15th Triennial World Congress, Barcelona, Spain

NONLINEAR IDENTIFICATION VIA VARIABLE STRUCTURE RECURRENT NEURAL NETWORKS

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Abstract: In this paper, we present a new approach for nonlinear identification using Variable Structure Recurrent Neural Networks (VSRNN). We propose a neural network identifier, whose structure changes depending on the error. In this way, a trade off between identification error and computational complexity is achieved.

Keywords: Dynamic Neural Networks, Variable Stucture Systems, Nonlinear systems, Identification, Lyapunov methodology.

1. INTRODUCTION

A main topic for nonlinear system identification is to select a suitable identifier structure. Recurrent neural networks offer the advantage of approximating a nonlinear system within any accuracy degree (Cotter N. E., 1990), provided that the neural identifier has sufficient synaptic connections (Kosmatopoulos E. B., 1997). However, it is quite difficult to determine the number of sufficient synaptic connections to approximate such a dynamical system.

If the neural identifier has no enough synaptic connections, it is not possible to assure that the parameters converge to their optimal values even using persistently excitation inputs, and in many cases the identification error does not converge to zero. On the other hand, if there are too many synaptic connections, computational resources are misused.

In this paper, to alleviate this situation, we propose a Variable Structure Neural Network (VSRNN) for nonlinear system identification. An initial configuration for the neural identifier is assumed, but if a pre-specified error criterion is not reached we add more synaptic connections, in order to satisfy such criterion.

2. MATHEMATICAL PRELIMINARIES

Before proceeding with the main subject of the paper, we introduce some required mathematical concepts.

2.1 *Input-to-State Stability*

Let consider the nonlinear system (Krstic M.,1998)

$$\dot{x} = f(x,t) + g_1(x,t)d \tag{1}$$

where $x \in \Re^n$ is the state space, $d \in \Re^r$ is an external disturbance, and $f(0,t) = 0$.

Definition 1. The system (1) is said to be input-to-state stable (ISS) if there exist a class $\mathcal{KL}$ function β and a $\mathcal{K}$ function χ, such that, for any $x(t_0)$ an for any input $d(\cdot)$ continuous on $[0,\infty)$ the solution exist for all $t \geq 0$ and satisfies

$$||x(t)|| \leq \beta(x(t_0), t - t_0) + \chi\left(\sup_{t_0 \leq \tau \leq t} ||d(\tau)||\right)$$

for any t_0, $0 \leq t_0 \leq t$.

Theorem 1. (Sontag) Suppose that for the system (1) there exist a C^1 function $V : \Re^n \times \Re^+$ such that for all $x \in \Re^n$ and for all $d \in \Re^r$,

$$\gamma_1(||x||) \leq V(x,t) \leq \gamma_2(||x||)$$

$$|x| \leq \delta(||d||) \Rightarrow \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(x,t) + \frac{\partial V}{\partial x} g_1(x,t) d \leq -\gamma_3(||x||)$$

where γ_1, γ_2 and δ are class $\mathcal{K}_\infty$ functions and γ_3 is a class $\mathcal{K}$ function, hence the system (1) is ISS with $\chi = \gamma_1^{-1} \circ \gamma_2 \circ \delta$.

2.2 *Switched Systems*

Let the family of vector fields $P = \{f^\sigma(x,u),\ f^\sigma : \Re^n \times \Re^m \to \Re^n,\ \sigma \in \Gamma\}$, where Γ is a index set, $f^\sigma(0,0) = 0\ \forall \sigma \in \Gamma$, and for each $\sigma \in \Gamma$ f^σ is locally Lispchitz (Mancilla J. L., 2000).

Given the family P, the switched system is considered

$$\dot{x} = f^s(x,u) \tag{2}$$

where $x \in \Re^n$, $u \in \Re^n$ $t \leq 0$ and s is the *switching signal* defined as a continuous function $[0,\infty) \to \Gamma$; associated with the signal s there is a sequence of real numbers $\Upsilon = t_0 < t_1 < ... < t_k < ...$ called switching time secuence and the sequence of indexes $\Sigma = \sigma_0, \sigma_1, ..., \sigma_k, ...$ such that $s(t) = \sigma_k$ for all $t_k \leq t < t_{k+1}$.

Theorem 2. Suppose that there exist a common ISS-Lyapunov pair (V,δ) for P. Then the system (2) is ISS (Mancilla J.L., 2000).

2.3 *Stability Lemmas*

Lemma 3. (Barbalat). Let $\phi : \Re \to \Re$ be a uniformly continuous on $[t_q,\infty)$. Suppose that $lim_{t\to\infty} \int_{t_q}^{\infty} \phi(\tau) d\tau$ exists and is finite (Khalil H., 1996), then,

$$\phi(t) \to 0 \text{ as } t \to \infty$$

with $t_q \geq 0$.

Lemma 4. Consider the system (Khalil H., 1996)

$$\dot{x} = f(t,x) \tag{3}$$

where $x \in \Re^n$ and $f(t,x)$ is locally Lipschitz in x and uniformly in t. Suppose that there exists a function $V : \Re^n \to \Re^+$ radially unbounded and continuously differentiable such that

$$\dot{V} = \frac{\partial V}{\partial x} f(t,x) \leq -W(x) \leq 0$$

$\forall t \geq t_0$ $\forall x \in \Re^n$, where $W(x)$ is a positive semidefinite function, then all trajectories of (3) are bounded for $t \geq t_0$ and $\|x(t_0)\| < c$, where c is a positive and bounded constant, moreover

$$\lim_{t\to\infty} W(x) = 0$$

3. RECURRENT HIGH ORDER NEURAL NETWORK

The recurrent neural network structure presented in (Kosmatopoulos E. B., 1997) is considered. For the sake of completeness, a brief description of this neural network is included here. In (Kosmatopoulos E. B., 1997), recurrent high order recurrent neural network (RHONN) are defined as

$$\dot{x}_i = -a_i x_i + \sum_{k=1}^{L_i} w_{ik} \prod_{j \in I_k} \eta_k^{d_j(k)},\ i = 1, ..., n \tag{4}$$

where: x_i is i-th neuron state, L_i is number of high order connections, $\{I_1, I_2, ..., I_{L_i}\}$ is a collection of no ordered subsets of $\{1, 2, ..., m+n\}$, $a_i > 0.$, w_{ik} are the adjustable weights of the neural network, $d_j(k)$ are no negative integers, and η is a vector defined as:

$$\eta = \begin{bmatrix} \eta_1 \\ \vdots \\ \eta_n \\ \eta_{n+1} \\ \vdots \\ \eta_{2n} \\ \eta_{2n+1} \\ \vdots \\ \eta_{2n+m} \end{bmatrix} = \begin{bmatrix} S(x_1) \\ \vdots \\ S(x_n) \\ S'(x_1) \\ \vdots \\ S'(x_n) \\ u_1 \\ \vdots \\ u_m \end{bmatrix}$$

with $u = [u_1, u_2, ..., u_m]^T$ the input to the neural networks, and $S(\cdot)$ a smooth sigmoid function formulated as:

$$S(x) = \frac{2}{1 + \exp(-\beta x)} - 1$$

For the sigmoid, β is a positive constant. Hence $S(x) \in (-1,1)$. and $S'(\cdot)$ is a smooth sigmoid function given by:

$$S'(x) = \frac{1}{1+\exp(-\beta x)}$$

As it can be seen, (4) allows the inclusion of high order terms.

Defining the vector

$$\rho_i = \begin{bmatrix} \rho_{i1} \\ \rho_{i2} \\ \vdots \\ \rho_{i,L_i} \end{bmatrix} = \begin{bmatrix} \prod_{j\in I_1} \eta_j^{d_j(1)} \\ \prod_{j\in I_2} \eta_j^{d_j(2)} \\ \vdots \\ \prod_{j\in I_{L_i}} \eta_j^{d_j(L_i)} \end{bmatrix}$$

(4) can be rewritten as

$$\dot{x}_i = -a_i x_i + \sum_{k=1}^{L_i} w_{ik}\rho_{ik}, \quad i = 1, ..., n$$

or

$$\dot{x}_i = -a_i x_i + w_i^\top \rho_i(x,u), \quad i = 1, ..., n \qquad (5)$$

where $w_i = [w_{i1} ... w_{i,L_i}]^\top$ is the vector of adaptive weights. It is clear that the synaptic weights depends on time. We name each $w_{ik}\rho_{ik}$ a high order connection, and each ρ_{ik} a high order term.

3.1 *Variable Structure Recurrent Neural Networks*

For simplicity, we define the dynamics for each neuron of a variable structure recurrent neural networks (VSRNN) as

$$\dot{x}_i = -a_i x_i + f_i^s(x,u,w_i), \quad i = 1, ..., n \qquad (6)$$

For the i-th neuron let a family of functions $P_i = \{f_i^\sigma(x,u,w_i),\ f_i^\sigma : \Re^n \times \Re^m \times \Re^{L_i+\Delta_i} \to \Re,\ \sigma \in \Gamma_i\}$, where $\Gamma_i = \{0,1,2,...,\Delta_i\}$, $s(t) : t \to \Gamma_i$ is the switching signal and Δ_i is called the maximal number, which is finite, of high order connections for the i-th neuron. We propose to select P_i such that $P_i = \{f_i^0 = w_i^{0T}\rho_i^0,\ f_i^1 = f_i^0 + w_{i,L_i^0+1}\rho_{i,L_i^0+1}, ...,$ $f_i^\sigma = f_i^{\sigma-1} + w_{i,L_i^0+\sigma}\rho_{i,L_i^0+\sigma}, \sigma = 2, ..., \Delta_i\}$, where $w_i^0, \rho_i^0 \in \Re^{L_i^0}$ and L_i^0 is the number of initial high order connections; hence, the maximal number of high order connections is $L_i^0 + \Delta_i$.

For simplicity, we can rewrite $P_i = \{f_i^0 = w_i^0\rho_i^0,$ $f_i^1 = w_i^{1\top}\rho_i^1,\ ...,\ f_i^\sigma = w_i^{\sigma\top}\rho_i^\sigma,\ \sigma = 2, ..., \Delta_i\}$, where $w_i^1 = [w_i^{0\top}\ w_{i,L_i^0+1}]^\top$, $\rho_i^1 = [\rho_i^{0\top}\ \rho_{i,L_i^0+1}]^\top$, $w_i^\sigma = [w_i^{(\sigma-1)\top}\ w_{i,(L_i^0+\sigma)}]^\top$ and $\rho_i^\sigma = [\rho_i^{(\sigma-1)\top}$ $\rho_{i,(L_i^0+\sigma)}]^\top$ with $\sigma = 2, ..., \Delta_i$. We name $f_i^0 = w_i^{0\top}\rho_i^0$ and $f_i^{\Delta_i} = w_i^\top \rho_i$ as the initial or minimal structure and the maximal structure, respectively, for the i-th neuron state. We define the high order connection which have not been connected at $t_\sigma \le t < t_{\sigma+1}$ as $\bar{w}_i^{\sigma\top}\bar{\rho}_i^\sigma$. It is worth mentioning that for the VSRNN, the function time s is a increasing one, with $s(0) = 0$.

The function s is determined by an external agent called supervisor; this agent evaluates the VSRNN performance and depending on this evaluation, it calculates, on-line, the value of s. Due to the way as we define the family of functions P_i, and the switching function s, the indexes secuence Σ_i, which is $0,1,2,...,\Delta_i$, is defined off-line. So that, only the switching time secuence Υ_i is given on-line by the supervisor.

As we stated before, the weights are time functions; hence, every w_{ik}, $k = 1,2,...,L_i^0$ for the initial structure is the solution of a differential equation as

$$\dot{w}_{ik} = v_{ik}(t),\ w_{ik}(0) \in \Re \qquad (7)$$

For $l = L_i^0+1, L_i^0+2, ..., L_i^0+\Delta_i$, the weights are given by

$$\dot{w}_{il} = \mu(t-t_s)v_{il}(t),\ w_{il}(0) = 0 \qquad (8)$$

where $v_{ij}(\cdot)$ is a bounded time function defined in the section 4.1 and $\mu(\cdot)$ is the well known unit step function. The above equations mean that all the weights not included are zeros until the respective high order connections are added.

4. NONLINEAR IDENTIFICATION

In this section, we consider the problem of identifying a nonlinear affine system given as

$$\dot{\chi} = f(\chi) + G(\chi)u \qquad (9)$$

where $\chi \in \Re^n$, $u \in \Re^m$, f is a smooth vector field, $G(\chi)$ is a matrix with columns g_i, $i = 1,2,...,n$, $G = [g_1\ g_2\ ...\ g_n]$. In order to identify this system (9), we use the VSRNN explained in Section 3.1. As discussed in (Kosmatopoulos E. B., 1997), it is assumed that the system (9) is fully described by a RHONN, with each neuron state given by

$$\dot{\chi}_i = -a_i\chi_i + w_i^{*\top}\rho_i^*(\chi,u) \qquad (10)$$

where $w_i^*, \rho_i^* \in \Re^{L_i}$. The optimal unknown parameters vector w_i^* is defined as

$$w_i^* = \arg\min_{w_i} \left\{ \begin{array}{l} \sup_{\chi,u} |f_i(\chi) + g_i(\chi)u \\ +a_i\chi_i - w_i^T\rho_i(\chi,u)| \end{array} \right\}$$

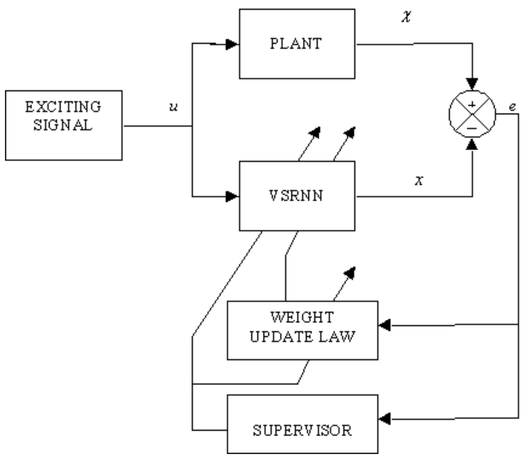

Fig. 1. Scheme proposed block diagram.

A1. We assume that the optimal weight vector w_i^* can be expressed as

$$w_i^* = [w_i^{*q\top} \ \ \bar{w}_i^{*q\top}]^\top$$

where the entries of w_i^{*q} can be any bounded value and the ones of $\bar{w}_i^{*q}$ are zeros.

This implies that there could exist a neural structure simpler than the maximal one, that can approximate arbitrary well the system (9); it is worth mentioning that the dimensions of w_i^{*q} and $\bar{w}_i^{*q}$ are unknown.

In order to identify the system (9), it is assumed that it can be represented by the proposed RHONN; then two possible model can be built.

- Parallel model $\dot{x}_i = -a_i x_i + w_i^\top \rho_i(x, u)$
- Series-Parallel model $\dot{x}_i = -a_i x_i + w_i^\top \rho_i(\chi, u)$

where x_i is the i-th component of the RHONN, and χ is the state of the system (9).

To develop the on line weight update law, we use the Series-Parallel model. The idea is to propose an initial structure for the RHONN and adapt the neural parameters, but if an error criterion is not satisfied we add an extra high order connection. We continue adding different high order connections until the error criterion is satisfied or the maximal neural structure is reached. Fig. 1 shows a block diagram of this scheme, where the supervisor evaluate the VSRNN's performance; if the error criterion is not satisfied, a high order connections will be added.

As in the definition of a VSRNN we can define $w_i^* = [w_i^{*s\top} \ \ \bar{w}_i^{*s\top}]^\top$, where $w_i^{*s}\rho_i^s$ are de connection added to the VSRNN and $\bar{w}_i^{*s\top}\bar{\rho}_i^s$ are the connections not added $\forall t_s \leq t < t_{s+1}$.

Let define the $i-$th identification error as

$$e_i = x_i - \chi_i$$

and the $i-$th parameter error at $t_s \leq t < t_{s+1}$

$$\tilde{w}_i^s = w_i^s - w_i^{*s}$$

Then from (6) and (10) we can obtain the error equation

$$\dot{e}_i = -a_i e_i + \tilde{w}_i^{s\top}\rho_i^s - \bar{w}_i^{*s\top}\bar{\rho}_i^s \qquad (11)$$

Lemma 5. The identification error e is ISS with respect to $d = [d_1 \ d_2 \ ... \ d_n]^\top$, $d_i = \tilde{w}_i^{s\top} z_i^s - \bar{w}_i^{*s\top}\bar{z}_i^s$, $i = 1, 2, ..., n$.

Proof. Let define the matrix $A = diag\{a_1, a_2, ..., a_n\}$. Now we propose the ISS-Lyapunov function $V = \frac{1}{2}||e||^2$.

If

$$||e|| \geq \delta(||d||) = \frac{||d||}{||A|| - b}$$

then

$$\dot{V} = e^\top A e + d^\top e < -b||e||^2$$

with $||A|| > b > 0$. Therefore, according with the lemma 1, e is ISS with respect to d, and $\chi(||d||) = \frac{||d||}{||A||-b}$. ■

Considering that the definition of d is valid for any structure, then according to (Mancilla J. L., 2000), the last Lemma states that the identification error can not diverge for any sequence of high order connections, hence e_i is always bounded for any structure.

4.1 *On-line Identification*

Using Barbalat's Lemma we can propose a definite positive and bounded by below function $V_i(e_i, w_i)$ and a parameter adaptive law such $\lim_{t\to\infty} V_i(e_i(t), w(t))$ exist; and $\dot{V}_i$ is uniformly continuous on $[t_q, \infty)$ and if $\dot{V}_i \to 0 \Rightarrow e_i \to 0$. Thereafter, we consider the Lyapunov function candidate

$$V_i = \frac{1}{2}(\gamma e_i^2 + \tilde{w}_i^\top \tilde{w}_i) \qquad (12)$$

where γ is the learning rate. Differentiating (12) along the trajectories of (11) we obtain

$$\dot{V}_i = -\gamma a_i e_i^2 + \gamma e_i \tilde{w}_i^{s\top}\rho_i^s - \gamma e_i \bar{w}_i^{*s\top}\bar{\rho}_i^s \qquad (13)$$

$$+\tilde{w}_i^\top \dot{\tilde{w}}_i \qquad (14)$$

If we define the weight adaptive law (Rovithakis G. A., 1994) as

$$\dot{w}_i^s = -\gamma e_i \rho_i^s \qquad (15)$$
$$\dot{\bar{w}}_i^s = 0$$

then (7) and (8) are satisfied, and the equation (13) becomes

$$\dot{V}_i = -\gamma a_i e_i^2 - \gamma e_i \bar{w}_i^{*s\top} \bar{\rho}_i^s$$

Considering **A1,** we are able to find some intermediate structure simpler than the maximal structure such that the term $\bar{w}_i^{*s\top} \bar{\rho}_i^s \cong 0$; hence, there exist some finite time t_q, such that

$$\dot{V}_i = -\gamma a_i e_i^2$$

for all $t \geq t_q$. That means that, after we add the $q-th$ connection to the VSRNN, we will not consider to add more high order terms.

Using Lemma 4 and supposing **A1**, it easy to see that if we reduce the modelling error term, by adding sufficient connections, at a finite time t_q, with the adaptive law (15) we guarantee that the weights are bounded and the identification error converges to zero after t_q, with $x = [e_i \; w_i^\top]^\top$, $V(x) = V_i(e_i, w_i)$, and $W(x) = \gamma a_i e_i^2$.

4.2 *Robust On-line Identification*

When the final parametrization has no been reached, the adaptive law (15) does not guarantee either the boundness of the weights or the convergence of the identification error to zero. Therefore the learning law (15) has to be modified in order to avoid the parameters drift problem. Then the well known σ-modification (Ioannou P. A., 1996), is applied to (15)

$$\dot{w}_i^s = -\gamma e_i \rho_i^s - \sigma_i w_i^s \qquad (16)$$

where σ_i is given as:

$$\sigma_i = \begin{cases} 0, & \text{if } ||w_i^s|| \leq M_i \\ \left(\dfrac{||w_i^s||}{M_i}\right)^q \sigma_{i_o}, & \text{if } M_i < ||w_i^s|| \leq 2M_i \\ \sigma_{i_o}, & \text{if } ||w_i^s|| > 2M_i \end{cases}$$

with integer $q \geq 1$, and σ_{i_0} and M_i positive constants.

Lemma 6. Consider the system (10) and the VSRNN (6) whose parameters are adapted using the law (16), then $e_i, w_i \in \mathcal{L}_\infty$.

Proof. The differential of V_i along the trajectories of (11) and (16) is given by

$$\dot{V}_i = -\gamma a_i e_i^2 - \sigma_i \tilde{w}_i^{s\top} w_i^s - \gamma e_i \bar{w}_i^{*s\top} \bar{\rho}_i^s$$

Applying the inequality

$$-\tilde{w}_i^{s\top} w_i^s \leq -(\tilde{w}_i^{s\top} \tilde{w}_i^s - \tilde{w}_i^{s\top} w_i^{*s})$$
$$\leq -\frac{1}{2}|\tilde{w}_i^s|^2 + \frac{1}{2}|w_i^{*s}|^2$$

and defining $d_0 = \max_{t \leq 0}(\bar{w}_i^{*s\top} \bar{\rho}_i^s)$, we have

$$\dot{V}_i \leq -\gamma a_i e_i^2 - \sigma_i \tilde{w}_i^{s\top} w_i^s + \frac{\gamma^2 e_i^2}{2} + \frac{{d_0}^2}{2}$$

Since $\tilde{w}_i^s = w_i^s - w_i^{*s}$, then

$$-\tilde{w}_i^{s\top} w_i^s \leq -(\tilde{w}_i^{s\top} \tilde{w}_i^s + \tilde{w}_i^{s\top} w_i^{*s})$$
$$\leq -\frac{1}{2}||\tilde{w}_i^s||^2 + \frac{1}{2}||w_i^{*s}||^2$$

Therefore

$$\dot{V}_i \leq -\gamma a_i e_i^2 + \frac{\gamma^2 e_i^2}{2} - \frac{1}{2}\sigma_i||\tilde{w}_i^s||^2$$
$$+\frac{1}{2}\sigma_i||w_i^{*s}||^2 + \frac{d_0^2}{2}$$

We define $\alpha = a_i - \frac{1}{2}\gamma$, hence

$$\dot{V}_i \leq -\gamma \alpha e_i^2 - \frac{1}{2}\sigma_i||\tilde{w}_i^s||^2$$
$$+\frac{1}{2}\sigma_i||w_i^{*s}||^2 + \frac{d_0^2}{2}$$

Substituting e_i from (12) in the above inequality, we have

$$\dot{V}_i \leq -\alpha V_i + \alpha \tilde{w}_i^\top \tilde{w}_i - \frac{1}{2}\sigma_i||\tilde{w}_i^s||^2$$
$$+\frac{1}{2}\sigma_i||w_i^{*s}||^2 + \frac{d_0^2}{2}$$

Taking the worst case, when $||w_i^s|| > 2M_i$, we can select $\sigma_{io} > 2\alpha$, then

$$\dot{V}_i \leq -\alpha V_i + \alpha||\bar{w}_i^{*s}||^2 + \frac{1}{2}\sigma_{io}||w_i^{*s}||^2 + \frac{d_0^2}{2}$$

therefore $V_i \in \mathcal{L}_\infty$ and the proof is complete. ■

5. SIMULATIONS

We consider the Duffing system to test the proposed identication scheme. The plant is defined as

$$\dot{\chi}_1 = \chi_2 \qquad (17)$$
$$\dot{\chi}_2 = (-k\chi_1 - ka^3\chi_1^3 - c\chi_2 + u)/m$$

with $m = 3$, $K = 0.1$, $a = 0.2$, $C = 0.05$, $u = A\sin(\omega t)$, $A = 0.5Nm$, $\omega = 1rad/s$. To identify the system (17) we consider a two states VSRNN given by

$$\dot{x}_1 = -a_1 x_1 + w_1^{s\top} \rho_1^s \qquad (18)$$
$$\dot{x}_2 = -a_2 x_2 + w_2^{s\top} \rho_2^s$$

where $a_1 = a_2 = 8$, $\beta = 0.5$, and $\gamma = 10$.

The results are presented in Fig. 1 and Fig. 2, for χ_1 and χ_2 respectively

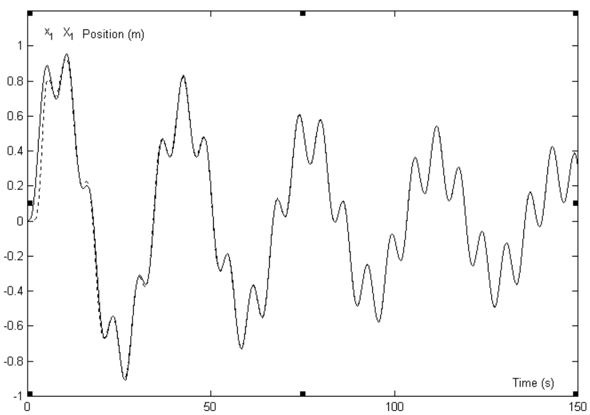

Fig. 1. Position tracking

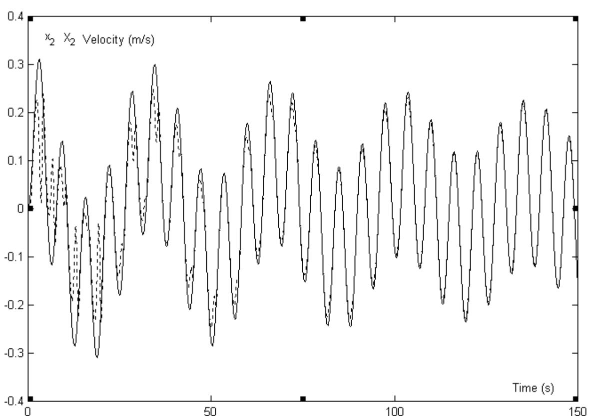

Fig. 2. Velocity tracking

6. CONCLUSIONS

The proposed identification scheme is suitable to deal with uncertainties in the plant. This scheme trades off between the identifier performance and the computational complexity. Research is being pursued to test the applicability of the proposed scheme via simulations. Some disadavantages of this identification scheme are; all states need to

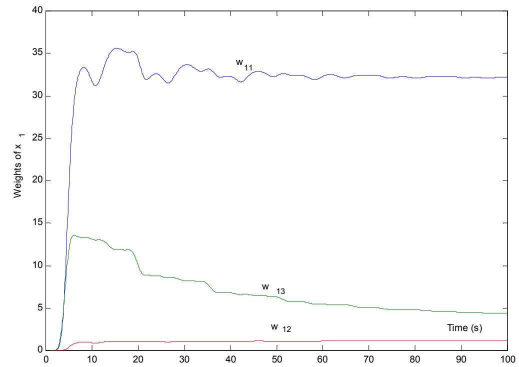

Fig. 2. Weigths w_{11}, w_{12} and w_{13}

be measurable and the order of the high order connections to be added is fixed and defined off-line. The most important topic for future works is to find some way to estimate the modelling error term, and then, the supervisor not only could give on-line the switching time secuence, but, the indexes secuence. It would be good to wipe away some connection if it is not needed to reduce the modelling error term.

Acknowledgements — The authors thank the support of CONACYT, Mexico, on Project 32059A.

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