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ON ESTIMATION FOR UNKNOWN INPUTS

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Abstract: The problem of unknown input estimation is considered in this paper. The problem can be viewed as a two-player zero-sum dynamic difference game. A game-theoretic approach that incorporates *maximum principle* arguments is adopted to solve this problem. The unknown input estimation is derived in terms of one discrete difference Riccati equation which is of a form suitable for recursive computation in online applications. *Copyright © 2002 IFAC*

Keywords: unknown input; game-theoretic approach; maximum principle; discrete difference Riccati equation; estimation.

1. INTRODUCTION

The estimation of unknown inputs is an important problem in control, communication and signal processing. For example, in control, the estimation of unknown inputs can be used for fault detection and isolation (FDI) and robust control design (Hou and Patton, 1998; Corless, 1998); in communication, the estimation of unknown inputs can be used for channel equalization (Wang and Balakrishnan, 1999); and in signal processing, it can be used for signal reconstruction, deconvolution and noise removal (Saberi et al., 2000; Shaked and Theodor, 1992; Weston and Norton, 1997). Various methodologies have been proposed for estimation of unknown inputs. Park and Stein (1988) have proposed a method to estimate unknown inputs by differentiating the output measurement. Hou and Patton (1998) have considered the problem of input observability and input reconstruction for linear time-invariant systems. The relations among input reconstruction, system inversion and disturbance-decoupled observers have been also explored. Park et al. (2000) have developed an estimate for unknown inputs using an optimal FIR (finite impulse response) filtering algorithm. Corless and Tu (1998) have presented an equivalent Lyapunov characterization of the unknown input observer existence conditions of Kudva et al.(1980) and designed the estimators which, using only a measured output, can asymptotically estimate the system state and the input to any desired accuracy. A random-walk process has been introduced to describe unknown inputs, and a two-stage Kalman filter has been used to estimate the unknown inputs together with the states of the system (Hsieh, 2000; Hsieh and Chen, 1999; Keller and Darouach, 1999; Keller et al., 1998). Recently, Saberi et al. (2000) have developed the exact unknown input estimation problem, which seeks to find a time-invariant linear stable proper or strictly proper filter that estimates the inputs while utilizing the measured outputs in such a way that the transfer function from all the inputs of the system to the estimation error is identically zero. This requires strong solvability conditions (algebraic constraints) to be satisfied. The optimal and sub-optimal unknown input estimation problems have also been presented in Saberi et al. (2000) and Shaked and Theodor (1992), where the transfer function from the unknown inputs to their estimation errors is minimized in either H_2 or H_∞ norm sense. It is worth noting that a number of literatures have been published to deal with the problem of designing an observer or a filter to estimate the state of the systems subject to unknown inputs (Bhattacharyya, 1978; Darouach and Zasadzinski, 1997; Darouach ea al., 1994; Hou and Patton, 1998; Kitanidis, 1987; Kobayashi and Nakamizo, 1982; Yang and Wilde, 1988). But here the unknown inputs of the system are directly estimated from the measurement.

In this paper, a game-theoretic framework that incorporates *maximum principle* arguments is proposed to treat the unknown input estimation problem. The problem can be viewed as a two-player zero-sum dynamic difference game. The difference game is defined in which the unknown input estimates and unintended inputs (unknown inputs, process noise, measurement noise and initial condition) have the conflicting objectives of, respectively, minimizing and maximizing the unknown input estimation error. The minimizer seeks the optimal unknown input estimates, and the maximizer seeks the worst-case intended inputs. It is worth emphasizing that game-theoretic approach has been applied to solve H_∞ control and filtering problems (Basar and Bernhard, 1995; Limebeer et al., 1989; Shen and Deng, 1997). In this paper, the game-theoretic approach incorporating *maximum principle* arguments is first applied to deal with the problem of unknown input estimation.

The paper is organized as follows. In Section 2, the problem of unknown input estimation is formulated, which is viewed as a two-player zero-sum dynamic difference game. The main results providing a solution to the problem of unknown input estimation are developed in Section 3 by a game-theoretic framework that incorporates maximum principle arguments. Some concluding remarks are given in Section 4.

2. STATEMENT OF THE PROBLEM

Consider the following class of discrete time-varying systems with unknown inputs defined on $k\in[0, N]$:

$$x_{k+1} = A_k x_k + B_{1,k} w_k + B_{2,k} u_k \qquad (1)$$

$$y_k = C_k x_k + D_k u_k + v_k \qquad (2)$$

where $x_k\in R^n$ is the state, $u_k\in R^p$ is the unknown input, $y_k\in R^m$ is the measured output, $w_k\in R^{q1}$ and $v_k\in R^{q2}$ are the process noise and the measurement noise, respectively, which are assumed to belong to $L_2[0, \infty]$. A_k, $B_{1,k}$, $B_{2,k}$, C_k, and D_k, are known real bounded matrix functions with compatible dimensions.

Remark 1: No particular assumption is made on the unknown input u_k. u_k may be any signal including impulsive noise. Furthermore, the unknown input is allowed to appear in the measurement equation. Thus, the systems (1) and (2) are of a general form. ■

Remark 2: For simplicity, terms of known inputs are not included in (1) and (2). Since the system parameter matrices are known, the known inputs will not affect the following analysis. ■

Remark 3: The process noise w_k and the measurement noise v_k are not required to be zero-mean white Gaussian sequences. They may be any disturbance signals with bounded energy and stationary power belonging to $L_2[0, \infty]$. ■

The objective of this paper is to estimate the unknown inputs from the measurement sequences y_k ($0\le k\le N$). The following performance is defined

$$J = \frac{\sum_{k=0}^{N-1}\|u_k - \hat{u}_k\|_{Q_k}^2}{\sum_{k=0}^{N-1}\|u_k\|_{R_k}^2 + \sum_{k=0}^{N-1}\|w_k\|_{W_k}^2 + \sum_{k=0}^{N-1}\|v_k\|_{V_k}^2 + \|x_0 - \hat{x}_0\|_S^2} \qquad (3)$$

where $\hat{x}_0$ is an a *priori* estimate of x_0, $\hat{u}_k$ is an estimate of the unknown input u_k. $Q_k\ge 0$, $R_k>0$, $W_k>0$, $V_k>0$ and $S>0$ are weighting matrices chosen by the designer, and $\|x_k\|_{A_k}^2 = x_k^T A_k x_k$. For all possible estimates $\hat{u}_k$, the performance (3) must satisfy the following inequality:

$$J \le \gamma^2 \qquad (4)$$

where $\gamma>0$ is a prescribed scalar. Therefore, our problem is to obtain an estimate $\hat{u}_k$ of u_k over the horizon $[0,N]$ such that the following performance index

$$\max_{u_k, w_k, v_k, x_0} \{\bar{J} = \sum_{k=0}^{N-1}[\|u_k - \hat{u}_k\|_{Q_k}^2 - \gamma^2(\|u_k\|_{R_k}^2 + \|w_k\|_{W_k}^2 + \|v_k\|_{V_k}^2)] - \gamma^2\|x_0 - \hat{x}_0\|_S^2\} \qquad (5)$$

is minimized using the measurement y_k. This problem can be stated as a deterministic linear quadratic game problem:

$$\min_{\hat{u}_k} \max_{u_k, w_k, v_k, x_0} \bar{J} \qquad (6)$$

The solution to this game problem will be developed in the next section.

3. THE ESTIMATION OF UNKNOWN INPUTS

Now, the maximum principle is applied to the performance index $\bar{J}$ in (5). Denoting the Lagrange multiplier by $2\gamma^2\lambda^T_{k+1}$, the Hamiltonian associated with the performance index $\bar{J}$ subject to (1) is given by

$$\begin{aligned} H = &\|u_k - \hat{u}_k\|_{Q_k}^2 - \gamma^2(\|u_k\|_{R_k}^2 + \|w_k\|_{W_k}^2 \\ &+ \|y_k - C_k x_k - D_k u_k\|_{V_k}^2) \\ &+ 2\gamma^2\lambda_{k+1}^T(A_k x_k + B_{1,k} w_k + B_{2,k} u_k) \end{aligned} \qquad (7)$$

By the maximum principle, the necessary conditions for H to be a maximum are

$$\frac{\partial H}{\partial u_k} = 2Q_k(u_k - \hat{u}_k) + 2\gamma^2 D_k^T V_k (y_k - C_k x_k - D_k u_k)$$
$$- 2\gamma^2 R_k u_k + 2\gamma^2 B_{2,k}^T \lambda_{k+1} = 0 \quad (8)$$

$$\frac{\partial H}{\partial w_k} = -2\gamma^2 W_k w_k + 2\gamma^2 B_{1,k}^T \lambda_{k+1} = 0 \quad (9)$$

$$\frac{\partial H}{\partial x_k} = 2\gamma^2 C_k^T V_k (y_k - C_k x_k - D_k u_k) + 2\gamma^2 A_k^T \lambda_{k+1}$$
$$= 2\gamma^2 \lambda_k \quad (10)$$

From (8), (9) and (10), u_k, w_k and v_k are obtained

$$u_k = T_{1,k}[Q_k \hat{u}_k - \gamma^2 B_{2,k}^T \lambda_{k+1} - \gamma^2 D_k^T V_k (y_k - C_k x_k)] \quad (11)$$

$$w_k = W_k^{-1} B_{1,k}^T \lambda_{k+1} \quad (12)$$

$$\lambda_k = C_k^T V_k (I + \gamma^2 D_k T_{1,k} D_k^T V_k)(y_k - C_k x_k) -$$
$$C_k^T V_k D_k T_{1,k} Q_k \hat{u}_k + (A_k^T + \gamma^2 C_k^T V_k D_k T_{1,k} B_{2,k}^T)\lambda_{k+1} \quad (13)$$

where

$$T_{1,k} = (Q_k - \gamma^2 R_k - \gamma^2 D_k^T V_k D_k)^{-1} \quad (14)$$

Substituting (11) and (12) into (1) yields

$$x_{k+1} = A_k x_k - \gamma^2 B_{2,k} T_{1,k} D_k^T V_k (y_k - C_k x_k) +$$
$$B_{2,k} T_{1,k} Q_k \hat{u}_k + (B_{1,k} W_k^{-1} B_{1,k}^T + \gamma^2 B_{2,k} T_{1,k} B_{2,k}^T)\lambda_{k+1} \quad (15)$$

(13) and (15) are a two-point boundary value problem with boundary conditions

$$x_0 = \hat{x}_0 + S^{-1}\lambda_0,\ \lambda_N = 0 \quad (16)$$

The solution to the problem is linear and is assumed to be of the form

$$x_k = \bar{x}_k + P_k \lambda_k \quad (17)$$

Substituting (17) into (13) gives

$$\lambda_k = T_{2,k} C_k^T V_k (I + \gamma^2 D_k T_{1,k} D_k^T V_k)(y_k - C_k \bar{x}_k)$$
$$- T_{2,k} C_k^T V_k D_k T_{1,k} Q_k \hat{u}_k + T_{2,k} \bar{A}_k^T \lambda_{k+1} \quad (18)$$

where

$$T_{2,k} = [I + C_k^T V_k (I + \gamma^2 D_k T_{1,k} D_k^T V_k) C_k P_k]^{-1} \quad (19)$$

$$\bar{A}_k = (A_k + \gamma^2 B_{2,k} T_{1,k} D_k^T V_k C_k) \quad (20)$$

Substituting (17) and (18) into (15) gives

$$\bar{x}_{k+1} - A_k \bar{x}_k - [\bar{A}_k P_k T_{2,k} C_k^T V_k (I + \gamma^2 D_k T_{1,k} D_k^T V_k)$$
$$- \gamma^2 B_{2,k} T_{1,k} D_k^T V_k]\bar{y}_k - (B_{2,k} - \bar{A}_k P_k T_{2,k} C_k^T V_k D_k) T_{1,k} Q_k \hat{u}_k$$
$$= (-P_{k+1} + \bar{A}_k P_k T_{2,k} \bar{A}_k^T + B_{1,k} W_k^{-1} B_{1,k}^T + \gamma^2 B_{2,k} T_{1,k} B_{2,k}^T)\lambda_{k+1} \quad (21)$$

where

$$\bar{y}_k = y_k - C_k \bar{x}_k \quad (22)$$

For (21) to hold true for arbitrary λ_{k+1}, both sides must be equal to zero identically, resulting in

$$\bar{x}_{k+1} = A_k \bar{x}_k + [\bar{A}_k P_k T_{2,k} C_k^T V_k (I + \gamma^2 D_k T_{1,k} D_k^T V_k)$$
$$- \gamma^2 B_{2,k} T_{1,k} D_k^T V_k]\bar{y}_k + (B_{2,k} - \bar{A}_k P_k T_{2,k} C_k^T V_k D_k) T_{1,k} Q_k \hat{u}_k \quad (23)$$

and

$$P_{k+1} = \bar{A}_k P_k T_{2,k} \bar{A}_k^T + B_{1,k} W_k^{-1} B_{1,k}^T + \gamma^2 B_{2,k} T_{1,k} B_{2,k}^T$$
$$P_0 = S^{-1} \quad (24)$$

Now, making use of (11), (12), (16), (17) and (18), the performance index $\bar{J}$ can be written

$$\bar{J} = \sum_{k=0}^{N-1} [\|u_k - \hat{u}_k\|_{Q_k}^2 - \gamma^2(\|u_k\|_{R_k}^2 + \|w_k\|_{W_k}^2 + \|v_k\|_{V_k}^2$$
$$- \gamma^2 \|x_0 - \hat{x}_0\|_S^2$$
$$= \sum_{k=0}^{N-1} \|x_1\|_{Q_k}^2 - \gamma^2 \sum_{k=0}^{N-1} \|x_2\|_{R_k}^2 - \gamma^2 \sum_{k=0}^{N-1} \|x_3\|_{V_k}^2]$$
$$- \gamma^2 \sum_{k=0}^{N-1} \|W_k^{-1} B_{1,k}^T \lambda_{k+1}\|_{W_k}^2 - \gamma^2 \|\lambda_0\|_{S^{-1}}^2 \quad (25)$$

where

$$x_1 = (T_{1,k} Q_k - I - \gamma^2 T_{1,k} D_k^T V_k C_k P_k T_{2,k} C_k^T V_k D_k T_{1,k} Q_k)\hat{u}_k$$
$$- \gamma^2 T_{1,k} D_k^T V_k T_{3,k} \bar{y}_k$$
$$+ \gamma^2 T_{1,k} (D_k^T V_k C_k P_k T_{2,k} \bar{A}_k^T - B_{2,k}^T)\lambda_{k+1} \quad (25a)$$

$$x_2 = (I - \gamma^2 T_{1,k} D_k^T V_k C_k P_k T_{2,k} C_k^T V_k D_k) T_{1,k} Q_k \hat{u}_k$$
$$- \gamma^2 T_{1,k} D_k^T V_k T_{3,k} \bar{y}_k$$
$$+ \gamma^2 T_{1,k} (D_k^T V_k C_k P_k T_{2,k} \bar{A}_k^T - B_{2,k}^T)\lambda_{k+1} \quad (25b)$$

$$x_3 = (T_{4,k} C_k P_k T_{2,k} C_k^T V_k - I) D_k T_{1,k} Q_k \hat{u}_k$$
$$+ T_{4,k} T_{3,k} \bar{y}_k$$
$$- (T_{4,k} C_k P_k T_{2,k} \bar{A}_k^T - \gamma^2 D_k T_{1,k} B_{2,k}^T)\lambda_{k+1} \quad (25c)$$

$$T_{3,k} = I - C_k P_k T_{2,k} C_k^T V_k (I + \gamma^2 D_k T_{1,k} D_k^T V_k) \quad (26)$$

$$T_{4,k} = I + \gamma^2 D_k T_{1,k} D_k^T V_k \quad (27)$$

Applying the identity

$$\gamma^2 (\sum_{k=0}^{N-1} \|\lambda_{k+1}\|_{P_{k+1}}^2 - \sum_{k=0}^{N-1} \|\lambda_k\|_{P_k}^2 + \lambda_0^T S^{-1} \lambda_0 - \lambda_N^T P_N \lambda_N) = 0 \quad (28)$$

to (25), it can be rewritten as

$$\bar{J} = \sum_{k=0}^{N-1} \|x_1\|_{Q_k}^2 - \gamma^2 \sum_{k=0}^{N-1} \|x_2\|_{R_k}^2 - \gamma^2 \sum_{k=0}^{N-1} \|x_3\|_{V_k}^2$$
$$- \gamma^2 \sum_{k=0}^{N-1} \|W_k^{-1} B_{1,k}^T \lambda_{k+1}\|_{W_k}^2 + \gamma^2 \sum_{k=0}^{N-1} \|\lambda_{k+1}\|_{P_{k+1}}^2 -$$
$$\gamma^2 \sum_{k=0}^{N-1} \|T_{2,k} C_k^T V_k T_{4,k} \bar{y}_k - T_{2,k} C_k^T V_k D_k T_{1,k} Q_k \hat{u}_k + T_{2,k} \bar{A}_k^T \lambda_{k+1}\|_{P_k}^2 \quad (29)$$

Making use of (24), a tedious but straightforward manipulation shows that the performance index (29) can be rewritten as follows

$$\bar{J} = \sum_{k=0}^{N-1} \|y_1\|_{Q_k}^2 - \gamma^2 \sum_{k=0}^{N-1} \|y_2\|_{R_k}^2 - \gamma^2 \sum_{k=0}^{N-1} \|y_3\|_{V_k}^2$$
$$- \gamma^2 \sum_{k=0}^{N-1} \|T_{2,k} C_k^T V_k T_{4,k} \bar{y}_k - T_{2,k} C_k^T V_k D_k T_{1,k} Q_k \hat{u}_k\|_{P_k}^2 \quad (30)$$

$$y_1 = (T_{1,k}Q_k - I - \gamma^2 T_{1,k}D_k^T V_k C_k P_k T_{2,k} C_k^T V_k D_k T_{1,k} Q_k)\hat{u}_k - \gamma^2 T_{1,k} D_k^T V_k T_{3,k} \bar{y}_k \tag{30a}$$

$$y_2 = (I - \gamma^2 T_{1,k} D_k^T V_k C_k P_k T_{2,k} C_k^T V_k D_k) T_{1,k} Q_k \hat{u}_k - \gamma^2 T_{1,k} D_k^T V_k T_{3,k} \bar{y}_k \tag{30b}$$

$$y_3 = (T_{4,k} C_k P_k T_{2,k} C_k^T V_k - I) D_k T_{1,k} Q_k \hat{u}_k + T_{4,k} T_{3,k} \bar{y}_k \tag{30c}$$

where the term λ_{k+1} has been eliminated.

(30) can be simplified as

$$\bar{J} = \sum_{k=0}^{N-1} [\gamma^2 \hat{u}_k^T Q_k T_{1,k} D_k^T V_k C_k P_k T_{2,k} C_k^T V_k D_k T_{1,k} Q_k \hat{u}_k + \hat{u}_k^T (Q_k - Q_k T_{1,k} Q_k) \hat{u}_k + \gamma^2 \hat{u}_k^T Q_k T_{1,k} D_k^T V_k T_{3,k} \bar{y}_k + \gamma^2 \bar{y}_k^T T_{3,k}^T V_k D_k T_{1,k}^T Q_k \hat{u}_k - \gamma^2 \bar{y}_k^T V_k T_{4,k} T_{3,k} \bar{y}_k] \tag{31}$$

Suppose

$$T_{1,k} < 0 \tag{32}$$

and

$$P_k + P_k C_k^T V_k (I + \gamma^2 D_k T_{1,k} D_k^T V_k) C_k P_k > 0 \tag{33}$$

If Assumptions (31) and (32) are satisfied, then it can be easily verified that

$$Q_k - Q_k T_{1,k} Q_k + \gamma^2 Q_k T_{1,k} D_k^T V_k C_k P_k T_{2,k} C_k^T V_k D_k T_{1,k} Q_k > 0 \tag{34}$$

and

$$V_k T_{4,k} T_{3,k} > 0 \tag{35}$$

Remark 4: Conditions (34) and (35) guarantee the existence of the solution to minimax problem (6) (Limebeer et al., 1989; Shen and Deng, 1997). Assumption (32) is easily satisfied by choosing Q_k, R_k and V_k under given γ. And Assumption (33) is also required for recursive computation in (24). ■

Next, a completing square operation (first on the $\hat{u}_k$-terms, then on the $\bar{y}_k$-terms) is performed to give:

$$\bar{J} = \sum_{k=0}^{N-1} [\| \hat{u}_k + \gamma^2 T_{5,k}^{-1} Q_k T_{1,k} D_k^T V_k T_{3,k} \bar{y}_k \|_{T_{5,k}}^2 - \gamma^2 \| \bar{y}_k \|_{T_{6,k}}^2] \tag{36}$$

where

$$T_{5,k} = \gamma^2 Q_k T_{1,k} D_k^T V_k C_k P_k T_{2,k} C_k^T V_k D_k T_{1,k} Q_k + Q_k - Q_k T_{1,k} Q_k > 0 \tag{37}$$

$$T_{6,k} = \gamma^2 T_{3,k}^T V_k D_k T_{1,k}^T Q_k T_{5,k}^{-1} Q_k T_{1,k} D_k^T V_k T_{3,k} + V_k T_{4,k} T_{3,k} > 0 \tag{38}$$

In view of (36), the optimal strategies of $\hat{u}_k$ and $\bar{y}_k$ are

$$\hat{u}_k^* = -\gamma^2 T_{5,k}^{-1} Q_k T_{1,k} D_k^T V_k T_{3,k} \bar{y}_k \tag{39}$$

and

$$\bar{y}_k^* = 0 \tag{40}$$

Thus

$$\min_{\hat{u}_k} \max_{\bar{y}_k} \bar{J} = 0 \tag{41}$$

i.e.

$$\bar{J}(\hat{u}_k^*, \bar{y}_k^*) = 0 \tag{42}$$

From (36), the optimal strategies $\hat{u}_k^*$ and $\bar{y}_k^*$ satisfy a saddle point inequality

$$\bar{J}(\hat{u}_k^*, \bar{y}_k) \le \bar{J}(\hat{u}_k^*, \bar{y}_k^*) \le \bar{J}(\hat{u}_k, \bar{y}_k^*) \tag{43}$$

Furthermore, according to (11), (12), (16), (22) and (2), (43) can be rewritten as

$$\bar{J}(\hat{u}_k^*, u_k, w_k, v_k, x_0) \le \bar{J}(\hat{u}_k^*, u_k^*, w_k^*, v_k^*, x_0^*) \le \bar{J}(\hat{u}_k, u_k^*, w_k^*, v_k^*, x_0^*) \tag{44}$$

The above results are summarized in the following theorem.

Theorem 1: Consider the system (1) and (2). Let γ>0 be a prescribed level and suppose $T_{1,k}$<0. If the discrete-time Riccati difference equation (24) has a symmetric positive-definite solution such that (33) is satisfied, then there exists an estimate $\hat{u}_k$ such that (4) is satisfied. Moreover, the estimate $\hat{u}_k$ is given by

$$\hat{u}_k = -\gamma^2 T_{5,k}^{-1} Q_k T_{1,k} D_k^T V_k T_{3,k} (y_k - C_k \bar{x}_k) \tag{45}$$

where the evolution of $\bar{x}_k$ is governed by the following equation

$$\bar{x}_{k+1} = A_k \bar{x}_k + [\bar{A}_k P_k T_{2,k} C_k^T V_k T_{4,k} - \gamma^2 B_{2,k} T_{1,k} D_k^T V_k - \gamma^2 (B_{2,k} - \bar{A}_k P_k T_{2,k} C_k^T V_k D_k) T_{1,k} Q_k T_{5,k}^{-1} Q_k T_{1,k} D_k^T V_k T_{3,k}](y_k - C_k \bar{x}_k) \tag{46}$$

Proof: According to (42) and (43), there exists an estimate $\hat{u}_k^*$ such that $\bar{J} \le 0$ which leads to (4). Substituting (22) into (39) yields (45), and substituting (45) into (23) yields (46). This completes the proof. ■

According to Theorem 1, the algorithm for estimating the unknown inputs is formulated as follows:

Step 1: Select weighting matrices Q_k, R_k, W_k, V_k, S.

Step 2: Computing $T_{1,k}$, $T_{2,k}$, $T_{3,k}$, $T_{4,k}$, $T_{5,k}$, $T_{6,k}$.

Step 3: Estimate $\hat{u}_k$ according to (46).

Step 4: Recursively solve the discrete-time Riccati difference equation (24) forward in time.

Step 5: Recursively compute $\bar{x}_{k+1}$ according to (46).

Step 6: k=k+1, go to Step 2.

Due to space limitation, the example is omitted.

4. CONCLUSIONS

In this paper, the problem of unknown input estimation has been considered, which can be viewed as a two-player zero-sum dynamic difference game. The solution is derived in terms of one discrete difference Riccati equation by a game-theoretic approach incorporating *maximum principle* arguments. The result can be used to estimate impulsive noise and hence remove the effect of impulsive noise in the system.

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