15th Triennial World Congress, Barcelona, Spain

MODELING AND DYNAMIC FEEDBACK LINEARIZATION OF A MULTI-STEERED N-TRAILER [1]

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Abstract: This paper develops the kinematic model of a class of multi steered general n-trailer system. The proposed kinematic model includes as particular cases the general and standard n-trailer systems commonly found in the literature. It is shown that the system is completely linearizable by dynamic state feedback and the explicit design of this dynamic compensator is also presented. *Copyright © 2002 IFAC*

Keywords: Mobile robots, Kinematics, Nonlinear systems, Feedback linearization, Models.

1. INTRODUCTION.

Wheeled mobile robots are nonholonomic systems because the velocities along the reference axis satisfy non integrable constraints. Trailer-like systems constitute a generalization of mobile robots. They are composed of a mobile robot, and several trailers pulled by the mobile robot. Trailer systems satisfy nonholonomic restrictions as well.

The development of mathematical models to describe the properties of wheeled mobile robots without trailers has been widely studied, see for instance, (Campion, 1996; Canudas de Witt, 1996). In these works the models were obtained under non slipping assumptions for the robot wheels.

There are two models of trailer systems frequently considered in the literature, the standard n-trailer and the general n-trailer (Rouchon, 1993; Bushnell, 1993; Altafini, 1998; Lamiraux, 1999). The standard n-trailer systems has been extensively studied because their kinematic model is written in a simple recursive form. The kinematic model of general n-trailer systems are closer to real trailers and they can be found also in the literature. These two models are widely studied and methods of exact linearization have been proposed for them. For example, in (Rouchon, 1993), the general 1-trailer system is shown to be differentially flat.

In this paper a class of multi steered general n-trailer system is introduced. This system consist of a wheeled mobile robot with fixed traction wheels and n trailers with steered direction. This system is a generalization of the standard n-trailer systems and general n-trailer systems.

This paper address two main topics. First the modeling of a class of multi steered general n-trailer model is addressed. Second, it is shown that the this system is completely linearizable by dynamic state feedback. The linearizability of the kinematic model is established by following the approach presented in (Aranda-Bricaire, 1995)

The paper is organized as follows. In Section 2, the modelling of a class of multi steered general n-trailer is presented. In Section 3 a structural

[1] Partially supported by CONACyT-MEXICO under grant No.34850-A.
[2] Supported by CONACyT-MEXICO

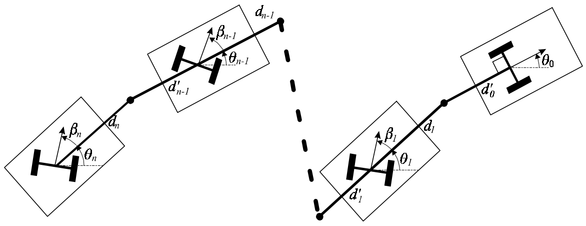

Fig. 1. Multi steered general n-trailer system.

analysis of this system is presented, showing that it admits a linearizing output. In section 4 the explicit design of the dynamic compensator is presented. Finally, some conclusions are given in Section 5.

2. MODELING.

Two models have been widely studied in the literature, the standard and general n-trailer systems. These models possess two control variables; namely, the linear and angular velocities of the tractor. The multi steered general n-trailer that is proposed in this work is intended to have further control variables, corresponding to steering direction of the trailers.

In this Section the model of a multi steered trailer is developed, for which *all* the trailers possess actuated wheels. The obtained model can be easily adapted to cope with the case of a n-trailer system without all the direction wheel actuated. The model also includes as particular cases the standard and general n-trailer models.

The *multi steered general n-trailer* considered in this work is shown in Figure 1. Where for simplicity, just the first two and last two trailers are depicted.The development of the general model will be done in two steps. First, the kinematic model of the steering mobile robot will be presented as an independent system. Second, a kinematic model corresponding to each trailer will be developed.

The kinematic model of *multi steered general n-trailer* consists of three state variables to represent the position and orientation of the tractor with respect to the reference axis, and two additional variables for each trailer to represent their orientation with respect to the previous trailer and their steering direction (orientation of the wheels axis). More precisely, a fully actuated multi steered general n-trailer possesses $n+2$ input variables and $2n+3$ state variables.

2.1 *Kinematic model of the tractor.*

In Figure 2 it is shown the tractor considered in this work. Simple geometric considerations produce

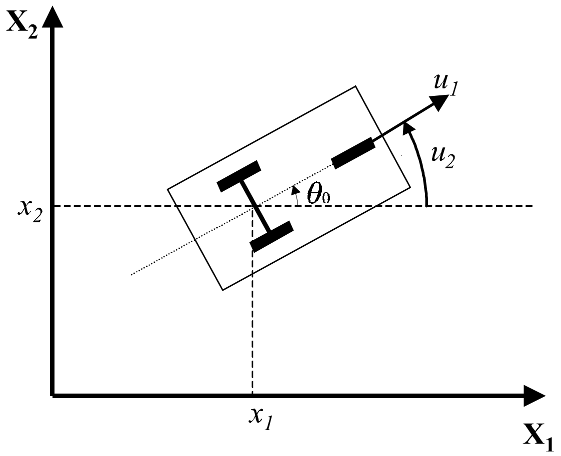

Fig. 2. Wheeled mobile robot with two fixed wheels.

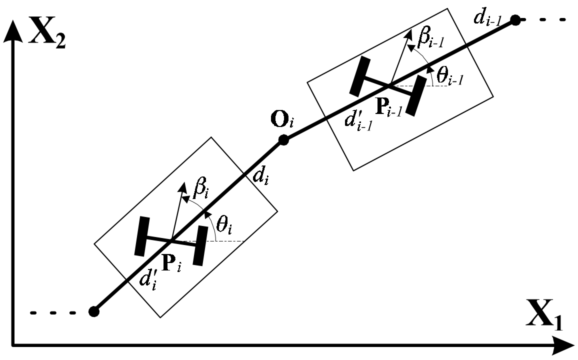

Fig. 3. $(i-1)$-th. and i-th. trailer of the multi steered general n-trailer system.

$$\begin{aligned} \dot{x}_1 &= u_1 \cos\theta_0 \\ \dot{x}_2 &= u_1 \sin\theta_0 \\ \dot{\theta}_0 &= u_2, \end{aligned} \tag{1}$$

where the inputs u_1, u_2 represent, respectively, the linear and angular velocities. The state variables (x_1, x_2, θ_0) correspond to the position and orientation of the robot.

2.2 *Trailers kinematic model.*

In order to obtain the kinematic model of the i-th trailer, consider the i-th and the $(i-1)$-th trailers in Figure 3.

The notations in Figure 3 are as follows: $\boldsymbol{v}_{\mathbf{O}_i}$, $\boldsymbol{v}_{\mathbf{P}_{i-1}}$, and $\boldsymbol{v}_{\mathbf{P}_i}$ represent, respectively, the linear velocities of the points $\mathbf{O}_i$, $\mathbf{P}_{i-1}$ and $\mathbf{P}_i$; v_{i-1} and v_i represent the magnitude of $\boldsymbol{v}_{\mathbf{P}_{i-1}}$ and $\boldsymbol{v}_{\mathbf{P}_i}$; $\boldsymbol{\omega}_{i-1}$ and $\boldsymbol{\omega}_i$ represent the angular velocities of the i − 1-th and i-th trailers; $\boldsymbol{r}_{\mathbf{O}_i/\mathbf{P}_{i-1}}$ and $\boldsymbol{r}_{\mathbf{O}_i/\mathbf{P}_i}$ represent, respectively, the position vector of the point $\mathbf{O}_i$ with respect to the points $\mathbf{P}_{i-1}$ and $\mathbf{P}_i$. Finally, $\hat{\jmath}$ and $\hat{\imath}$ are unitary vectors along the axis $\mathbf{X}_1$ and $\mathbf{X}_2$, and $\hat{k}$ is a unitary vector perpendicular to the plane on which the vehicle moves.

The linear velocity $\boldsymbol{v}_{\mathbf{O}_i}$ of the point $\mathbf{O}_i$ considered as a part of the $(i-1)$-th trailer is given by

$$\boldsymbol{v}_{\mathbf{O}_i} = \boldsymbol{v}_{\mathbf{P}_{i-1}} + \boldsymbol{\omega}_{i-1} \times \boldsymbol{r}_{\mathbf{O}_i/\mathbf{P}_{i-1}} \tag{2}$$

where,

$$\begin{aligned} \boldsymbol{\omega}_{i-1} &= \hat{k}\dot{\theta}_{i-1} \\ \boldsymbol{r}_{\mathbf{O}_i/\mathbf{P}_{i-1}} &= -d'_{i-1}(\hat{\imath}\cos\theta_{i-1} + \hat{\jmath}\sin\theta_{i-1}), \end{aligned}$$

and

$$\begin{aligned} \boldsymbol{v}_{\mathbf{P}_{i-1}} = v_{i-1}[&\hat{\imath}\cos\left(\beta_{i-1} + \theta_{i-1}\right) \\ &+ \hat{\jmath}\sin\left(\beta_{i-1} + \theta_{i-1}\right)]. \end{aligned}$$

In a similar manner, the linear velocity of the point $\mathbf{O}_i$, considered as part of the i-th trailer is given by

$$\boldsymbol{v}_{\mathbf{O}_i} = \boldsymbol{v}_{\mathbf{P}_i} + \boldsymbol{\omega}_i \times \boldsymbol{r}_{\mathbf{O}_i/\mathbf{P}_i} \tag{3}$$

where,

$$\begin{aligned} \boldsymbol{\omega}_i &= \hat{k}\dot{\theta}_i \\ \boldsymbol{r}_{\mathbf{O}_i/\mathbf{P}_i} &= -d'_i\left(\hat{\imath}\cos\theta_i + \hat{\jmath}\sin\theta_i\right) \end{aligned}$$

and

$$\boldsymbol{v}_{\mathbf{P}_i} = v_i\left[\hat{\imath}\cos\left(\beta_i + \theta_i\right) + \hat{\jmath}\sin\left(\beta_i + \theta_i\right)\right].$$

Combining the expressions (2) and (3) it is possible to obtain

$$v_i = \frac{v_{i-1}\cos\left(\phi_i + \beta_{i-1}\right) + \dot{\theta}_{i-1}d'_{i-1}\sin\phi_i}{\cos\beta_i} \tag{4}$$

$$\dot{\theta}_i = \frac{v_{i-1}\sin\left(\alpha_i + \beta_{i-1}\right) - \dot{\theta}_{i-1}d'_{i-1}\cos\alpha_i}{d_i\cos\beta_i}, \tag{5}$$

where $\phi_i = \theta_{i-1} - \theta_i$, $\alpha_i = \phi_i - \beta_i$ and $\beta_0 = 0$.

Finally, the kinematic model of the i-th trailer can be written in recursive form as,

$$\begin{aligned} \dot{\theta}_i &= \frac{v_{i-1}\sin\left(\alpha_i + \beta_{i-1}\right) - \dot{\theta}_{i-1}d'_{i-1}\cos\alpha_i}{d_i\cos\beta_i} \\ \beta_i &= u_{i+2}. \end{aligned}$$

Proposition 2.1. *The recursive representation of $\dot{\theta}_i$ can be rewritten as*

$$\dot{\theta}_i = a_i(\cdot)u_1 + b_i(\cdot)u_2,$$

where $a_i(\cdot)$ and $b_i(\cdot)$ are functions of $(\theta_0, ..., \theta_i, \beta_1, ..., \beta_i)$ for $i = 1, 2, ..., n$.

Proof. The proof will be done by induction. First, for $i = 1$, equation (4) and (5) produce,

$$\begin{aligned} \dot{\theta}_1 &= \frac{\sin\alpha_1}{d_1\cos\beta_1}u_1 - \frac{d'_0\cos\alpha_1}{d_1\cos\beta_1}u_2 \\ v_1 &= \frac{\cos\phi_1}{\cos\beta_1}u_1 + \frac{d'_0\sin\phi_1}{\cos\beta_1}u_2, \end{aligned}$$

that can be rewritten as

$$\begin{aligned} \dot{\theta}_1 &= a_1u_1 + b_1u_2 \\ v_1 &= r_1u_1 + q_1u_2. \end{aligned}$$

For the k-step assume that,

$$\begin{aligned} \dot{\theta}_k &= a_ku_1 + b_ku_2 \\ v_k &= r_ku_1 + q_ku_2. \end{aligned} \tag{6}$$

Then at the $(k+1)$-step, considering again equations (4) and (5),

$$\begin{aligned} \dot{\theta}_{k+1} &= \frac{v_k\sin\left(\alpha_{k+1} + \beta_k\right) - \dot{\theta}_kd'_k\cos\alpha_{k+1}}{d_{k+1}\cos\beta_{k+1}} \\ v_{k+1} &= \frac{v_k\cos\left(\phi_{k+1} + \beta_k\right) + \dot{\theta}_kd'_k\sin\phi_{k+1}}{\cos\beta_{k+1}}. \end{aligned}$$

Substitution of (6) in the above equation produces,

$$\begin{aligned} \dot{\theta}_{k+1} = &\frac{r_k\sin\left(\alpha_{k+1} + \beta_k\right) - a_kd'_k\cos\alpha_{k+1}}{d_{k+1}\cos\beta_{k+1}}u_1 \\ &+ \frac{q_k\sin\left(\alpha_{k+1} + \beta_k\right) - b_kd'_k\cos\alpha_{k+1}}{d_{k+1}\cos\beta_{k+1}}u_2 \\ v_{k+1} = &\frac{r_k\cos\left(\phi_{k+1} + \beta_k\right) + a_kd'_k\sin\phi_{k+1}}{\cos\beta_{k+1}}u_1 \\ &+ \frac{q_k\cos\left(\phi_{k+1} + \beta_k\right) + b_kd'_k\sin\phi_{k+1}}{\cos\beta_{k+1}}u_2, \end{aligned}$$

or equivalently,

$$\begin{aligned} \dot{\theta}_{k+1} &= a_{k+1}u_1 + b_{k+1}u_2 \\ v_{k+1} &= r_{k+1}u_1 + q_{k+1}u_2. \end{aligned} \tag{7}$$

Thus, the proposition is proven. ■

2.3 *Complete kinematic model.*

From the tractor model (1) and the Proposition 2.1, the complete kinematic model of the multi steered general n-trailer system can now be given by

$$\dot{x} = g\left(x\right)u, \tag{8}$$

where

$$\begin{aligned} x &= \begin{bmatrix} x_1 & x_2 & \theta_0 & \theta_1 & \cdots & \theta_n & \beta_1 & \ldots & \beta_n \end{bmatrix}^T \\ u &= \begin{bmatrix} u_1 & \cdots & u_{n+2} \end{bmatrix}^T \\ g\left(x\right) &= \begin{bmatrix} \cos\theta_0 & 0 & \mathbf{0}_{2\times n} \\ \sin\theta_0 & 0 & \\ a_0 & b_0 & \\ \vdots & \vdots & \mathbf{0}_{(n+1)\times n} \\ a_n & b_n & \\ \mathbf{0}_{n\times 2} & & \mathbf{I}_{n\times n} \end{bmatrix}. \end{aligned}$$

and

$$
\begin{aligned}
a_0 &= 0, \quad b_0 = 1, \quad q_0 = 0, \quad r_0 = 1 \\
a_i &= \frac{r_{i-1}\sin\left(\alpha_i+\beta_{i-1}\right) - a_{i-1}d'_{i-1}\cos\alpha_i}{d_i\cos\beta_i} \\
b_i &= \frac{q_{i-1}\sin\left(\alpha_i+\beta_{i-1}\right) - b_{i-1}d'_{i-1}\cos\alpha_i}{d_i\cos\beta_i} \\
r_i &= \frac{r_{i-1}\cos\left(\phi_i+\beta_{i-1}\right) + a_{i-1}d'_{i-1}\sin\phi_i}{\cos\beta_i} \\
q_i &= \frac{q_{i-1}\cos\left(\phi_i+\beta_{i-1}\right) + b_{i-1}d'_{i-1}\sin\phi_i}{\cos\beta_i}
\end{aligned}
$$

Note that the recursive system (8) has been written as a nonlinear control affine system. The properties of this class of systems have been widely treated in the literature (Isidori, 1995). Also notice that system (8) is driftless. This property comes from the fact that any position in an equilibrium point if $u_i = 0$, for $i = 1, ..., n+2$.

Remark 2.1. *The model (8) include as a particular cases:*

(1) *The general n-trailer system without all steered trailers is obtained by letting $\beta_j = 0$ for each j-th trailer without actuated wheels.*
(2) *The model of the general n-trailer is obtained by letting $\beta_i = 0$, for $i = 1, 2, ..., n$.*
(3) *The model of the standard n-trailer is obtained by letting $\beta_i = 0$, $d'_i = 0$, for $i = 1, 2, ..., n$.*

3. LINEARIZATION PROPERTIES.

This Section addresses the feedback linearization problem for the kinematic model of the multi steered general n-trailer system (8). In particular, it will be shown that this system has a linearizing output.

Definition 3.1. *An output function is said to be a linearizing output if it satisfies the following conditions:*

(1) *The system is right-invertible with respect to this output function.*
(2) *The system possesses trivial zero dynamics with respect to this output.*

For more details about the notion of linearizing output the reader is referred to (Aranda-Bricaire, 1995; Isidori, 1986).

In the event that a system admits a linearizing output, the so-called *standard noninteracting feedback* linearizes both the input-output and input-state responses of the system.

In what follows it will be established the linearization properties of the multi steered general n-trailer.

In order to establish the main result, the following technical results are needed.

Lemma 3.1. *The functions $\frac{\partial\dot\theta_i}{\partial\beta_i}$ are generically different from zero.*

Corollary 3.1. *The second order time derivatives $\ddot x_1$ and $\ddot x_2$ are linear functions of u_2 and $\dot u_1$. The second order time derivatives $\ddot\theta_i$ are linear functions of $u_1, \ldots, u_{i+2}, \dot u_1$, and $\dot u_2$.*

Lemma 3.1 and Corollary 3.1 can be proved directly by obtaining the second order derivatives of x_1, x_2 and θ_i.

Theorem 3.1. *Consider the multi steered general n-trailer system (8), and the output function*

$$
y = h(x) = \left[x_1, x_2, \theta_1, ..., \theta_n\right]^T. \tag{9}
$$

Then, the system (8)-(9) is right-invertible and possesses trivial zero dynamics. As a consequence system (8)-(9) is fully linearizable by dynamic state feedback.

Proof. Recall that $\dim x = 2n+3$, and $\dim u = n+2$. Define the sequence of subspaces $E_k = span\left\{dx, dy, ..., dy^{(k)}\right\}$, $k \geq 0$. Following (Di Benedetto, 1989), system (8)-(9) is invertible if there exist an integer $0 \leq k \leq 2n+3$, such that

$$
\dim E_k - \dim E_{k-1} = n+2.
$$

Consider now the following sets of one-forms:

$$
\begin{aligned}
dx &= \{dx_1, dx_2, d\theta_0, ..., d\theta_n, d\beta_1, ..., d\beta_n\} \\
dy &= \{dx_1, dx_2, d\theta_1, ..., d\theta_n\} \\
d\dot y &= \left\{d\dot x_1, d\dot x_2, d\dot\theta_1, ..., d\dot\theta_n\right\} \\
d\ddot y &= \left\{d\ddot x_1, d\ddot x_2, d\ddot\theta_1, ..., d\ddot\theta_n\right\}.
\end{aligned}
$$

From equation (8) it is possible to write,

$$
d\dot y \in span\left\{d\theta_0, ..., d\theta_i, d\beta_1, ..., d\beta_i, du_1, du_2\right\}.
$$

To obtain $d\ddot y$, consider first

$$
\begin{aligned}
\ddot x_1 &= -u_1u_2\sin\theta_0 + \dot u_1\cos\theta_0 \\
\ddot x_2 &= u_1u_2\cos\theta_0 + \dot u_1\sin\theta_0 \\
\ddot\theta_i &= b_i\dot u_2 + a_i\dot u_1 + \sum_{j=0}^{i}\frac{\partial\dot\theta_i}{\partial\theta_j}\dot\theta_j + \sum_{j=1}^{i}\frac{\partial\dot\theta_i}{\partial\beta_j}u_{j+2}, \\
&\quad i = 1, ..., n,
\end{aligned}
$$

where,

$$
\frac{\partial\dot\theta_i}{\partial\beta_i} = u_1\frac{\partial a_i}{\partial\beta_i} + u_2\frac{\partial b_i}{\partial\beta_i}.
$$

Then, it is possible to see that,

$$
d\ddot y \in span\left\{d\dot y, du_3, ..., du_{n+2}, d\dot u_1, d\dot u_2\right\}.
$$

Therefore, from the above developments the subspaces E_k are obtained as,

$$\begin{aligned}
E_0 &= span\{dx, dy\} \\
&= span\{dx_1, dx_2, d\theta_0, ..., d\theta_n, d\beta_1, ..., d\beta_n\} \\
E_1 &= span\{dx, dy, d\dot{y}\} \\
&= E_0 \oplus span\{du_1, du_2\} \\
E_2 &= span\{dx, dy, d\dot{y}, d\ddot{y}\} \\
&= E_1 \oplus span\{du_3, ..., du_{n+2}, d\dot{u}_1, d\dot{u}_2\},
\end{aligned}$$

that produce,

$$\begin{aligned}
\dim E_0 &= 2n+3 \\
\dim E_1 &= 2n+5 \\
\dim E_2 &= 3n+7.
\end{aligned}$$

Since, for $k = 2$,

$$\dim E_2 - \dim E_1 = n+2$$

then, system (8)-(9) is right invertible.

Define $X = span\{dx\}$, and $Y = span\left\{dy^{(k)}, k \geq 0\right\}$. It is well known (see for instance (Aranda-Bricaire, 1995)), that the system (8)-(9) does not possess zero dynamics if

$$\dim(X \cap Y) = 2n+3.$$

Noting that,

$$\begin{aligned}
X &= span\{dx\} \\
&= span\{dx_1, dx_2, d\theta_0, ..., d\theta_n, d\beta_1, ..., d\beta_n\} \\
Y_1 &= span\{dy, d\dot{y}\} \\
&= span\{dx_1, dx_2, d\theta_0, ..., d\theta_n, d\beta_1, ..., d\beta_n, du_1, du_2\},
\end{aligned}$$

since $Y_1 \subset Y$, it follows that

$$\dim(X \cap Y) = 2n+3.$$

Then, system (8)-(9) has trivial zero dynamics. ■

4. LINEARIZING FEEDBACK DESIGN.

This section addresses the design of a linearizing feedback for the kinematic model (8)-(9) of the multi steered general n-trailer system.

The decupling matrix of the system is given by

$$D(\theta_0, ..., \theta_n, \beta_1, ..., \beta_n) = \begin{bmatrix} \cos\theta_0 & 0 & \\ \sin\theta_0 & 0 & \\ a_1 & b_1 & \mathbf{0}_{n+2\times n} \\ \vdots & \vdots & \\ a_n & b_n & \end{bmatrix}.$$

Clearly the matrix $D(\cdot)$ is noninvrtible. Therefore, it is not possible to decuple the input-output response of the system by static feedback. Then a dynamic extension of the system is necessary.

The dynamic extension consist in adding two pure integrators in front of the input u_1, and one pure integrator in front of the input u_2. That is:

$$\begin{aligned}
&u_1 = \xi_1,\ \dot{\xi}_1 = \xi_3,\ \dot{\xi}_3 = w_1, \\
&u_2 = \xi_2,\ \dot{\xi}_2 = w_2, \\
&u_{i+2} = w_{i+2}, \qquad i = 1, 2, ..., n
\end{aligned} \tag{10}$$

where $w = (w_1, ..., w_{n+2})^T$ is the new input vector for the extended system. This dynamic extension produce a simpler singular manifold that the one produced by classic dynamic extension algorithm (Isidori, 1995).

Taking successive time-derivatives of the output function (9) along the trayectories of the extended system (8)-(10) produces

$$\begin{bmatrix} y_1^{(3)} \\ y_2^{(3)} \\ y_3^{(2)} \\ \vdots \\ y_{n+2}^{(2)} \end{bmatrix} = \begin{bmatrix} x_1^{(3)} \\ x_2^{(3)} \\ \theta_1^{(2)} \\ \vdots \\ \theta_n^{(2)} \end{bmatrix} = \mathbf{A}(x, \xi) w + \mathbf{F}(x, \xi) \tag{11}$$

where

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{0}_{2\times n} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} -2\xi_2\xi_3 \sin\theta_0 - \xi_1\xi_2^2 \cos\theta_0 \\ 2\xi_2\xi_3 \cos\theta_0 - \xi_1\xi_2^2 \sin\theta_0 \\ a_1\xi_3 + \sum_{j=0}^{1} \frac{\partial\dot{\theta}_1}{\partial\theta_j}\dot{\theta}_j \\ \vdots \\ a_n\xi_3 + \sum_{j=0}^{n} \frac{\partial\dot{\theta}_1}{\partial\theta_j}\dot{\theta}_j \end{bmatrix},$$

and

$$\mathbf{A}_{11} = \begin{bmatrix} \cos\theta_0 & \xi_1 \sin\theta_0 \\ \sin\theta_0 & \xi_1 \cos\theta_0 \end{bmatrix}$$

$$\mathbf{A}_{21} = \begin{bmatrix} 0 & b_1 \\ \vdots & \vdots \\ 0 & b_n \end{bmatrix}$$

$$\mathbf{A}_{22} = \begin{bmatrix} \frac{\partial\dot{\theta}_1}{\partial\beta_1} & 0 & ... & 0 \\ \frac{\partial\dot{\theta}_2}{\partial\beta_1} & \frac{\partial\dot{\theta}_2}{\partial\beta_2} & ... & 0 \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial\dot{\theta}_n}{\partial\beta_1} & \frac{\partial\dot{\theta}_n}{\partial\beta_2} & ... & \frac{\partial\dot{\theta}_n}{\partial\beta_n} \end{bmatrix}.$$

Proposition 4.1. *The matrix $A(x)$ is generically nonsingular.*

Proof. First note that

$$\det \mathbf{A}_{11} = \xi_1.$$

Next, from Lemma 3.1, it can be conclude that

$$\det \mathbf{A}_{22} = \prod_{j=1}^{n} \frac{\partial \dot{\theta}_j}{\partial \beta_j}$$

is generically different from zero, then

$$\det A(x,\xi) = \xi_1 \prod_{j=1}^{n} \frac{\partial \dot{\theta}_j}{\partial \beta_j}$$

is generically different from zero. ■

Then from (11) the dynamic linearizing feedback is given by

$$w = \mathbf{A}(x,\xi)^{-1} [v - \mathbf{F}(x,\xi)], \tag{12}$$

where $v = (v_1, ..., v_{n+2})$ is a new control variable.

Remark 4.1. *The relative degree of the extended system is* $\{3, 3, 2, ..., 2\}$*, and its sum is* $2n+6$*, then since the dimension of the system (8)-(9) is* $2n+3$*, and the dimension of the dynamic compensator is* 3*, then the linearizing feedback (12) linearize completely the system.*

Remark 4.2. *The control law (12) presents a singularity or, more precisely, is undefined when the state belongs to the so called singular manifold. The singular manifold is defined by*

$$S = \left\{ (x,\xi) \in \mathbf{R}^{2n+6} \mid \xi_1 \prod_{j=1}^{n} \frac{\partial \dot{\theta}_j}{\partial \beta_j} = 0 \right\}. \tag{13}$$

Due to the existence of the singular manifold (13), the control law (12) is not globally defined. Since this paper is devoted to the structural generic properties of the multisteeered general n-trailer, we have decided not to deal with the problem of singularities. This problem was addressed in a future paper, where a discontinuous control law was proposed which allows global motion planning for the multisteered general n-trailer.

5. CONCLUSIONS.

In this paper it is presented the kinematic model of a multi steered general n-trailer system. It is shown that this model includes as a particular cases the general and the standard n-trailer systems previously studied in the literature. The multi steered general n-trailer system presents a general form of a model for trailer systems. This can be used to obtain another trailer models with less steered directions and study their properties. It is shown that this trailer systems can be written as nonlinear control affine systems. It also is proven that the system is completely linearizable by dynamic state feedback under the assumption that all the steering wheels are actuated.

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