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# HOMOGENEOUS STABILIZATION OF THE EXTENDED CHAINED FORM SYSTEM

**N.P.I. Aneke** [1] **D.A. Lizárraga** [2] **H. Nijmeijer** [3]

*Dynamics and Control Group, Department of Mechanical Engineering, Eindhoven University of Technology, P.O. Box 513, 5600 MB Eindhoven, The Netherlands,*
[1] `n.p.i.aneke@tue.nl`, [2] `d.lizarraga@tue.nl`, [3] `h.nijmeijer@tue.nl`

Abstract: A controller is presented for point stabilization of the extended chained form system. The extended chained form system can not be stabilized by means of continuous pure-state feedback. Moreover, it can not be exponentially stabilized by smooth feedback, since the linearization around equilibrium points is uncontrollable. In this paper, a controller is given that ρ-exponentially stabilizes the system. The controller is derived by using a combined averaging technique for homogeneous systems and a backstepping approach. The controller is illustrated in a benchmark example by application to the V/STOL aircraft without gravity.

Keywords: Dynamics, Stability, Constraints, Nonlinear control

## 1. INTRODUCTION

In this contribution we consider the feedback stabilization problem for a specific class of second-order nonholonomic systems, or systems subject to non-integrable acceleration constraints. This class consists of second-order nonholonomic systems that can be transformed by coordinate and feedback transformation into the extended chained form. Examples of such systems are the planar horizontal underactuated PPR and RRR manipulators, see (Yoshikawa *et al.*, 2000), and the V/STOL aircraft without gravity (Hauser *et al.*, 1992).

The extended chained form can be seen as a generalization of the chained form (Murray and Sastry, 1993), in the sense that it contains a drift vector field while the chained form is a drift-less system. In (Astolfi, 1996) exponential convergence towards the desired equilibrium point of the extended chained form system was obtained using discontinuous control. In (Laiou and Astolfi, 1999) this result was extended to obtain a weakened Lyapunov stability result called quasi-smooth exponential stability. It is not yet clear whether the extended chained form system can be stabilized by means of smooth time-varying feedback.

In this contribution we consider the feedback stabilization problem by continuous periodic time-varying feedback. The idea is to use a continuous periodic time-varying feedback to stabilize a subsystem, and use a backstepping approach to stabilize the complete system. Our approach follows that of (Morin and Samson, 1997).

## 2. PROBLEM FORMULATION

Consider the extended chained form system

$$\begin{aligned} \ddot{\xi}_1 &= u_1 \\ \ddot{\xi}_2 &= u_2 \\ \ddot{\xi}_3 &= \xi_2 u_1. \end{aligned} \tag{1}$$

Since the image of the mapping $(\xi, \dot{\xi}, u) \mapsto (\dot{\xi}_1, \dot{\xi}_2, \dot{\xi}_3, u_1, u_2, \xi_2 u_1)$ does not contain any point $(0,0,0,0,0,\varepsilon)$ for $\varepsilon \neq 0$, this system does not satisfy Brockett's condition (Brockett, 1983). Therefore there does not exist a continuous time-invariant state feedback that asymptotically stabilizes the system to the origin. To our knowledge the feedback stabilization problem for (1) has not been solved yet. Although controllers have been developed that achieve exponential convergence to the origin (Astolfi, 1996), it is not even clear whether the system (1) can be ρ-exponentially stabilized. In this paper we will show that (1) can be ρ-exponentially stabilized by continuous periodic time-varying feedback.

Consider the dynamics (1) in state-space form:

$$\begin{aligned} \dot{x}_1 &= x_2 & \dot{x}_2 &= u_1 \\ \dot{x}_3 &= x_4 & \dot{x}_4 &= u_2 \\ \dot{x}_5 &= x_6 & \dot{x}_6 &= x_3 u_1, \end{aligned} \tag{2}$$

with state-vector $x = [x_1, x_2, \ldots, x_6]^T$ given by $x_i = \xi_i$, $x_{i+1} = \dot{\xi}_i$, $i = 1,3,5$. The feedback stabilization problem can be formulated as follows.

*Problem 2.1.* (*Point stabilization problem*). The feedback stabilization problem is solvable if we can design appropriate continuous time-varying state feedback controllers of the form

$$u_1 = u_1(t,x), \qquad u_2 = u_2(t,x) \tag{3}$$

such that the equilibrium $x = 0$ of the closed-loop system (2,3) is globally asymptotically stable.

## 3. PRELIMINARIES

In this paper we use the elements of $C^0(\mathbb{R}^n \times \mathbb{R}; \mathbb{R}^n)$, the set of continuous mappings from $\mathbb{R}^n \times \mathbb{R}$ to $\mathbb{R}^n$, to represent continuous (time-varying) vector fields on $\mathbb{R}^n$. Let us now recall some definitions and properties related to homogeneous systems.

Given a weight vector $r = (r_1, \ldots, r_n)$ of real parameters $r_i > 0$ $(i = 1, \ldots, n)$ and a real number $\lambda > 0$, the mapping $\delta^r_\lambda : \mathbb{R}^n \to \mathbb{R}^n$ defined by

$$\delta^r_\lambda(x) = (\lambda^{r_1} x_1, \ldots, \lambda^{r_n} x_n)$$

is called a dilation of weight $r$.

A homogeneous norm associated with a dilation $\delta^r_\lambda$ is a continuous positive-definite function $\rho : \mathbb{R}^n \to \mathbb{R}$ which is homogeneous of degree one with respect to $\delta^r_\lambda$. For example, a homogeneous norm associated with the dilation $\delta^r_\lambda$ is given by

$$\rho^r_p(x) = \left( \sum_{j=1}^{n} |x_j|^{p/r_j} \right)^{1/p}, \qquad p > 0.$$

A continuous function $f : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}$ is said to be homogeneous of degree $\tau$ with respect to $\delta^r_\lambda$ if

$$f(\delta^r_\lambda(x), t) = \lambda^\tau f(x,t)$$

for every couple $(x,t) \in \mathbb{R}^n \times \mathbb{R}$.

*Definition 3.1.* A (time-varying) vector field $f : \mathbb{R}^n \to \mathbb{R}^n$ given by $f(x,t) = \sum_{i=1}^n f_i(x,t)\partial/\partial x_i$, is said to be homogeneous of degree $\tau \geq 0$ with respect to $\delta^r_\lambda$ if, for each $i = 1, \ldots, n$, the $i$-th component $f_i$ is a homogeneous function of degree $\tau + r_i$ with respect to $\delta^r_\lambda$. More precisely, for $i = 1, \ldots, n$,

$$f_i(\delta^r_\lambda(x), t) = \lambda^{\tau + r_i} f_i(x,t)$$

for all $\lambda > 0$ and every couple $(x,t) \in \mathbb{R}^n \times \mathbb{R}$.

*Definition 3.2.* Consider a homogeneous norm ρ associated with a dilation $\delta^r_\lambda$. The origin of the system $\dot{x} = f(x,t)$ with $f(0,t) = 0$, $\forall t$, is said to be locally ρ-exponentially stable (with respect to a dilation $\delta^r_\lambda$) if there exist strictly positive constants $\delta$, $K$ and $\gamma$ such that for any $t_0 \in \mathbb{R}$ and any solution $x(t)$, with $x(t_0) = x_0$,

$$\rho(x_0) < \delta \quad \Longrightarrow \quad \rho(x(t)) \leq K\rho(x_0)e^{\gamma(t-t_0)}.$$

Note that (local) ρ-exponential stability implies (local) $\mathcal{K}$-exponential stability as defined in (Sørdalen and Egeland, 1995). Let us recall a result that will be used to deduce ρ-exponential stability of the controlled system.

*Proposition 3.1.* (Pomet and Samson, 1994) Let $\delta^r_\lambda$ be a dilation and assume that the vector fields $f, g \in C^0(\mathbb{R}^n \times \mathbb{R}; \mathbb{R}^n)$ are $T$-periodic in their second argument, $f$ is homogeneous of degree zero with respect to $\delta^r_\lambda$, and $h$ can be written as a (possibly infinite) sum of homogeneous vector fields, of strictly positive degree, with respect to $\delta^r_\lambda$. If the origin is an asymptotically stable point for

$$\dot{x} = f(x,t)$$

then

(i) it is also globally ρ-exponentially stable
(ii) the origin of the 'perturbed' system

$$\dot{x} = f(x,t) + h(x,t)$$

is locally ρ-exponentially stable.

The following averaging result for (fast) time-varying homogeneous systems will also be used to prove ρ-exponential stability of the controlled system.

*Proposition 3.2.* (M'Closkey and Murray, 1993) Consider the system

$$\dot{x} = f(x, t/\varepsilon), \tag{4}$$

with $f : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ a continuous $T$-periodic vector field ($f(x,t+T) = f(x,t)$) and $f(0,t) = 0$, $\forall t$. Assume that (4) is homogeneous of degree zero with respect to a dilation $\delta^r_\lambda(x)$ and that the origin $y = 0$ of the "averaged system"

$$\dot{y} = \bar{f}(y), \qquad \bar{f}(y) = 1/T \int_0^T f(y,t)dt, \tag{5}$$

is asymptotically stable. Then there exists $\varepsilon_0 > 0$ such that, for any $\varepsilon \in (0, \varepsilon_0)$, the origin $x = 0$ is exponentially stable with respect to the dilation $\delta^r_\lambda(x)$.

The main result that will be used to prove ρ-exponential stability of the controlled system is a result for cascaded high-gain control for a class of homogeneous systems, given in (Morin and Samson, 1997). It concerns the classical problem of integrator backstepping for homogeneous time-varying systems, and is given by the following proposition:

*Proposition 3.3.* (Morin and Samson, 1997) Consider the following system:

$$\dot{x} = f(x, v(x^1, t), t) \tag{6}$$

with $f : \mathbb{R}^n \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}^n$ a continuous $T$-periodic function, $x^1 = (x_1, \ldots, x_m), m \leq n$ and $v : \mathbb{R}^m \times \mathbb{R} \to \mathbb{R}$ a continuous $T$-periodic function, differentiable with respect to $t$, of class $C^1$ on $(\mathbb{R}^m \setminus \{0\}) \times \mathbb{R}$, homogeneous of degree $q$ with respect to the dilation $\delta^r_\lambda(x)$.

Assume that (6) is homogeneous of degree zero with respect to the dilation $\delta^r_\lambda(x)$ and that the origin $x = 0$ is an asymptotically stable equilibrium point. Then for $k$ positive and large enough, the origin $(x = 0, y = 0)$ is an asymptotically stable equilibrium point of the system

$$\begin{aligned} \dot{x} &= f(x, y, t) \\ \dot{y} &= -k(y - v(x^1, t)). \end{aligned} \tag{7}$$

*Remark 3.1.* Proposition 3.3 can be applied recursively to the asymptotic stabilization of the system

$$\begin{aligned} \dot{x} &= f(x, y_1, t) \\ \dot{y_1} &= y_2 \\ &\vdots \\ \dot{y_n} &= u. \end{aligned} \tag{8}$$

Suppose that the feedback $y_1 = v_1(x^1, t)$ asymptotically stabilizes the system, *i.e.* , the first equation in (8). Then by recursive application of Proposition 3.3 it follows that the feedback

$$u = -k_n(y_n - v_{n-1}(x^1, y_1, \ldots, y_{n-1}, t)), \tag{9}$$

where $v_i = -k_i(y_i - v_{i-1}(x^1, y_1, \ldots, y_{i-1}, t))$ for $i = 2, \ldots, n-1$, asymptotically stabilizes the origin.

*Remark 3.2.* Note that if the system (8) is homogeneous of degree zero with respect to some dilation $\bar{\delta}^r_\lambda$, then Proposition (3.1) implies that the closed-loop system (8,9) is *globally* $\rho$-exponentially stable.

## 4. STABILIZATION OF THE EXTENDED CHAINED FORM SYSTEM

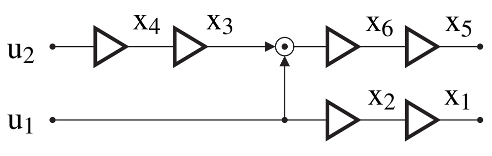


Fig. 1. The extended chained form system.

Consider the equilibrium $x = 0$ of (2). The dynamics of the $(x_5, x_6, x_3, x_4)$-part are in strict feedback form as illustrated in Figure 1. Therefore we can apply a backstepping approach to stabilize the dynamics of $(x_5, x_6, x_3, x_4)$.

The idea of using a combined homogeneous and backstepping approach has already been proposed in (Morin and Samson, 1997). In the following sections this result will be extended to the case of the extended chained form system. First we rewrite the system into

$$\Delta_1 \begin{cases} \dot{x}_5 = x_6 \\ \dot{x}_6 = x_3 u_1 \\ \dot{x}_1 = x_2 \\ \dot{x}_2 = u_1 \end{cases} \Delta_2 \begin{cases} \dot{x}_3 = x_4 \\ \dot{x}_4 = u_2. \end{cases} \tag{10}$$

In the first part of the approach we consider $x_3$ as a "virtual input" and use it, along with the input $u_1$ to stabilize the origin of the $\Delta_1$ subsystem. The second part of the approach consists of using a backstepping technique to stabilize the origin of the complete system $(\Delta_1, \Delta_2)$. This approach is described in the following two sections.

### 4.1 *Stabilizing the $\Delta_1$ subsystem*

The subsystem $\Delta_1$ with $v = x_3$ as a virtual input is given by

$$\Delta_1 \begin{cases} \dot{x}_5 = x_6 \\ \dot{x}_6 = v u_1 \\ \dot{x}_1 = x_2 \\ \dot{x}_2 = u_1 \end{cases} \tag{11}$$

Notice that the vector field $x \mapsto f(x, (u_1, v))$, which defined the $\Delta_1$ subsystem, is not affine in the control variables $(u_1, v)$. Nevertheless, if we define a dilation $\bar{\delta}^r_\lambda$ with weight $\bar{r} = (2, 2, 1, 1)$ and apply feedback functions $u_1 = \alpha_1(x, t)$ and $v = \alpha_2(x, t)$, with $(\alpha_1, \alpha_2) \in C^0(\mathbb{R}^4 \times \mathbb{R}; \mathbb{R}^1)$ r-homogeneous of degree one, then the closed-loop vector field $(x, t) \mapsto f(x, \alpha_1(x, t), \alpha_2(x, t))$ becomes r-homogeneous of degree zero.

Define $x^1 = (x_1, x_2, x_5, x_6)$. Consider the feedback laws $(u_1, v) \in C^0(\mathbb{R}^4 \times \mathbb{R}; \mathbb{R}^1)$ given by

$$\begin{aligned} u_1 &= -k_1 x_1 - k_2 x_2 + h(x^1) g(t/\varepsilon) \\ v &= -\frac{k_5 x_5 + k_6 x_6}{h(x^1)} g(t/\varepsilon) \end{aligned} \tag{12}$$

with $k_i > 0, i \in \{1, 2, 5, 6\}$ and $g : \mathbb{R} \to \mathbb{R}$ a $T$-periodic function satisfying $\int_0^T g(\tau) d\tau = 0$ and $\int_0^T g^2(\tau) d\tau > 0$. The continuous function $h : \mathbb{R}^4 \to \mathbb{R}$ is positive-definite and homogeneous of degree one with respect to $\bar{\delta}^r_\lambda$, for example $h(x^1) = \bar{\rho}(x)$, with $\bar{\rho}(x)$ being a homogeneous norm associated with the dilation $\bar{\delta}^r_\lambda$ given by

$$\bar{\delta}^r_\lambda(x^1) = (\lambda^2 x_5, \lambda^2 x_6, \lambda x_1, \lambda x_2), \tag{13}$$

*Proposition 4.1.* Consider the closed-loop system (11, 12) with $g : \mathbb{R} \to \mathbb{R}$ a continuous $T$-periodic function satisfying $\int_0^T g(\tau) d\tau = 0$ and $\sigma = \frac{1}{T} \int_0^T g^2(\tau) d\tau > 0$. Assume that the continuous function $h : \mathbb{R}^n \to \mathbb{R}$ is homogeneous of degree one with respect to the dilation $\bar{\delta}^r_\lambda(x^1, t)$. Then there exists $\varepsilon_0 > 0$ such that, for all $\varepsilon \in (0, \varepsilon_0)$, the origin of the closed-loop system (11,12) is $\bar{\rho}$-exponentially stable.

**PROOF.** The closed-loop system is given by

$$\begin{aligned}
\dot{x}_5 &= x_6 \\
\dot{x}_6 &= -\frac{(k_5x_5+k_6x_6)}{h(x^1)}\,(-(k_1x_1+k_2x_2)g(t/\varepsilon) \\
&\quad +h(x^1)g(t/\varepsilon)^2) \\
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -k_1x_1-k_2x_2+h(x^1)g(t/\varepsilon)
\end{aligned}$$

This system is homogeneous of order zero with respect to the dilation $\bar{\delta}^r_\lambda$ and can be written as $\dot{x}=f(x,t)$ where $f(x,t)$ is $T$-periodic in $t$. By assumption $h(x^1)$ is homogeneous of degree one with respect to $\delta^r_\lambda$. Therefore the closed-loop system is homogeneous of degree zero with respect to $\bar{\delta}^r_\lambda$. The "averaged system", see (5), is given by

$$\begin{aligned}
\dot{x}_5 &= x_6 \\
\dot{x}_5 &= \sigma(-k_5x_5-k_6x_6) \\
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -k_1x_1-k_2x_2
\end{aligned} \qquad (14)$$

which is globally exponentially stable. The conclusion follows by application of Proposition 3.2.

*Remark 4.1.* The input $v(t,x^1)$ in (12) is not defined for $x^1=0$, *i.e.* $h(x^1)=0$. However, any function $g(x,t)$ that is homogeneous of degree $\tau>0$ with respect to a dilation $\bar{\delta}^r_\lambda(x)$ and continuous for all $x\neq 0$, can be extended by continuity to be continuous at $x=0$. Therefore $v(t,x^1)$ becomes continuous and bounded at $x^1=0$ by defining $v(t,x^1)=\lim_{x^1\to 0} v(t,x^1)=0$ for $x^1=0$.

### 4.2 *Stabilizing the* $(\Delta_1,\Delta_2)$ *subsystem*

We now consider the $\Delta_2$ subsystem. In the previous section we designed a feedback $u_1=\alpha_1(x,t)$ and a virtual feedback $v=\alpha_2(x,t)$ that exponentially stabilized the $\Delta_1$ subsystem w.r.t. the dilation $\bar{\delta}^r_\lambda(x)$. The input $u_2$ can be obtained by using the backstepping approach given in Proposition 3.3.

By Proposition 4.1 the $\Delta_1$ subsystem is asymptotically stabilized by $x_3:=v(x^1,t)$, with $v$ given by (12). The $(\Delta_1,\Delta_2)$ system can be written as

$$\begin{aligned}
\dot{x}^1 &= f(x^1,x_3,t) \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= u_2.
\end{aligned}$$

By recursive application of Proposition 3.3, see Remark 3.1, we conclude that the equilibrium $x=0$ can be asymptotically stabilized by the controller

$$u_2=-k_4(x_4+k_3(x_3+(k_5x_5+k_6x_6)\frac{g(t/\varepsilon)}{h(x^1)})). \qquad (15)$$

Thus, the origin of the extended chained form system (2) can be asymptotically stabilized by the control laws:

$$\begin{aligned}
u_1 &= -k_1x_1-k_2x_2+h(x^1)g(t/\varepsilon) \qquad (16)\\
u_2 &= -k_4(x_4+k_3(x_3+(k_5x_5+k_6x_6)\frac{g(t/\varepsilon)}{h(x^1)})),
\end{aligned}$$

with $k_i>0$, $i\in 1,2,\ldots,6$.

*Corollary 4.2.* The extended chained form (2) is globally exponentially stable with respect to the dilation $\delta^r_\lambda$ given by

$$\delta^r_\lambda(x)=(\lambda x_1,\lambda x_2,\lambda x_3,\lambda x_4,\lambda^2x_5,\lambda^2x_6). \qquad (17)$$

**PROOF.** The weight vector is $r=(1,1,1,1,2,2)$. The vector field $f(x)=(x_2,0,x_4,0,x_6,0)^{\mathrm{T}}$ and the input vector fields $g_1(x)=(0,1,0,0,0,x_3)^{\mathrm{T}}$ and $g_2(x)=(0,0,0,1,0,0)^{\mathrm{T}}$ of (2) are of degree 0, $-1$ and $-1$ respectively with respect to the dilation $\delta^r_\lambda$. The control laws $u_1$ and $u_2$ given in (16) are of degree one with respect to $\delta^r_\lambda(x)$. The closed-loop system is therefore of degree zero with respect to $\delta^r_\lambda$. By application of Proposition 3.1 we conclude that the origin of the closed-loop system is globally $\rho$-exponentially stable with respect to the dilation $\delta^r_\lambda(x^1)$.

## 5. THE V/STOL AIRCRAFT WITHOUT GRAVITY

In this section we study the standard planar V/STOL (vertical/short take off and landing) aircraft (Hauser *et al.*, 1992). The V/STOL aircraft is an underactuated system and can be viewed as a special case of a second-order nonholonomic control system, in the sense that the system is subject to a non-integrable acceleration constraint. If the system is not influenced by gravity, the linear approximation around any equilibrium point is not controllable, and the system can not be stabilized by strict static state feedback. Moreover, due to non-controllability of the linear approximation, it is not possible to exponentially stabilize the system by smooth feedback.

In this section we will use the V/STOL aircraft *without gravity* as a benchmark example and show that it can be globally $\rho$-exponentially stabilized using the developed control laws. The global stabilization of the V/STOL aircraft *with gravity* can be solved using the same control laws.

### 5.1 *Equations of motion*

A longitudinal or planar model of the V/STOL aircraft in hover mode has been developed in (Hauser *et al.*, 1992). Let $(x,y)$ be denote the horizontal position and altitude (inertial coordinates) of the center of mass and let $\theta$ denote the roll angle. The control inputs are the thrust acceleration $v_1$ and the roll acceleration $v_2$. The equations of motion, without gravity, become:

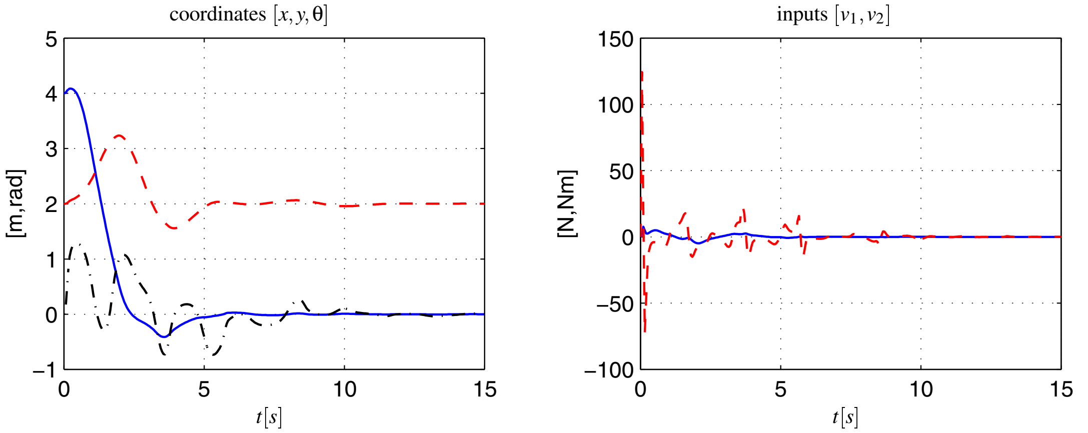


Fig. 2. V/STOL aircraft: $x$ (*solid*), $y$ (*dashed*), $\theta$ (*dash-dotted*), inputs $v_1$ (*solid*), $v_2$ (*dashed*)

$$\begin{aligned}\ddot{x} &= -\sin(\theta)v_1 + \gamma\cos(\theta)v_2 \\ \ddot{y} &= \cos(\theta)v_1 + \gamma\sin(\theta)v_2 \\ \ddot{\theta} &= v_2\end{aligned} \tag{18}$$

where $\gamma$ is the coefficient related to the coupling between the rolling acceleration and the lateral acceleration of the aircraft. Using a coordinate and feedback transformation the V/STOL aircraft dynamics can be transformed into the extended chained form. We will map an arbitrary equilibrium point $q_d = [x_d, y_d, \theta_d]$ to the origin $\xi = 0$ of the extended chained form.

Assume that $\gamma > 0$. The (x,y)-dynamics can be linearized by choosing

$$\begin{aligned}v_1 &= -\sin(\theta)u_x + \cos(\theta)u_y \\ v_2 &= \frac{1}{\gamma}\left(\cos(\theta)u_x + \sin(\theta)u_y\right).\end{aligned} \tag{19}$$

The coordinate and feedback transformations

$$\begin{aligned}\zeta_x &= y + \gamma(\cos(\theta) - \cos(\theta_d)) \\ \zeta_y &= -x + \gamma(\sin(\theta) - \sin(\theta_d)) \\ u_x &= -\sin(\theta)v_1 + \gamma\cos(\theta)v_2 - \gamma\sin(\theta)\dot{\theta}^2 \\ u_y &= \cos(\theta)v_1 + \gamma\sin(\theta)v_2 + \gamma\cos(\theta)\dot{\theta}^2\end{aligned} \tag{20}$$

bring the system into

$$\begin{aligned}\ddot{\zeta}_x &= \cos(\theta)v_1 \\ \ddot{\zeta}_y &= \sin(\theta)v_1 \\ \ddot{\theta} &= v_2\end{aligned} \tag{21}$$

The coordinates $(\zeta_x, \zeta_y)$ have been identified as flat outputs for the system, see (Olfati-Saber, 2000). Finally, for $|\theta - \theta_d| < \pi/2$, the coordinate transformation

$$\begin{aligned}\xi_1 &= \cos(\theta_d)(\zeta_x - y_d) + \sin(\theta_d)(\zeta_y + x_d) \\ \xi_2 &= \tan(\theta - \theta_d) \\ \xi_3 &= -\sin(\theta_d)(\zeta_x - y_d) + \cos(\theta_d)(\zeta_y + x_d)\end{aligned} \tag{22}$$

and input transformation

$$\begin{aligned}v_1 &= \frac{u_1}{\cos(\theta - \theta_d)} \\ v_2 &= \cos^2(\theta - \theta_d)u_2 - 2\tan(\theta - \theta_d)\dot{\theta}^2\end{aligned} \tag{23}$$

bring the system into the extended chained form (1). Concluding, the planar model of a V/STOL aircraft without gravity can be transformed into the extended chained form.

### 5.2 *Simulation*

Define $q = [x, y, \theta]$. Our control objective is to stabilize the equilibrium $q_d = [0, 2, 0]$ starting from an initial condition $(q(0), \dot{q}(0)) = (4, 2, 0, 0, 0, 0)$. We select $\gamma = 0.1$. The controllers are given by (16) with $g(t) = \sin(t)$, $\varepsilon = 1/2$ and the control parameters

$$\begin{aligned}k_1 = 1,\ \ k_2 = 1,\ \ k_3 = 20 \\ k_4 = 20,\ \ k_5 = 2,\ \ k_6 = 2.\end{aligned}$$

The function $h(x^1)$ is chosen equal to the homogeneous norm $\bar{\rho}(x)$ given by (13), *i.e.*

$$h(x_1, x_2, x_5, x_6) = \sqrt{x_1^2 + x_2^2 + |x_5| + |x_6|}$$

The result of stabilizing the equilibrium $q = q_d$ of the V/STOL aircraft is shown in Figure 2. The logarithm of the homogeneous norms $\bar{\rho}(x)$ and $\rho(x)$ associated with (13) and (17) respectively, are shown in Figure 3.

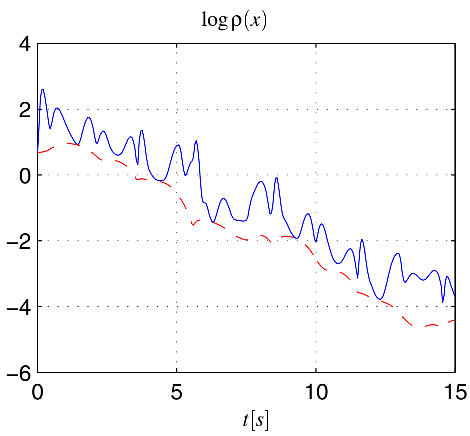


Fig. 3. The logarithm of $\rho(x)$ (-) and $\bar{\rho}(x)$ (- -)

## 6. CONCLUSIONS

We have presented a continuous periodic time-varying controller for stabilization of the extended chained form system. The design approach is inspired by the results in (Morin and Samson, 1997). The controller achieves global $\rho$-exponential stability of the origin of the extended chained form system. The simulations show the effectiveness of our approach. It should be noted that we can only guarantee $\rho$-exponential stability for the extended chained form system, *i.e.* the $\xi$ coordinates. The mechanical system, *i.e.* the $q$ coordinates, is only $\rho$-exponentially stable on a set where the input and coordinate transformation are valid.

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