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MAXIMIZING PERFORMANCE AND ROBUSTNESS OF PI AND PID CONTROLLERS BY GLOBAL OPTIMIZATION

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Abstract: The paper reports a new algorithm for global constrained optimization and its application to the design of PI and PID controllers. The algorithm is described in detail and the features which make it suited for controller design are emphasized. Various design criteria and constraints are considered. The numerical results show good performance in all tests: its flexibility and ease of use make it an alternative to more classical design procedures.

Keywords: PI/PID parameter tuning, Global Optimization.

1. INTRODUCTION

Several recent papers dealing with design methods for PI and PID controllers witness the lasting interest on this subject. Two main lines of research are present.

The first, in the spirit of Ziegler and Nichols work, leads to formulas relating the parameters of the step response of the process to the parameters of the controller. See, for instance, Åström and Hagglund (1995a), Ho *et al.* (1999).

In the second approach the controller parameters are found solving a constrained optimization problem: given the transfer function of the plant, find a controller that minimizes a functional of the error due to a load disturbance, under constraints that ensure robustness of the overall system. Along this line Åström *et al.* (1998) propose a new design strategy for PI controllers: the Integral Error (IE) due to a step load disturbance is minimized under a constraint on the peak value of the sensitivity function. The method is elegant and effective, but it relies on the special choice of the performance index and the structure of the sensitivity constraint, so that the results and the solution algorithm cannot be extended to other integral indices, to different constraints, to the PID controller.

In this paper PI and PID controllers are tuned using a new Global Optimization method, thus retaining the idea of Åström *et al.* (1998), but taking advantage of its generality and flexibility.

Global Optimization (GO) algorithms such as Branch and Bound, Genetic, Simulated Annealing, Randomized, have been used extensively in the past decade to solve problems arising in systems analysis and design. The algorithm here proposed belongs to the family of *information algorithms* (Strogin, 1989). It is an extension to the multivariable case of the algorithm presented in Sergeyev and Markin (1995) for constrained single variable problems. A convergence proof is given in Sergeyev *et al.* (2001).

After describing in details the algorithm and the design problems, several numerical experiments are reported and discussed. The experiments involve: a) the reproduction of several examples on PI design from Åström *et al.* (1998), to compare the respective results; b) the solution of new PI design problems: minimization of the IE with a constraint on the phase margin, minimization of the Integral Squared Error (ISE) of the output to a step load disturbance with a constraint either on the sensitivity or on the phase margin; c) the solution of similar problems for PID controllers.

2. THE OPTIMIZATION ALGORITHM

Consider the problem

$$\begin{array}{cc} \min\limits_{p \in P} & J(p) \\ \text{subject to: } \{F_i(p) \le 0,\ i = 1, \ldots, m, & \end{array}$$

where P is a hyper-rectangle of $\mathbf{R}^n$ and the functions $J(\cdot)$ and $F_i(\cdot)$ are assumed Lipschitz continuous over P. The first step of the proposed method transforms the n-dimensional problem into a one-dimensional one. Following Strongin (1989), this is accomplished using space-filling fractal Peano curves, that establish a continuous mapping $p(\cdot)$ from the real interval $I = [0, 1]$ to the hyper-rectangle P. Any Lipschitz continuous function $f(\cdot)$ from P to $\mathbf{R}$ can thus be transformed into a continuous function $\phi(\cdot)$ from I to $\mathbf{R}$ by the composite mapping $\phi(x) = f(p(x)),\ x \in I$.

Let $f_i(x) = F_i(p(x)),\ i = 1, \ldots, m,\ f_{m+1}(x) = J(p(x)),\ x \in I$; the original problem is replaced by:

$$\begin{array}{cc} \min\limits_{x \in I} & f_{m+1}(x) \\ \text{subject to: } \{f_i(x) \le 0,\ i = 1, \ldots, m. & \end{array}$$

To take constraints into account define the integer valued function $\nu(x)$ which gives the index of the first constraint not satisfied at x, i.e., $f_i(x) \le 0$ for $i = 1, \ldots, \nu(x) - 1$, $f_i(x) > 0$ for $i = \nu(x)$. If all the constraints are satisfied, set $\nu(x) = m + 1$.

Suppose that k iterations of the algorithm have been executed (initial iterations are done at $x^0 = 0$ and $x^1 = 1$). The choice of the point x^{k+1} is made by:

Step 1. The points $x^0, \ldots, x^k$ of the previous iterations are ordered and renumbered by subscripts

$$0 = x_0 < x_1 < \cdots < x_i < \cdots < x_k = 1.$$

Step 2. At each point x_i associate the index $\nu_i = \nu(x_i)$, and the value

$$z_i = f_{\nu_i}(x_i) - \begin{cases} 0 & \text{if } \nu_i < m + 1, \\ z_k^* & \text{if } \nu_i = m + 1, \end{cases}$$

where $z_k^* = \min\{f_{m+1}(x_i), \nu(x_i) = m + 1\}$ is an estimate of the minimum of $f_{m+1}(\cdot)$.

Step 3. Calculate lower bounds

$$\mu_j = \max_{0 \le q < p \le k} \left\{ \frac{|z_p - z_q|}{(x_p - x_q)^{1/n}}, \nu_p = \nu_q = j \right\},$$

$j = 1, \ldots, m + 1$, for the Hölder constant of $f_j(\cdot)$. Where μ_j can not be calculated, set $\mu_j = 0$.

Step 4. Let $\Delta_i = (x_i - x_{i-1})^{1/n}$; for each interval $(x_{i-1}, x_i), i = 1, \ldots, k$, calculate the values

$$M_i = \max\{\lambda_i, \gamma_i\},$$

that estimate the local Hölder constant over that interval. Values λ_i and γ_i reflect the influence on M_i of the local and global information obtained during the previous k iterations; they are

$$\lambda_i = \max\{l_i, c_i, r_i\},$$

$$c_i = \begin{cases} \dfrac{|z_i - z_{i-1}|}{\Delta_i}, & \text{if } \nu_i = \nu_{i-1} \\ 0 & \text{otherwise} \end{cases}$$

$$l_i = \begin{cases} \dfrac{|z_{i-1} - z_{i-2}|}{\Delta_{i-1}}, & \text{if } \begin{cases} i \ge 2 \\ \nu_{i-2} = \nu_{i-1} \\ \nu_{i-1} \ge \nu_i \end{cases} \\ 0 & \text{otherwise} \end{cases}$$

$$r_i = \begin{cases} \dfrac{|z_{i+1} - z_i|}{\Delta_{i+1}}, & \text{if } \begin{cases} i \le k - 1 \\ \nu_{i+1} = \nu_i \\ \nu_i \ge \nu_{i-1} \end{cases} \\ 0, & \text{otherwise} \end{cases}$$

$$\gamma_i = \frac{\mu_j\,\Delta_i}{X_j^{\mathrm{m}}}, j = \max\{\nu_i, \nu_{i-1}\}$$

$$X_j^{\mathrm{m}} = \max_{1 \le i \le k} \{\Delta_i : \max\{\nu_i, \nu_{i-1}\} = j\}.$$

If $M_i < \xi$ set $M_i = \xi$, where $\xi > 0$ reflects our supposition that the functions $f_j(x)$ are not constant over the interval (x_{i-1}, x_i).

Step 5. For each interval $(x_{i-1}, x_i), i = 1, \ldots, k$, calculate the *characteristic of the interval*

$$R_i = \begin{cases} rM_i\Delta_i + \dfrac{(z_i - z_{i-1})^2}{rM_i\Delta_i} \\ \qquad - 2\,(z_i + z_{i-1}), & \nu_i = \nu_{i-1} \\ 2\,rM_i\Delta_i - 4\,z_i, & \nu_i > \nu_{i-1} \\ 2\,rM_i\Delta_i - 4\,z_{i-1}, & \nu_{i-1} > \nu_i \end{cases}$$

where $r > 1$ is a real value, denoted as the *reliability parameter* of the method.

Step 6. Compute the point

$$x^{k+1} = -\frac{\mathrm{sign}(z_t - z_{t-1})}{2\,r}\,\frac{|z_t - z_{t-1}|^n}{M_t^n} + \frac{x_t + x_{t-1}}{2}, \quad \text{if } \nu_t = \nu_{t-1}$$

$$x^{k+1} = \frac{x_t + x_{t-1}}{2}, \qquad \text{otherwise}$$

where $t = \min\{\arg\max\{R_i, i = 1, \ldots, k\}\}$.

Step 7. If $(x_t - x_{t-1})^{1/n} > \varepsilon$ go to **Step 1**, else set $x^* = \arg\min\{f_{m+1}(x_i), \nu(x_i) = m + 1\}$.

The estimate of the minimizer is given by $p^* = p(x^*)$.

Remark 1 The tolerance ε must be larger than n-th root of the machine precision: this reflects the change of metric induced by the Peano transformation. Parameter ξ is not critical, unless 'flat' functions occur. As for the reliability parameter r, it has been shown (Sergeyev *et al.*, 2001) that the sequence generated by the algorithm converges to the solution of the problem for sufficiently large r for any given Lipschitz problem: however, large r may cause a slow convergence.

Remark 2 The ordering of the constraints is important: an appropriate hierarchy may save computation time, and may allow to relax the hypothesis that all the constraints are Lipschitz continuous in the box P.

3. THE DESIGN PROBLEM

A two degrees of freedom PID (Åström and Hägglund, 1995b) with derivative action on the filtered output and weight b on the set-point is considered (Fig. 1).

Let $G_\mathrm{c} = k_\mathrm{p} + k_\mathrm{i}/s + G_\mathrm{d}$, $S = 1/(1 + GG_\mathrm{c})$ (sensitivity), $T = 1 - S$ (complementary sensitivity), $e_y(t)$ the error due to a step load disturbance $l_\mathrm{d}(t)$ and

$$\mathrm{IE} = \int_0^\infty e_y(t)dt = 1/k_\mathrm{i},$$
$$\mathrm{ISE} = \int_0^\infty e_y^2(t)dt = \|E_y\|_2^2.$$

3.1 *PI design*

The first test reproduces some experiments from Åström *et al.* (1998); the optimization problem is:

- PB 1-a – given M_s, find $k_\mathrm{p}, k_\mathrm{i}$ such that:
  1) the closed loop system is stable;
  2) $\|S\|_\infty \le M_\mathrm{s}$;
  3) IE is minimum.

Then the flexibility of the algorithm is tested in two directions: measuring the performance by the ISE index, and constraining the phase margin m_φ:

- PB 1-b – given M_s, find $k_\mathrm{p}, k_\mathrm{i}$ such that:
  1) the closed loop system is stable;
  2) $\|S\|_\infty \le M_\mathrm{s}$;
  3) ISE is minimum.
- PB 2-a,b – find $k_\mathrm{p}, k_\mathrm{i}$ such that:
  1) the closed loop system is stable;
  2) $m_\varphi \ge 60°$;
  3) either IE is minimum or ISE is minimum.

The above problems do not depend on b. Four rational and delay test processes are considered:

$$G_1(s) = \frac{1}{(s+1)^3},$$
$$G_2(s) = \frac{9}{(s+1)(s^2+2s+9)},$$
$$G_3(s) = \frac{\exp(-15s)}{(s+1)^3},$$
$$G_4(s) = \frac{\exp(-s)}{s}.$$

The stability constraint is checked first. Then, if the system is stable the robustness constraint is computed. If this last is satisfied, the criterion is evaluated.

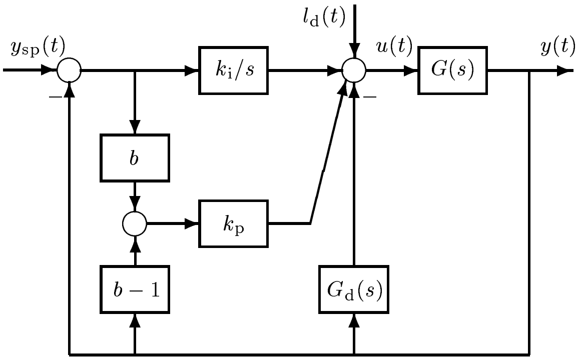

Fig. 1. PID control scheme. $G(s)$ is the process transfer function; $G_\mathrm{d}(s) = k_\mathrm{d}s/(1 + sk_\mathrm{d}/N)$. If $k_\mathrm{d} = 0$ the PI scheme is obtained.

The tolerance is set to $\varepsilon = 10^{-5}$, and the lower bound on Holder constants at $\xi = 10^{-8}$ in all experiments. The search region P is chosen in the first instance as a rectangle containing the stability region: the rectangle is determined by Routh criterion for rational $G(s)$, by Nyquist criterion for plants with delay.

The algorithm is run using a small value of r, typically $r = 1.4$. On the basis of plots such as that in Fig. 2, a second trial is performed with a reduced box and a larger r, *e.g.*, $r = 2$. The algorithm shows in almost all experiments a satisfactory performance: only in few cases the limit of 1 000 iterations is reached, whereas less than 100 iterations are often sufficient to localize the minimum. The first estimate is almost always confirmed by the second run.

Once the optimal pair $(k_\mathrm{p}, k_\mathrm{i})$ is obtained, b is chosen as the largest value in $[0, 1]$ for which $\|G_\mathrm{sp}\|_\infty = 1$, where $G_\mathrm{sp} = G(bk_\mathrm{p} + k_\mathrm{s}/s)/(1 + GG_\mathrm{c})$ is the transfer function between the set-point and the output. If no such value exists, then $b = 0$ is set. Table 1 summarizes the results of the tests. The values of the IE and of $(k_\mathrm{p}, k_\mathrm{i})$ found in Åström *et al.* (1998) are reported (underlined) for comparison.

Figures 3–6 show the responses to a step set-point, followed by a step load disturbance: for a given constraint, no striking difference arises by the use of the IE or ISE criterion, whereas the constraints have a relevant effect. An increasing tendency to oscillations from the controllers designed with the constraint $\|S\|_\infty \le 1.4$ to $\|S\|_\infty \le 2.0$ and finally $m_\varphi \ge 60°$ is observed. As for G_4, the design with $m_\varphi \ge 60°$ gives a very slow decay of the disturbance response.

3.2 *PID design*

At the beginning the two problems considered are:

- PB 3-a,b – given M_s, find $k_\mathrm{p}, k_\mathrm{i}, k_\mathrm{d}$ such that:

1) the closed loop system is stable;
2) $\|S\|_\infty \le M_\mathrm{s}$;
3) either IE is minimum or ISE is minimum.

Due to the increased number of variables, convergence is more difficult to attain. The tolerance is set at $\varepsilon = 10^{-3}$, the maximum number of iterations at 10 000, and the filter parameter at $N = 10$.

Table 2 shows the trials to obtain the optimal controller with the ISE index and $M_s = 1.4$ for $G_1(s)$. The first two runs of the algorithm do not find the minimum, so it is necessary to further modify the search region and increase the reliability parameter. The same procedure is followed for the IE index: final results are shown in Table 3 and time responses in Figure 7.

Acting on b at the same way as in the PI design does not remove the large oscillations in the set point response. However, the flexibility of the algorithm allows to introduce a further constraint on the complementary sensitivity $T(s)$ to reduce the resonance peak of the set-point to output response.

Then the optimization problem becomes:

• PB 4-a,b - -given $M_\mathrm{s}, M_\mathrm{p}$, find $k_\mathrm{p}, k_\mathrm{i}, k_\mathrm{d}$ such that:

1) the closed loop system is stable;
2) $\|S\|_\infty \le M_\mathrm{s}$;
3) $\|T\|_\infty \le M_\mathrm{p}$;
4) either IE is minimum or ISE is minimum.

The set of numerical experiments is performed for $G_1(s)$ and $N = 10,\ 100, \infty$ ($N = \infty$ represents unfiltered derivative action), with $M_s = 1.4$ and $M_p = 1.4$. The final estimates are reported in Table 4, and the time responses are shown in Fig. 8.

The tables and figures show interesting features. It is apparent that the constraint on the complementary sensitivity causes a decrease of the overshoot from 37% (Fig. 7, ISE plot) or 45% (Fig. 7, IE plot) to 13% (Fig. 8, $N = 10$ plot). On the other hand the difference between the peaks of the response to the load disturbance is negligible. When two constraints are imposed, there is no difference between the PID parameters obtained by minimizing the IE or the ISE criterion. The results of Table 4 show that the optimal values of the PID parameters are strongly influenced by N, that is by the bandwidth of the filter acting on the output signal. Moreover the IE and ISE indices increase when N is decreased: this effect is also seen in the time responses to the load disturbance, whereas the influence on the set-point response is negligible.

4. DISCUSSION AND CONCLUSIONS

A new algorithm for constrained global optimization has been described and applied to the classical problem of tuning PI/PID parameters. Its main features are:

- it only requires that a box search region is given: the search is performed along a space filling curve that maps the interval $[0, 1]$ into the region;
- the constraints are ordered: when a constraint is not satisfied, the next constraints and the objective function are not computed;
- it is able to deal with black-box objective function, i.e., the analytic expression of the function to be minimized is not required;
- the behaviour of the algorithm depends almost exclusively on the reliability parameter r.

The problems here considered can be all stated as: given any transfer function as a model of the plant, find the parameters of a PI/PID controller that

- make the closed loop system stable;
- guarantee, to a certain degree, robustness against model mismatch and good set-point response;
- minimize a functional of the error due to a unit step disturbance acting on the plant input.

Such design objectives are not new: several cited papers discuss them and propose solution algorithms, but they seem to be more or less specialized. The novelty of our proposal is a general algorithm, which poses no restriction on the plant model, on the number and type of constraints, on the function to be minimized.

The characteristic features mentioned above make it particularly suited to the present application:

- the small number of decision variables avoids the degradation of performance which affects optimization methods using space filling curve when the number of variables increases;
- the rectangular region is defined by bounds imposed on the parameters of the controller: in the paper the bounds have been found on the basis of a rough stability analysis, but in practical instances they can be given on the basis of technical specifications on the components;
- constraints ordering plays a more important role: since the prerequisite is closed loop stability, if the stability constraint is not satisfied, the computation of other quantities, which may be unbounded or meaningless, is not even attempted;
- to refine a first estimate, it is important that the behaviour of the algorithm only depends on r;
- the MATLAB version of the algorithm uses, in the computation of constraints and criteria, all the MATLAB tools for LTI systems analysis.

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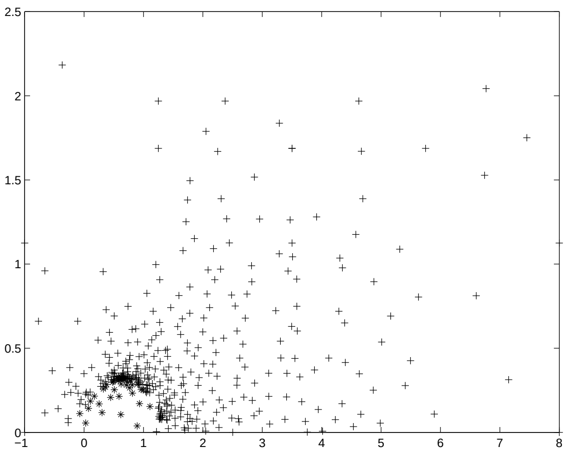

Fig. 2. Plot on the $(k_\mathrm{p}, k_\mathrm{i})$ plane of the search points generated by the algorithm for Problem PIES. $G(s) = G_1(s), M_s = 1.4$ and $r = 1.4$: $+$ non-feasible points; $*$ feasible points.

Table 1. Summary of the best estimates of the PI parameters obtained by minimizing the IE or ISE index.

Crit.	Constraint	$k_{\rm p}$	$k_{\rm i}$	IE	ISE
		$G_1(s)$			
IE	$\|\|S\|\|_\infty \leq 1.4$	0.634	0.325	3.075	1.243
		0.633	0.325	3.070	
IE	$\|\|S\|\|_\infty \leq 2.0$	1.214	0.685	1.459	0.559
		1.220	0.685	1.450	
ISE	$\|\|S\|\|_\infty \leq 1.4$	0.820	0.317	3.154	1.108
ISE	$\|\|S\|\|_\infty \leq 2.0$	1.826	0.578	1.731	0.436
IE	$m_\varphi \geq 60°$	1.200	0.454	2.205	0.689
ISE	$m_\varphi \geq 60°$	1.531	0.409	2.446	0.623
		$G_2(s)$			
IE	$\|\|S\|\|_\infty \leq 1.4$	0.321	0.843	1.187	0.617
		0.313	0.839	1.190	
IE	$\|\|S\|\|_\infty \leq 2.0$	0.460	1.540	0.649	0.426
		0.482	1.540	0.648	
ISE	$\|\|S\|\|_\infty \leq 1.4$	0.391	0.805	1.243	0.599
ISE	$\|\|S\|\|_\infty \leq 2.0$	0.784	1.260	0.794	0.356
IE	$m_\varphi \geq 60°$	1.139	1.416	0.705	0.291
ISE	$m_\varphi \geq 60°$	1.337	1.330	0.752	0.281
		$G_3(s)$			
IE	$\|\|S\|\|_\infty \leq 1.4$	0.165	0.027	37.59	24.88
		0.164	0.027	37.50	
IE	$\|\|S\|\|_\infty \leq 2.0$	0.278	0.048	20.75	19.52
		0.266	0.048	20.80	
ISE	$\|\|S\|\|_\infty \leq 1.4$	0.201	0.026	38.46	24.39
ISE	$\|\|S\|\|_\infty \leq 2.0$	0.419	0.041	24.33	18.17
IE	$m_\varphi \geq 60°$	0.626	0.053	18.98	18.64
ISE	$m_\varphi \geq 60°$	0.530	0.048	22.37	17.66
		$G_4(s)$			
IE	$\|\|S\|\|_\infty \leq 1.4$	0.279	0.042	23.92	51.72
		0.282	0.042	23.90	
IE	$\|\|S\|\|_\infty \leq 2.0$	0.502	0.130	7.675	11.63
		0.488	0.131	7.630	
ISE	$\|\|S\|\|_\infty \leq 1.4$	0.305	0.040	25.00	48.37
ISE	$\|\|S\|\|_\infty \leq 2.0$	0.567	0.118	8.496	10.53
IE	$m_\varphi \geq 60°$	0.347	0.021	47.16	72.80
ISE	$m_\varphi \geq 60°$	0.399	0.020	51.02	67.36

Table 2. The ISE index is minimized for the plant $G_1(s)$ with respect $k_{\rm p}, k_{\rm i}, k_{\rm d}$ with $N = 10$. The constraint is $||S||_\infty \leq 1.4$.

Box	r	$k_{\rm p}$	$k_{\rm i}$	$k_{\rm d}$	ISE
$[-1, 8] \times [0, 2.25] \times [0, 10]$	1.4	2.186	0.249	1.240	0.678
$[1, 3] \times [0, 1] \times [0, 3]$	2	1.990	0.983	1.583	0.230
$[1.5, 2.5] \times [1, 2] \times [1, 2.5]$	4	1.734	1.702	1.791	0.195
$[1.5, 2] \times [1.5, 2] \times [1.5, 2]$	4	1.728	1.758	1.848	0.192

Table 3. The IE and ISE are minimized for the plant $G_1(s)$ with respect $k_{\rm p}, k_{\rm i}, k_{\rm d}$ with $N = 10$. The constraint is $||S||_\infty \leq 1.4$.

Criterion	$k_{\rm p}$	$k_{\rm i}$	$k_{\rm d}$	IE	ISE
IE	1.4773	1.8539	2.1380	0.5394	0.2047
ISE	1.7277	1.7585	1.8477	0.5687	0.1917

Table 4. The IE and ISE are minimized for the plant $G_1(s)$, with $N = 10, 100, \infty$. Constraints are $||S||_\infty \leq 1.4, ||T||_\infty \leq 1.4$.

N	Crit.	$k_{\rm p}$	$k_{\rm i}$	$k_{\rm d}$	IE	ISE
10	IE	1.920	1.216	1.595	0.823	0.210
10	ISE	1.930	1.216	1.583	0.823	0.209
100	IE	3.177	1.849	3.149	0.541	0.089
100	ISE	3.163	1.845	3.266	0.542	0.089
∞	IE	3.983	2.265	4.234	0.442	0.059
∞	ISE	3.990	2.263	4.388	0.442	0.058

Fig. 3. PI design. Closed-loop time responses of process $G_1(s)$ to step set-point variation and load disturbance. Parameters are optimized with respect to IE (upper plot) or to ISE (lower plot); robustness constraints are $||S||_\infty \leq 1.4$, $||S||_\infty \leq 2$, $m_\varphi \geq 60°$.

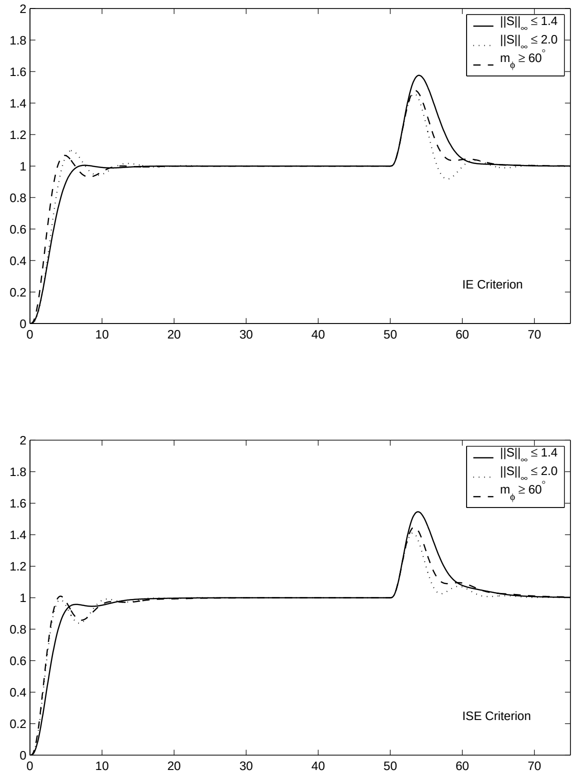

Fig. 4. PI design. Closed-loop time responses of process $G_2(s)$ to step set-point variation and load disturbance. Parameters are optimized with respect to IE (upper plot) or to ISE (lower plot); robustness constraints are $||S||_\infty \leq 1.4$, $||S||_\infty \leq 2$, $m_\varphi \geq 60°$.

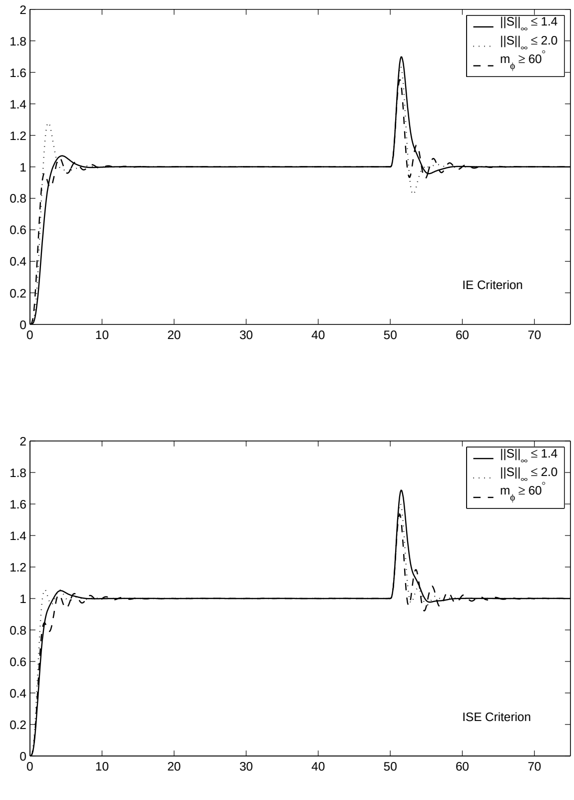

Fig. 5. PI design. Closed-loop time responses of process $G_3(s)$ to step set-point variation and load disturbance. Parameters are optimized with respect to IE (upper plot) or to ISE (lower plot); robustness constraints are $||S||_\infty \leq 1.4$, $||S||_\infty \leq 2$, $m_\varphi \geq 60°$.

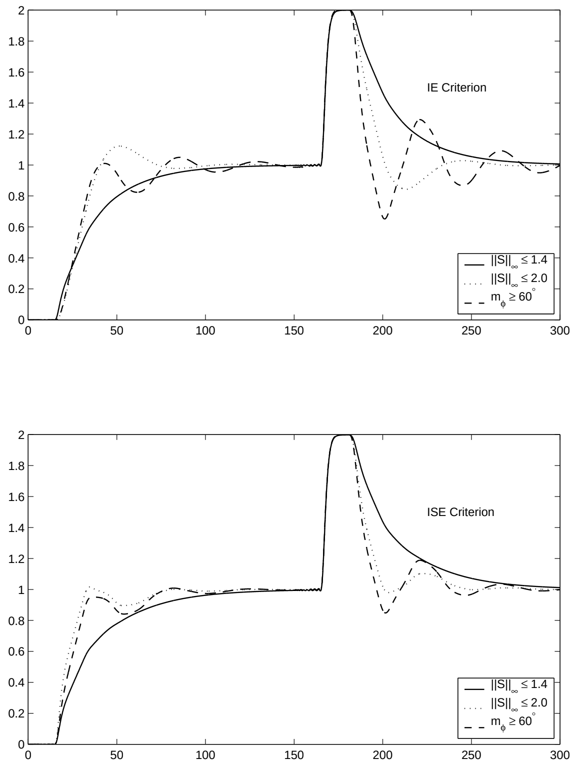

Fig. 6. PI design. Closed-loop time responses of process $G_4(s)$ to step set-point variation and load disturbance. Parameters are optimized with respect to IE (upper plot) or to ISE (lower plot); robustness constraints are $||S||_\infty \leq 1.4$, $||S||_\infty \leq 2.$, $m_\varphi \geq 60°$.

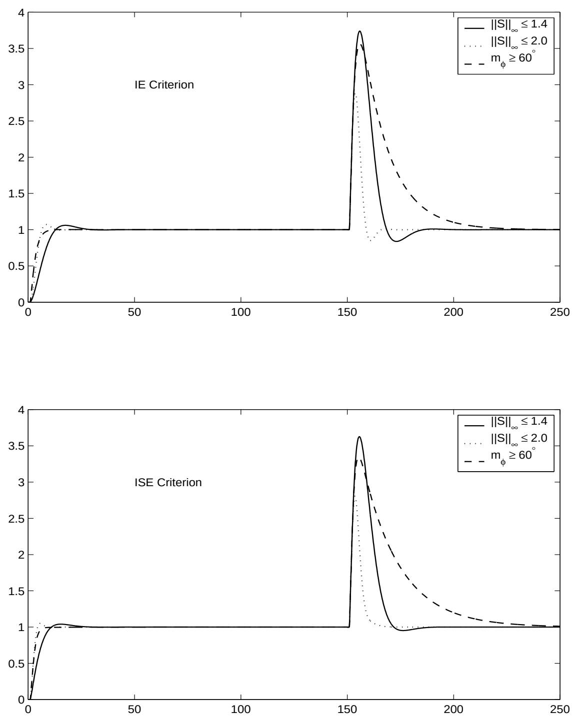

Fig. 7. PID design. Closed-loop time responses of process $G_1(s)$ to step set-point variation and load disturbance. Parameters (with $N = 10$) are optimized with respect to IE or to ISE; the constraint is $||S||_\infty \leq 1.4$.

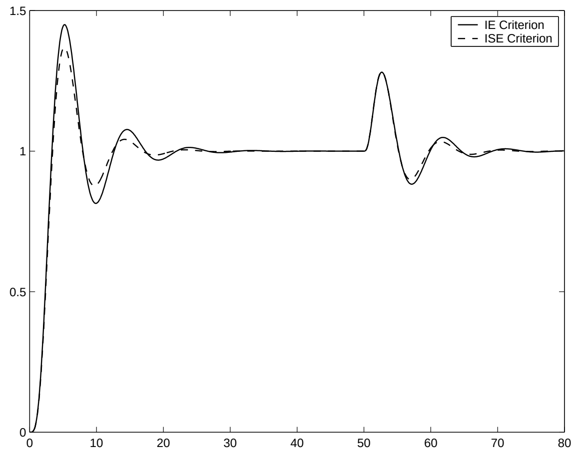

Fig. 8. PID design. Closed-loop time responses of process $G_1(s)$ to step set-point variation and load disturbance. Parameters are optimized with respect to IE or to ISE, for some values of the filter parameter N. Constraints are $||S||_\infty \leq 1.4$ and $||T||_\infty \leq 1.4$.