upsilon^k(t)\dot{\tilde{\vartheta}}(t)$$

with

$$\xi^k(t) = \Lambda[f(\Lambda^{-1}\hat{z}^k(t)) - f(\Lambda^{-1}z^k(t))] + \Lambda[g(\Lambda^{-1}\hat{z}^k(t)) - g(\Lambda^{-1}z^k(t))]u^k(t) \quad (23)$$

Now define

$$\eta^k(t) = \tilde{z}^k(t) - \Upsilon^k(t)\tilde{\vartheta}(t)$$

then we get

$$\begin{aligned}\dot{\eta}^k(t) &= \rho(A_o - k_oc_o)[\eta^k(t) + \Upsilon^k(t)\tilde{\vartheta}(t)] + \xi^k(t) + \rho\Lambda\Psi^k(t)\Omega\tilde{\vartheta}(t) - \dot{\Upsilon}^k(t)\tilde{\vartheta}(t)\\ &= \rho(A_o - k_oc_o)\eta^k(t) + \xi^k(t) + [\rho(A_o - k_oc_o)\Upsilon^k(t) + \rho\Lambda\Psi^k(t)\Omega - \dot{\Upsilon}^k(t)]\tilde{\vartheta}(t)\end{aligned}$$

Recall that $\Upsilon(t)$ is generated by (19) and $\Upsilon^k(t)$ is defined by (20), thus $\Upsilon^k(t)$ satisfies

$$\dot{\Upsilon}^k(t) = \rho(A_o - k_oc_o)\Upsilon^k(t) + \rho\Lambda\Psi^k(t)\Omega$$

Therefore, we obtain

$$\dot{\eta}^k(t) = \rho(A_o - k_oc_o)\eta^k(t) + \xi^k(t)$$

The definition of $\tilde{\vartheta}(t)$ implies $\dot{\tilde{\vartheta}} = \rho^{-1}\Omega^{-1}\dot{\tilde{\theta}}(t)$. Then, from (22b) and the fact that $\dot{\theta} = 0$, we obtain

$$\begin{aligned}\dot{\tilde{\vartheta}}(t) &= \Gamma\sum_{i=0}^{m-1}\Upsilon^{i^T}(t)c_o^T[y^i(t) - c_o\hat{x}^i(t)]\\ &= -\Gamma\sum_{i=0}^{m-1}\Upsilon^{i^T}(t)c_o^Tc_o\tilde{x}^i(t)\\ &= -\Gamma\sum_{i=0}^{m-1}\Upsilon^{i^T}(t)c_o^Tc_o\tilde{z}^i(t)\\ &= -\Gamma\sum_{i=0}^{m-1}\Upsilon^{i^T}(t)c_o^Tc_o[\eta^i(t) + \Upsilon^i(t)\tilde{\vartheta}(t)]\end{aligned}$$

Putting together the equations of η and $\tilde{\vartheta}$ yields

$$\dot{\eta}^k(t) = \rho(A_o - k_oc_o)\eta^k(t) + \xi^k(t) \quad (24a)$$

$$\dot{\tilde{\vartheta}}(t) = -\Gamma\sum_{i=0}^{m-1}\Upsilon^{i^T}(t)c_o^Tc_o[\eta^i(t) + \Upsilon^i(t)\tilde{\vartheta}(t)] \quad (24b)$$

Consider the Lyapounov function candidate

$$V(t) = \sum_{k=0}^{m-1}\eta^{k^T}(t)S\eta^k(t) + \tilde{\vartheta}^T(t)\Gamma^{-1}\tilde{\vartheta}(t)$$

$$\begin{aligned}\frac{dV(t)}{dt} &= -\rho\sum_{k=0}^{m-1}\eta^{k^T}(t)S\eta^k(t) + 2\sum_{k=0}^{m-1}\eta^{k^T}(t)S\xi^k(t) - 2\tilde{\vartheta}^T(t)\sum_{i=0}^{m-1}\Upsilon^{i^T}(t)c_o^Tc_o[\eta^i(t) + \Upsilon^i(t)\tilde{\vartheta}(t)]\\ &= 2\sum_{k=0}^{m-1}\eta^{k^T}(t)S\xi^k(t) - \sum_{k=0}^{m-1}\left(\begin{bmatrix}\eta^k(t)\\ \tilde{\vartheta}(t)\end{bmatrix}^T M_k(\rho)\begin{bmatrix}\eta^k(t)\\ \tilde{\vartheta}(t)\end{bmatrix}\right)\end{aligned} \quad (25)$$

with

$$M_k(\rho) = \begin{bmatrix}\rho S & c_o^Tc_o\Upsilon^k(t)\\ \Upsilon^{k^T}(t)c_o^Tc_o & \frac{2}{m}\sum_{i=0}^{m-1}\Upsilon^{i^T}(t)c_o^Tc_o\Upsilon^i(t)\end{bmatrix}$$

The matrix S has been designed to be positive definite. By Assumption 4, the matrix

$$\frac{2}{m}\sum_{i=0}^{m-1}\Upsilon^{i^T}(t)c_o^Tc_o\Upsilon^i(t)$$

is positive definite and uniformly bounded from below. Moreover, $\Upsilon^k(t)$ is uniformly bounded from above. It follows that, for sufficiently large ρ, the matrix $M_k(\rho)$ is positive definite.

Because f and g are globally Lipschitz and have the triangular form, it can be shown that ξ^k as defined in (23) satisfies the inequality

$$\begin{aligned}\|\xi^k(t)\| &\leq \kappa(\rho^{-1})\|\tilde{z}^k(t)\|\\ &= \kappa(\rho^{-1})\|\eta^k(t) + \Upsilon^k(t)\tilde{\vartheta}(t)\|\end{aligned}$$

with $\kappa(\rho^{-1}) > 0$ a polynomial in ρ^{-1} depending on the Lipschitz constants of the functions f and g and on the bound of u. Therefore, there exist polynomials $\mu_1(\rho^{-1})$ and $\mu_2(\rho^{-1})$ such that

$$\eta^{k^T}(t)S\xi^k(t) \leq \mu_1(\rho^{-1})\|\eta^k(t)\|^2 + \mu_2(\rho^{-1})\|\eta^k(t)\|\cdot\|\tilde{\vartheta}(t)\|$$

It follows that

$$\frac{dV(t)}{dt} \leq 2\sum_{k=0}^{m-1}\Big(\mu_1(\rho^{-1})\|\eta^k(t)\|^2 + \mu_2(\rho^{-1})\|\eta^k(t)\|\cdot\|\tilde{\vartheta}(t)\|\Big) - \sum_{k=0}^{m-1}\left(\begin{bmatrix}\eta^k(t)\\ \tilde{\vartheta}(t)\end{bmatrix}^T M_k(\rho)\begin{bmatrix}\eta^k(t)\\ \tilde{\vartheta}(t)\end{bmatrix}\right)$$

The first sum at the right hand side of the inequality does not increase with ρ. For sufficiently large ρ, each term in the second sum is positive definite and increases linearly with ρ. Therefore, for sufficiently large ρ, $dV(t)/dt$ is negative definite. We can then conclude that $\eta^k(t) \to 0$ and $\tilde{\vartheta}(t) \to 0$, and thus $\tilde{z}^k(t) = \eta^k(t) + \Upsilon^k(t)\tilde{\vartheta}(t) \to 0$. It follows that $\tilde{x}^k(t) \to 0$ and $\tilde{\theta}(t) \to 0$. □

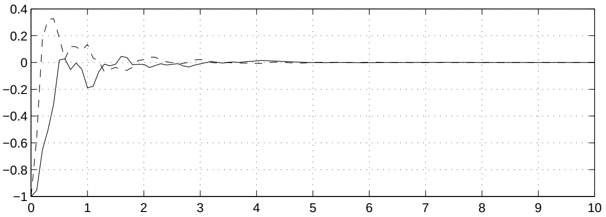

Figure 1. Simulation example: state estimation errors $\tilde{x}_1(t)$ (solid line) and $\tilde{x}_2(t)$ (dashed line).

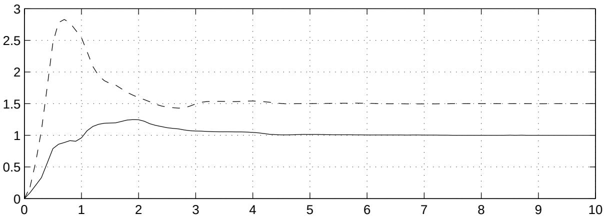

Figure 2. Simulation example: parameter estimates $\hat{\theta}_1(t)$ (solid line) and $\hat{\theta}_2(t)$ (dashed line). The true parameter values are $\theta_1 = 1$ and $\theta_2 = 1.5$.

5. NUMERICAL EXAMPLE

In this section we show a simulation example of the proposed adaptive observer. The simulated system is

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) + \psi_{1,1}(t)\theta_1 \\ \dot{x}_2(t) &= 10 \sin x_2(t) + \psi_{2,1}(t)\theta_1 + \psi_{2,2}(t)\theta_2 \\ y(t) &= x_1(t) \end{aligned}$$

with the parameters $\theta_1 = 1$, $\theta_2 = 1.5$. The excitation signals are $\psi_{1,1}(t) = \sin 2t + \cos 20t$, $\psi_{2,1}(t) = \sin 5t + \cos 16t$ and $\psi_{2,2}(t) = \sin 3t + \cos 27t$. Note that, in principle, adding some nonlinear functions of x_1 into the state equations would not increase any difficulty, since x_1 is directly measured by y.

The parameters of the adaptive observer are: $m = 5$, $\Delta = 0.1$, $\rho = 3$, $\Gamma = \text{diag}([3, 0.5])$, $\Omega = \text{diag}([1, 3])$. The initial values are $\hat{x}^k(0) = [0, 0]^T$, $\hat{\theta}(0) = [0, 0]^T$.

The state estimation errors are plotted in Figure 1, and the parameter estimates in Figure 2. The convergence of the state and parameter estimation is practically achieved at about the 5th second.

6. CONCLUSION

We have proposed a method for the joint estimation of state and parameters for a class of nonlinear systems. The method is *constructive* and guarantees *global convergence* under appropriate assumptions. It extends an earlier result to systems with a more parametrization.

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