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DESIGN OF ROBUST CONTROLLERS FOR UNCERTAIN TRANSFER FUNCTIONS IN FACTORED FORM

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Abstract: The paper presents a new method for computations of the magnitude and phase envelopes of uncertain transfer functions. The idea is to factor the transfer function into its real and complex pair roots and find the maximum and the minimum magnitudes of the gain and phase of each factor. The Bode envelopes of the given uncertain system are then found from those of the individual factors. This approach, which is different from those based on the interval polynomial method of Kharitonov, has the major advantage that the representation is more applicable to practical situations where typically the coefficients of the various factored terms relate to physical parameters of a mathematical model. Further the method results in narrower envelopes and therefore improved designs as illustrated in the examples which consider, lead, PI and PID controller designs.

Keywords: Uncertain linear systems; Robust control; Bode diagrams; Envelopes; Frequency responses; Robust stability

1. INTRODUCTION

Frequency response based methods play a fundamental role in the field of classical control theory. Based on the frequency domain properties of control systems, powerful graphical tools such as the Nyquist plot, Bode plots and Nichols chart have been developed in classical control theory for the analysis and design of control systems. In recent years, motivated by the well known Kharitonov and the edge theorems (Kharitonov, 1979; Bartlett, *et al.*, 1988), several results have been published for the extensions of these methods to control systems with parametric uncertainty for robust control system design.

The purpose of this paper is to study the computation of the Bode envelopes and to discuss the design of robust controllers for real uncertain processes using the frequency response approach of classical controller design methods. Frequency response design methods are important in classical control theory since the transient response performance, which is often most important in a control system design, is related in an indirect manner to the frequency response method. For example, the gain and phase margins give a rough estimate of the system damping. Although the relationship between the transient response and the frequency response is indirect, frequency domain specifications can be easily obtained using the Bode plot of a transfer function. The Bode plot of an open loop transfer function of a control system provides a clear indication of how the frequency response should be modified to meet given specifications. Therefore, controller design based on the

Bode plot is simple and straightforward. However, in order to apply this simple design technique to uncertain systems, it is necessary to compute the Bode envelope of a given uncertain system.

It is well known that some process dynamics can be approximated by simple first order models or by a standard second order system. In a more general form the open-loop transfer function of a real physical model can normally be written as

$$G(s) = \frac{\prod_{i=1}^{a}(s^2+2\zeta_{ni}\omega_{ni}s+\omega_{ni}^2)/\omega_{ni}^2}{\prod_{j=1}^{b}(s^2+2\zeta_{dj}\omega_{dj}s+\omega_{dj}^2)/\omega_{dj}^2} \cdot \frac{K\prod_{k=1}^{m}(1+sL_k)}{s^N\prod_{l=1}^{n}(1+sT_l)}e^{-\tau s} \quad (1)$$

where s^N in the denominator represents a pole of multiplicity N at the origin and $2a+m \leq N+n+2b$. It is assumed that the parameters K, L_k, ζ_{ni}, ω_{ni}, ζ_{dj}, ω_{dj}, T_l and τ are not known exactly but vary within intervals as follows

$$\begin{aligned} &K \in [\underline{K}, \overline{K}], L_k \in [\underline{L_k}, \overline{L_k}], \zeta_{ni} \in [\underline{\zeta_{ni}}, \overline{\zeta_{ni}}], \\ &\omega_{ni} \in [\underline{\omega_{ni}}, \overline{\omega_{ni}}], \zeta_{dj} \in [\underline{\zeta_{dj}}, \overline{\zeta_{dj}}], \omega_{dj} \in [\underline{\omega_{dj}}, \overline{\omega_{dj}}], \\ &T_l \in [\underline{T_l}, \overline{T_l}], \tau \in [\underline{\tau}, \overline{\tau}] \end{aligned} \quad (2)$$

Although frequency response computations for uncertain systems such as the Nyquist, Nichols and Bode envelopes has been extensively studied in the literature, the computation of the Bode envelopes of the uncertain transfer functions of the form of Eq.(1), which is the form of most real process systems and leads to simpler results than using the more general formulations usually considered, has not been specifically investigated. It is of course important to point out that computation of the exact boundary of the Nyquist and Nichols envelopes of an uncertain transfer function of the form of Eq.(1) is a difficult and challenging problem. However, for the computation of the Bode envelope the scenario is different since for construction of the Bode envelope one needs to find the magnitude and phase extremums at each frequency. In this paper, a simple procedure is given to construct the boundary of the Bode envelope of such transfer functions. Using the Bode envelope and classical controller design methods, robust controllers are designed for uncertain systems.

The paper is organized as follows: In Section 2, a review of some fundamental results related to frequency response computation of uncertain systems which have been developed in the field of parametric robust control is given. The construction of the Bode envelope of an uncertain transfer function of the form of Eq.(1) is given in Section 3. Robust stability analysis and the design of robust controllers are discussed in Section 4. Section 5 gives some concluding remarks.

2. FREQUENCY RESPONSE OF UNCERTAIN SYSTEMS

The computation of the frequency response of uncertain transfer functions plays a major role for the application of frequency domain methods for the analysis and design of robust control systems. However, the main problem is to find the boundary of the Bode, Nyquist or Nichols envelopes without computing them by simply gridding all the uncertain parameters. The Kharitonov and Kharitonov like results such as the edge theorem and the box theorem have triggered a large amount of very promising research on the computation of frequency responses of uncertain systems. Although it is not possible to review all of these results here, a short review of some important results developed in this direction are summarized below. The extensions of the Kharitonov and Kharitonov like results as well as a discussion of the extensive literature on the area of parametric robust control can be found in the books (Barmish, 1994; Bhattacharyya, 1995).

The Bode, Nyquist and Nichols envelopes of interval transfer functions are studied in (Bailey and Panzer, 1988; Bartlett, *et al.*, 1993; Hollot and Tempo, 1994; Keel and Bhattacharyya, 1994 and references therein). However, the interval uncertainty structure is the simplest one. In a general case, the coefficients of an uncertain polynomial may be affine linear, multilinear or polynomic functions. There are some important results in the literature when the uncertain transfer function has a more complex uncertainty structure than the interval one. For example, in Bailey and Hui (1989), it was assumed that the numerator and the denominator polynomials of the transfer function of the system were independent polynomials with affine linear uncertainty structure and an angle sweeping technique was proposed in order to compute the Nichols template boundary. A $2q$-convex parpolygonal approach was given in Tan and Atherton (2000a) for frequency response computation of control systems with affine linear uncertainty. In (Fu, 1990; Bartlett, 1990), it was assumed that the coefficients of the numerator and the denominator polynomials of a transfer function with affine linear uncertainty were correlated with each other and it was shown that the Nyquist envelope was contained in the set obtained by mapping the exposed edges of the uncertainty box in the complex plane. Some improvements of the results of (Fu, 1990; Bartlett, 1990) were given in Bartlett (1993) for computing the Bode and Nyquist envelopes of uncertain systems. The concept of tree structured decomposition has been introduced in Barmish, *et al.* (1990) to compute the value set of complicated transfer functions. The construction of the value set of uncertain transfer functions in factored real form has been studied in Gutman, *et al.* (1994). The computation of the Bode envelopes and design of robust controllers for uncertain transfer functions has been studied in Tan and Atherton (2000b).

3. CONSTRUCTION OF THE BODE ENVELOPES

This section gives a procedure for the construction of the magnitude and phase envelopes of the uncertain transfer function of Eq.(1). Write Eq.(1) as

$$G(s) = G_1(s) \cdot G_2(s) \tag{3}$$

where

$$G_1(s) = \frac{\prod_{i=1}^{a}(s^2 + 2\zeta_{ni}\omega_{ni}s + \omega_{ni}^2)/\omega_{ni}^2}{\prod_{j=1}^{b}(s^2 + 2\zeta_{dj}\omega_{dj}s + \omega_{dj}^2)/\omega_{dj}^2} \tag{4}$$

and

$$G_2(s) = \frac{K\prod_{k=1}^{m}(1 + sL_k)}{s^N \prod_{l=1}^{n}(1 + sT_l)} e^{-\tau s} \tag{5}$$

The numerator and denominator polynomials of $G_2(s)$ of Eq. (5) are multiplies of independent first order uncertain polynomials. Consider an uncertain real pole $p(s) = 1 + sT$ where $T \in [\underline{T}, \overline{T}]$. For $s = j\omega^*$ ($\omega^* \in [0, \infty)$), the real and imaginary parts of this uncertain pole can be written as $Re[p(\omega^*)] = 1$ and $\underline{T}\omega^* \leq Im[p(\omega^*)] \leq \overline{T}\omega^*$. Therefore, the value set of $p(s)$ at $s = j\omega^*$ is a straight line parallel to the imaginary axis with end(vertex) points $v_1(j\omega^*) = 1 + j\underline{T}\omega^*$ and $v_2(j\omega^*) = 1 + j\overline{T}\omega^*$. Thus, the magnitude and phase extremums of $p(s)$ at $s = j\omega^*$ can be written as

$$\begin{aligned}
max|p(j\omega^*)| &= max(|v_1(j\omega^*)|, |v_2(j\omega^*)|) \\
min|p(j\omega^*)| &= min(|v_1(j\omega^*)|, |v_2(j\omega^*)|) \\
maxarg[p(j\omega^*)] &= maxarg(arg[v_1(j\omega^*)], \\
&\quad arg[v_2(j\omega^*)]) \\
minarg[p(j\omega^*)] &= minarg(arg[v_1(j\omega^*)], \\
&\quad arg[v_2(j\omega^*)])
\end{aligned} \tag{6}$$

For uncertain real zeros the same result can be obtained. Then the magnitude extremums of $G_2(s)$ are

$$\begin{aligned}
max|G_2(s)| &= \frac{\overline{K}.max|1 + L_1 s| \cdots max|1 + L_m s|}{|s|^N.min|1 + T_1 s| \cdots min|1 + T_n s|} \\
min|G_2(s)| &= \frac{\underline{K}.min|1 + L_1 s| \cdots min|1 + L_m s|}{|s|^N.max|1 + T_1 s| \cdots max|1 + T_n s|}
\end{aligned} \tag{7}$$

and the phase extremums are

$$\begin{aligned}
maxarg[G_2(js)] = {}& (maxarg[1 + L_1 s] + \cdots + \\
& maxarg[1 + L_m s] - \omega\underline{\tau}) - (90N + minarg[1 + \\
& T_1 s] + \cdots + minarg[1 + T_n s])
\end{aligned} \tag{8}$$

$$\begin{aligned}
minarg[G_2(js)] = {}& (minarg[1 + L_1 s] + \cdots + \\
& minarg[1 + L_m s] - \omega\overline{\tau}) - (90N + maxarg[1 + \\
& T_1 s] + \cdots + maxarg[1 + T_n s])
\end{aligned} \tag{9}$$

It can be seen that $G_1(s)$ of Eq. (4) has uncertain complex poles and zeros. For clarity of presentation, consider a standard second order system as

$$G(s) = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2} \tag{10}$$

where $\zeta \in [\underline{\zeta}, \overline{\zeta}]$ and $\omega_0 \in [\underline{\omega_0}, \overline{\omega_0}]$. Then the magnitude and phase extremums of this standard second order system can be computed as follows:

If $\zeta > 0.707$ then

$$max|G_1(j\omega)| = |\frac{\overline{\omega_0^2}}{-\omega^2 + 2\underline{\zeta}\overline{\omega_0}j\omega + \overline{\omega_0^2}}| \tag{11}$$

If $\zeta < 0.707$ then from $\omega = 0$ to $\omega_{pmin} = \overline{\omega_0}\sqrt{1 - 2\underline{\zeta}^2}$

$$max|G_1(j\omega)| = |\frac{\underline{\omega_0^2}}{-\omega^2 + 2\underline{\zeta}\underline{\omega_0}j\omega + \underline{\omega_0^2}}| \tag{12}$$

The maximum value of the gain at ω_{pmin} is

$$\frac{1}{2\underline{\zeta}\sqrt{1 - 2\underline{\zeta}^2}} \tag{13}$$

and the gain remains constant at this value until $\omega_{pmax} = \overline{\omega}\sqrt{1 - 2\underline{\zeta}^2}$, and then for $\omega \in [\omega_{pmax}, \infty)$

$$max|G_1(j\omega)| = |\frac{\overline{\omega_0^2}}{-\omega^2 + 2\underline{\zeta}\overline{\omega_0}j\omega + \overline{\omega_0^2}}| \tag{14}$$

The minimum magnitude for all ω is

$$min|G_1(j\omega)| = |\frac{\underline{\omega_0^2}}{-\omega^2 + 2\overline{\zeta}\underline{\omega_0}j\omega + \underline{\omega_0^2}}| \tag{15}$$

The maximum phase for $\omega \in [0, \underline{\omega_0})$ is

$$maxarg[G_1(j\omega)] = arg[\frac{\underline{\omega_0^2}}{-\omega^2 + 2\overline{\zeta}\underline{\omega_0}j\omega + \underline{\omega_0^2}}] \tag{16}$$

and for $\omega \in [\underline{\omega_0}, \infty)$

$$maxarg[G_1(j\omega)] = arg[\frac{\underline{\omega_0^2}}{-\omega^2 + 2\underline{\zeta}\underline{\omega_0}j\omega + \underline{\omega_0^2}}] \tag{17}$$

The minimum phase for $\omega \in [0, \overline{\omega_0})$ is

$$minarg[G_1(j\omega)] = arg[\frac{\overline{\omega_0^2}}{-\omega^2 + 2\underline{\zeta}\overline{\omega_0}j\omega + \overline{\omega_0^2}}] \quad (18)$$

and for $\omega \in [\overline{\omega_0}, \infty)$

$$minarg[G_1(j\omega)] = arg[\frac{\overline{\omega_0^2}}{-\omega^2 + 2\overline{\zeta}\overline{\omega_0}j\omega + \overline{\omega_0^2}}] \quad (19)$$

From these results, the Bode envelopes of $G(s)$ of Eq. (1) can be obtained from

$$\begin{aligned} max|G(s)| &= max|G_1(s)| \cdot max|G_2(s)| \\ min|G(s)| &= min|G_1(s)| \cdot min|G_2(s)| \\ maxarg[G(s)] &= maxarg[G_1(s)] + maxarg[G_2(s)] \\ minarg[G(s)] &= minarg[G_1(s)] + minarg[G_2(s)] \end{aligned} \quad (20)$$

Example 1: Let the plant transfer function be

$$G(s) = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2} \quad (21)$$

where $\omega_0 \in [0.8, 1.6]$ and $\zeta \in [0.5, 1.2]$. Overbounding the parameters, the following interval transfer function $G(s) = ([0.64, 2.56])/(s^2 + [0.8, 3.84]s + [0.64, 2.56])$ is obtained. The Bode envelopes of the system using the method given above(solid line) and the Bode envelopes of the overbounded system(dotted line) are shown in Figure ??. From Figure ??, it is clear that the method of this paper gives a much narrower Bode envelopes.

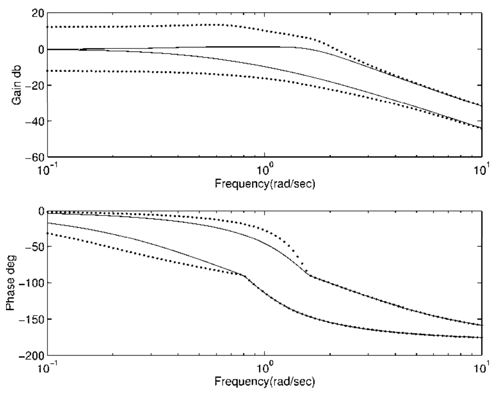

Fig. 1. Bode envelopes

4. APPLICATION OF THE RESULTS

Once the Bode envelope of an uncertain process has been constructed, it can then be used to check the robust stability and to design robust controllers by classical methods as shown in the following examples.

Example 2: In this example the application of the proposed method for determining the maximum gain of transfer function

$$G(s) = \frac{K\omega_0^2}{s(s^2 + 2\zeta\omega_0 s + \omega_0^2)} \quad (22)$$

to maintain stability for $\omega_0 \in [1.5, 1.8]$ and $\zeta \in [0.3, 0.8]$ is illustrated. For this example, one can use the Routh stability criterion and get $K < 2\zeta\omega_0^3/\omega_0^2$ which gives $K < 0.625$ after overbounding the parameters (using interval arithmetic). However, for $K = 0.85$, the Bode envelopes are shown in Figure ?? where one can see that the system just becomes unstable for this value of K. Therefore, the approach of this paper gives maximum value of K as 0.85 which is greater than 0.625. However, one may say that the difference is not big. Let assume that there is an uncertain real pole of the form $1 + Ts$ where $T \in [0.2, 0.5]$ instead of integrator. Then, the Routh criterion gives $K < ((2\zeta\omega_0 T + 1)(2\zeta\omega_0 + \omega_0^2 T)/\omega_0^2 T) - 1$. After overbounding the uncertain parameters, it is computed that $K < -0.016$ for stability. Therefore, there is not any positive value of K for stability. On the other hand, the Bode envelopes for $K = 1.19$ are shown in Figure ?? which clearly show that the system is stable for $K < 1.19$. It is clear that the method presented in this paper gives a large value for K compared with the overbounding one.

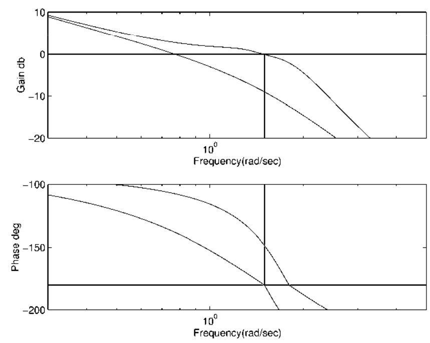

Fig. 2. Bode envelopes for $K = 0.85$

Example 3: Let the plant transfer function be

$$G(s) = \frac{K}{s(T_1 s + 1)(T_2 s + 1)} \quad (23)$$

where $K \in [2.6, 3.5]$, $T_1 \in [0.32, 0.44]$ and $T_2 \in [0.1, 0.14]$. The objective is to design a lead controller of the form

$$C(s) = \frac{Ts + 1}{\alpha Ts + 1}, \quad 0 < \alpha < 1 \quad (24)$$

for which the compensated overall system has a phase margin of 45°. The Bode envelope of

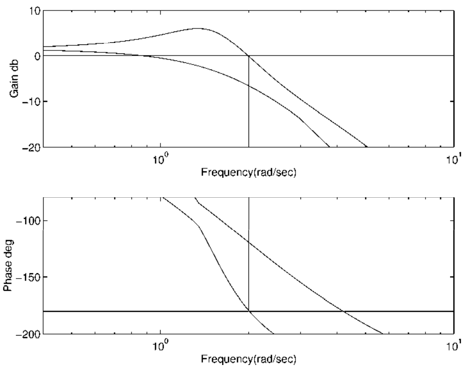

Fig. 3. Bode envelopes for $K = 1.19$

the uncompensated system is shown in Figure ?? where the minimum phase margin is equal to 20°. In order to obtain the required phase margin then

$$45^\circ = 20 + \phi_m - 15^\circ \quad (25)$$

where 15° is the additional phase estimate. Eq.(25) gives $\phi_m = 40^\circ$ and α is found from $\alpha = (1 - sin\phi_m)/(1 + sin\phi_m)$ giving $\alpha = 0.22$. From

$$max|G(j\omega_m)| = -10log_{10}(1/\alpha) = -6.57db \quad (26)$$

giving $\omega_m \approx 4.15rad/sec$. Since

$$\omega_m = \frac{1}{T\sqrt{\alpha}} \quad (27)$$

this gives $T = 0.5$ and the lead controller is

$$C(s) = \frac{0.5s + 1}{0.11s + 1} \quad (28)$$

From Figure ?? it can be seen that the minimum phase margin of the compensated system is greater than 42°.

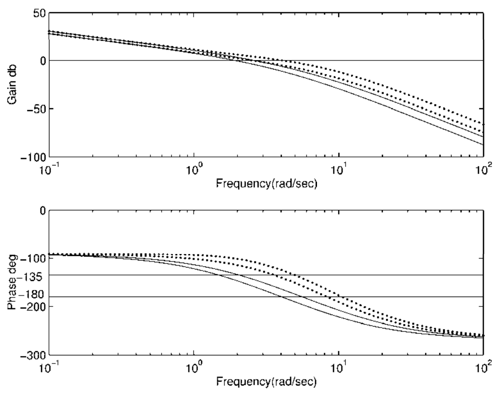

Fig. 4. Bode envelopes(-uncomp., ...comp)

Example 4: Consider

$$G(s) = \frac{K(L_1 s + 1)}{(T_1 s + 1)(T_2 s + 1)(T_3 s + 1)(T_4 s + 1)} \quad (29)$$

where $K \in [8, 10]$, $L_1 \in [0.9, 1.3]$, $T_1 \in [1.5, 2.3]$, $T_2 \in [0.5, 0.7]$, $T_3 \in [0.1, 0.3]$ and $T_4 \in [0.15, 0.2]$. The aim is to design a PI controller of the form

$$C(s) = \frac{K_p s + K_i}{s} \quad (30)$$

which guarantees that the entire family has a phase margin of at least 45°. From Figure ?? it can be seen that the new maximum gain crossover frequency of the family should be moved to $\omega_1 = 2.21rad/sec$ where

$$180 + minarg[G(j\omega_1)] \approx 50^\circ \quad (31)$$

The maximum magnitude of $G(s)$ at $s = j\omega_1$ is $14.8db$. Thus, from $-20log_{10}K_p = max|G(j\omega_1)| = 14.8db$, $K_p = 0.18$. One can choose the corner frequency K_i/K_p to be one decade below ω_1 in order to ensure that the phase lag of the PI compensator only affects the phase of the compensated system at ω_1 by approximately 5° allowed in Eq.(31). Therefore, from $K_i = (\omega_1/10)K_p = (2.21/10)0.18 = 0.04$. Thus, the designed PI controller is

$$C(s) = \frac{0.18s + 0.04}{s} \quad (32)$$

From Figure ?? it can be seen that the minimum phase margin of the compensated system is greater than 45°.

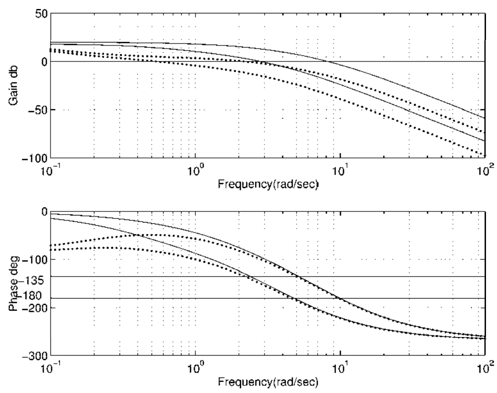

Fig. 5. Bode envelopes(-uncomp., ...comp)

Example 5: Consider a plant with

$$G(s) = \frac{K}{(T_1 s + 1)(T_2 s + 1)} e^{-\tau s} \quad (33)$$

where $K \in [0.6, 1.4]$, $T_1 \in [0.4, 0.6]$, $T_2 \in [1.6, 2.4]$ and $\tau \in [0.3, 0.6]$. The objective is to find the parameters of a PID controller of the form

$$C(s) = K_p(1 + \frac{1}{sT_i} + sT_d) \quad (34)$$

using the critical point information, critical frequency ω_c and critical gain K_c, only. There are several approaches to designing(tuning) a controller given ω_c and K_c for a process. One of the simplest approaches, although not recommended if the set point response has to have small overshoot, is the Ziegler-Nichols approach which suggests that for a PID controller of the form of Eq.(34), $K_p = 0.6K_c$, $T_i = 0.5T_c$ and $T_i = 4T_d$ where $T_c = 2\pi/\omega_c$.

From the uncompensated Bode envelope shown in Figure ??, it can be seen that the frequency where the minimum gain margin is achieved is equal to $1.71 rad/sec$ and at this frequency the minimum gain margin of the family is equal to $7.2db$. From, $20log_{10}K_c = 7.2db$, one gets $K_c = 2.3$. Thus, using the Z-N approach, the following PID controller is designed

$$C(s) = \frac{1.17s^2 + 2.54s + 1.38}{1.84s} \tag{35}$$

From the Bode envelopes of Figure ?? it was found that the phase margin of the compensated system is greater than $46°$ and the gain margin is greater than $6.7db(2.16)$.

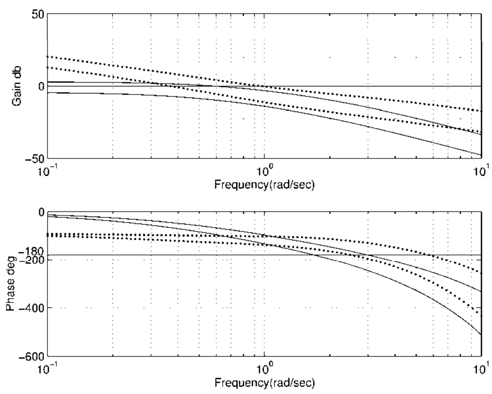

Fig. 6. Bode envelopes(-uncomp., ...comp)

5. CONCLUSION

Lag/lead, PI and PID controllers are popular and widely used in industrial applications. The popularity of these types of controllers is due to their simplicity and satisfactory performance for a wide class of systems. In classical control engineering, design methods for these types of controllers are based on a plant with fixed parameters. However, most practical systems contain uncertainties. In this paper, the design of robust controllers for real uncertain systems has been studied. A new method has been presented for the computation of the Bode envelopes of an uncertain transfer function of the form of Eq.(1). Based on the Bode envelope, robust lead, PI and PID controllers have been designed for different uncertain systems.

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