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OUTPUT REGULATION OF REACTOR SYSTEMS WITH ACTUATOR CONSTRAINTS

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Abstract: This paper deals with a catalytic continuous stirred tank reactor (CCSTR) example, modeled as a two-input constrained nonlinear system. Based on the Lyapunov-based linearization strategy, the saturation-type parameterized control design can reduce the effect of actuator subject to amplitude and rate constraints, and the integrated two-input control framework can ensure the asymptotic output regulation and robustness against unknown disturbances. Our results are illustrated via numerical simulation. *Copyright © 2002 IFAC*

Keywords: Feedback linearization; Lyapunov methods; Actuators; Constraints; Non-minimum phase systems.

1. INTRODUCTION

In every physically meaningful process, process variables are naturally bounded. Generally, the magnitude of a manipulated variable can only vary between the upper and lower limits of the corresponding actuator. In fact, it is questionable whether the control design procedure ignores the physical constraint of the actuator. For instance, developments in differential geometric control technique fall in the class of analytical model-based control; their approaches may reach the limit of control action so that the linearization fails and performance degradation or instability may occur. In the earlier anti-windup methods, Calvet and Arkun (1988) opted for the reconstruction of the external linear input by observing the current states as well as the original input so that the linear internal model control-based framework can enhance the regulation performance. Valluri and Soroush (1998) developed analytical nonlinear schemes that offer great flexibility to a desirable closed-loop output response in the presence of input constraints. Recently, integrating robustness and constraints in control designs have received significant attention. Alvarez-Ramirez (1999) provided a novel stability analysis of a class of CSTR systems in the presence of input saturations and uncertain chemical kinetics. El-Farra and Christofides (2001) have developed a nonlinear control framework that integrates robustness, optimality and explicit constraint-handling capabilities.

In fact, these controller constraint problems cannot accurately describe the actual actuator action, so that it arises in the case of controllers that include undesired oscillations and overshoots that which cannot reflect the real opening magnitude and rate of control valves. Recently, the relevant developments for dual (magnitude and rate) actuator constraints have been mentioned. Stoorvogel and Saberi (1999) have proposed that dual input constraints can be modeled as an operator and incorporated into the controller design. Except for active input constraints, the geometric nonlinear control frequently encounters other challenging problems such as nonlinear separation principle or non-minimum phase systems (Wu, 1999a, b; Kanter et al., 2001). Besides, the control of non-minimum phase processes usually induces approximate feedback linearization so that the magnified plant/model mismatch needs to be minimized by virtue of the very large control action (Wu, 1999a,b). Under

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the limits of actuator manipulation, the constraints can make the system drift away from the operating region to undesired attractive points. Kanter et al. (2001) have shown that the new model predictive control law could perform optimally in the presence of input constraints.

Recent process control problems have concentrated on disturbance rejection, partial state feedback, actuator constraints, non-minimum phase and so on. In this article, a catalytic continuous stirred tank reactor (CCSTR) whose dynamics are strongly nonlinear and accompanied with non-minimum phase behavior is considered as a proper example. While this example can be treated as a two-input uncertain process in the presence of unavailable input constraints, we confirm that the integrated two-input control framework is effective to avoid actuator overload and takes disturbance compensation. The main technical contribution is a Lyapunov- based linearization strategy for temperature stabilization of such a reactor. Notably, it is verified that the novel two-input control scheme can ensure satisfactory output regulation and all results are illustrated via numerical simulation.

2. PROBLEM STATEMENT

2.1 Catalytic Continuous Stirred Tank Reactor

Let the catalytic continuous stirred tank reactor (CCSTR) described in Christofides and Daoutidis (1996) be considered in which endothermic, homogenous reaction $A \rightarrow B$ and exothermic, catalytic reaction $A \rightarrow C$ take place. The inlet flow rate F with pure species A of concentration C_{A0} and temperature T_{A0} is processed. The homogenous reaction generates the side-product *B*, and the second reaction would lead to the desired product *C*. Reactions heat can be removed through a coolant wall temperature T_w. The process model can be expressed as,

$$\frac{dC_{Ah}}{dt} = \frac{F}{V_h}(C_{A0} - C_{Ah}) - k_h \exp\left(\frac{-E_h}{RT_h}\right) - \frac{K_c A_c}{V_h}(C_{Ah} - C_{Ac}) \tag{1a}$$

$$\frac{dT_h}{dt} = \frac{F}{V_h}(T_{A0} - T_h) + \frac{(-\Delta H_h)}{r_h c_{ph}} k_h \exp\left(\frac{-E_h}{RT_h}\right) - \frac{U_w A_w}{r_h c_{ph} V_h}(T_h - T_w) - \frac{U_c A_c}{r_h c_{ph} V_h}(T_h - T_c) \tag{1b}$$

$$\frac{dC_{Ac}}{dt} = \frac{K_c A_c}{V_c}(C_{Ah} - C_{Ac}) - k_c \exp\left(\frac{-E_c}{RT_c}\right) C_{Ac} \tag{1c}$$

$$\frac{dT_c}{dt} = \frac{U_c A_c}{r_c c_{pc} V_c}(T_h - T_c) + \frac{(-\Delta H_c)}{r_c c_{pc}} k_c \exp\left(\frac{-E_c}{RT_c}\right) C_{Ac} \tag{1d}$$

The meanings of above symbols have been defined in notation section. The characteristics of CCSTR systems are stated:

- The inputs F and T_w usually can be treated as the manipulated variables.
- The states C_{Ah} and C_{Ac} remain nonnegative for all *t*. According to the principle of mass conservation, it admits $C_{Ac} \le C_{Ah} \le C_{A0}$.
- The temperature representations T_{A0}, T_h, T_c and T_w are nonnegative, bounded in the feasible set.
- Inlet composition C_{A0} and temperature T_{A0} are probably affected by external disturbances.
- In the effective cost and safe manner, the satisfactory operating conditions are based on a low concentration of species A in the catalytic phase but the reactor temperature is not high.
- Under the system parameters and the corresponding initial values of the system variables listed in Table 1, it was verified that the process exhibits non-minimum phase behavior.

Table 1 Process parameter values

$c_{ph} = 0.231$, $c_{pc} = 2.31$ ($\mathrm{kcal \cdot kg^{-1} \cdot K^{-1}}$)
$r_h = 0.9$, $r_c = 9$ ($\mathrm{kg \cdot lt^{-1}}$)
$K_c A_c = 1618$ ($\mathrm{lt \cdot min^{-1}}$)
$U_c A_c = 6667$, $U_w A_w = 3340$ ($\mathrm{kcal \cdot min^{-1} \cdot K^{-1}}$)
$R = 1.987$ ($\mathrm{kcal \cdot kmol^{-1} \cdot K^{-1}}$)
$k_h = 164.68$ $\mathrm{mol \cdot lt^{-1} \cdot min^{-1}}$
$k_c = 2000$ $\mathrm{min^{-1}}$ ($\mathrm{kcal \cdot kgmol^{-1}}$)
$E_h = 8000$, $E_c = 9000$ ($\mathrm{kcal \cdot kgmol^{-1}}$)
$\Delta H_h = 500$, $\Delta H_c = -350$ ($\mathrm{kcal \cdot kgmol^{-1}}$)
$V_c = 150$, $V_h = 1000$ (lt)
$C_{A0} = 10$ ($\mathrm{mol \cdot lt^{-1}}$)
$T_{A0} = 320$ (K)
$F_s = 100$ ($\mathrm{lt \cdot min^{-1}}$)

2.1 State-space model

To add reliable input action in the case of the CCSTR process, eq. (1) can be described as a nonlinear model with actuators subject to amplitude and rate constraints

$$
\begin{aligned}
\dot{x} &= f(x) + g_1(x)\mathrm{sat}(\omega) + g_2(x)F_s \\
\dot{\omega} &= -\theta\omega + \theta u_1 \\
y &= h(x)
\end{aligned} \tag{2}
$$

where the state variables $x = [C_{Ah}, T_h, C_{Ac}, T_c]^T$, the input variable $\omega = T_w$, and the saturation function $\mathrm{sat}(\cdot)$ describes the actuator action between the upper and lower bound,

$$
\mathrm{sat}(\omega) = \begin{cases} \omega_{\max}, & \omega \geq \omega_{\max} \\ \omega, & \omega_{\min} < \omega < \omega_{\max} \\ \omega_{\min}, & \omega \leq \omega_{\min} \end{cases} \tag{3}
$$

Besides, the dynamic $\dot{\omega}$ represents the actuator model in which the parameter θ is unknown whose value can affect the rate of actuator action.

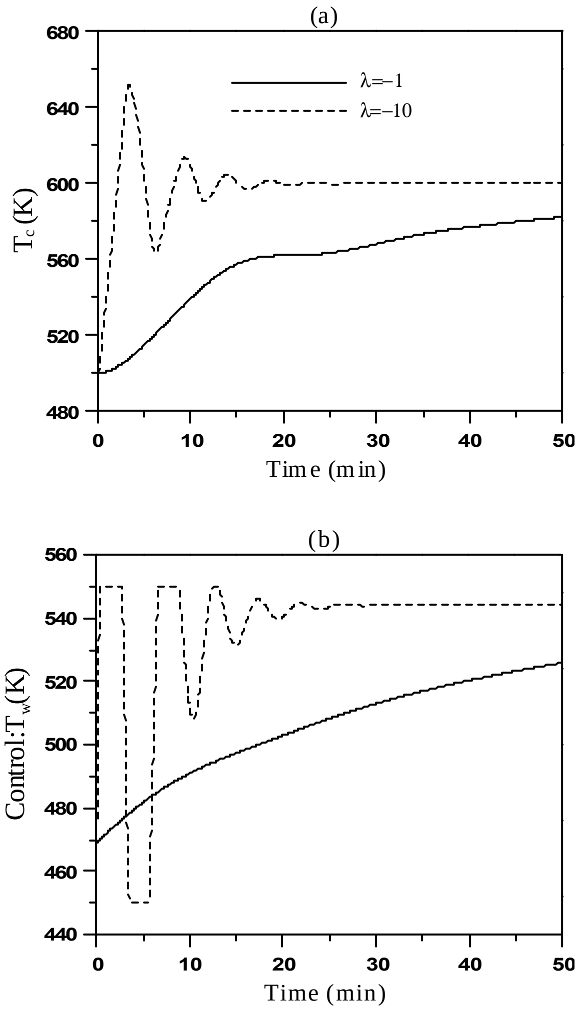

Fig. 1. Closed-loop state profiles by manipulating the input T_w on the basis of I/O linearization

Remark 1. Obviously, the statement of process model is imposed on actuator's constraints. Since the actuator model is unavailable, the developed nonlinear control scheme for anti-windup design cannot include the accurate model of actuator. Besides, the regulation of catalytic temperature x_4 is feasible because it can be measured and also maintain desired product quality specification. Thus, the output function $h(x) = x_4$ is valid and the variable u_1 for temperature feedback design is treated as the primary control.

Remark 2. Considering that the manipulated variable T_w is based on the conventional feedback linearization design associated with linear pole assignment (λ) in Henson and Seborg (1997), Fig. 1(a) shows that the output tracking for a desired setpoint (600 K) would be degraded by a far away pole design ($\lambda = -10$). It is owing to the actuator saturation that prevents the nonlinear control action depicted in Fig 1(b). Moreover, the nonlinear controller associated with anti-windup design concept should be reflected in the following design procedures.

3. LYAPUNOV-BASED LINEARIZATION STRATEGY

3.1 Single actuator design

In this section, a stable, asymptotic output regulation for this constrained system will be addressed step-by-step. First, the error variable is defined as $e_1 = h(x) - y_{d1}$, where y_{d1} is the desired temperature setpoint. Moreover, the time derivative of e_1 is shown as

$$
\begin{aligned}
\dot{e}_1 &= \frac{\partial h}{\partial x}[f(x) + g_1(x)\mathrm{sat}(\omega) + g_2(x)F_s] \\
&= L_f h + \mathrm{sat}(\omega)L_{g_1}h + F_s L_{g_2}h
\end{aligned} \tag{4}
$$

Since the terms $L_{g_1}h$ and $L_{g_2}h$ are zero, the primary input does not appear. Moreover, the new error variable could be set as $e_2 = \varepsilon(L_f h + e_1)$, where ε is a positive parameter, and eq. (4) can be written as

$$
\varepsilon\dot{e}_1 = -\varepsilon e_1 + e_2 \tag{5}
$$

Considering the time derivative of e_2

$$
\begin{aligned}
\dot{e}_2 &= \varepsilon[L_f h + L_f^2 h + u_1 L_{g_1} L_f h + F_s L_{g_2} L_f h \\
&+ (\mathrm{sat}(\omega) - u_1)L_{g_1} L_f h]
\end{aligned} \tag{6}
$$

If the term $L_{g_1}L_f h$ is nonzero, a parameterized state feedback law is given as

$$
\begin{aligned}
u_1 &= (\varepsilon L_{g_1} L_f h)^{-1} \\
&\times \left(-\frac{e_2}{\varepsilon^2} - \varepsilon(L_f^2 h + L_f h) - \varepsilon F_s L_{g_2} L_f h \right)
\end{aligned} \tag{7}
$$

Moreover, eq. (6) can be written as

$$
\varepsilon\dot{e}_2 = -\frac{e_2}{\varepsilon} + \varepsilon^2(\mathrm{sat}(\omega) - u_1)L_{g_1} L_f h \tag{8}
$$

Under the Lyapunov function $V_1 = (1/2)\varepsilon(e_1^2 + e_2^2)$, the time derivative of V_1 can be shown as

$$\dot{V}_1 \le -\left(\sqrt{\boldsymbol{e}}e_1 - \frac{1}{2\sqrt{\boldsymbol{e}}}e_2\right)^2 - \frac{3}{4\boldsymbol{e}}|e_2|^2 + \boldsymbol{e}^2|e_2|\left|(\mathrm{sat}(\boldsymbol{w}) - u_1)L_{g_1}L_f h\right| \tag{9}$$

Furthermore, the closed-loop system can be written as

$$\begin{aligned} \boldsymbol{e}\dot{e}_1 &= -\boldsymbol{e}e_1 + e_2 \\ \boldsymbol{e}\dot{e}_2 &= -\frac{e_2}{\boldsymbol{e}} + \boldsymbol{e}^2(\mathrm{sat}(\boldsymbol{w}) - u_1)L_{g_1}L_f h \\ \dot{\boldsymbol{w}} &= -\boldsymbol{q}\boldsymbol{w} + \boldsymbol{q}u_1 \\ \dot{\boldsymbol{h}} &= q(e, \boldsymbol{h}) \end{aligned} \tag{10}$$

where $\boldsymbol{h} \in \Re^2$ represents the internal dynamic. According to the above design algorithm and validation, the output regulation of this constrained system can be stated as follows.

***Proposition 1*:** Let the constrained CCSTR system in eq. (2) be considered if the non-singular condition $L_{g_1}L_f h \neq 0$ and the stable internal dynamic in $\boldsymbol{h}$ can hold, the parameterized state feedback control with appropriately small parameters $\boldsymbol{e}$ can conditionally ensure the stable closed-loop system as well as the asymptotic output tracking.

Remark 3. Under the above Lyapunov-based design procedures, the stability of singularly perturbed closed-loop system in eq. (10) can be guaranteed. The tracking performance can be recovered while the error term for actuator action, $\dot{\boldsymbol{w}} = -\boldsymbol{q}\boldsymbol{w} + \boldsymbol{q}\mathrm{sat}(u_1)$, in the fast mode can be reduced as $\boldsymbol{e} \to 0$. Notably, this high-gain state feedback approach still induces the undesired actuator action. Nevertheless, we can adopt a simple manner to avoid the controller saturation, while the saturation function for u_1 is shown as

$$\mathrm{sat}(u_1) = \begin{cases} \boldsymbol{w}_{\max}, & u_1 \ge \boldsymbol{w}_{\max} \\ u_1, & \boldsymbol{w}_{\min} < u_1 < \boldsymbol{w}_{\max} \\ \boldsymbol{w}_{\min}, & u_1 \le \boldsymbol{w}_{\min} \end{cases} \tag{11}$$

The saturation-type parameterized state feedback law

$$\mathrm{sat}(u_1) = (\boldsymbol{e}L_{g_1}L_f h)^{-1} \times \left(-\frac{e_2}{\boldsymbol{e}^2} - \boldsymbol{e}(L_f^2 h + L_f h) - \boldsymbol{e}F_s L_{g_2}L_f h\right) \tag{12}$$

can reduce the undesired actuator response due to the saturated first-order model

$$\dot{\boldsymbol{w}} = -\boldsymbol{q}\boldsymbol{w} + \boldsymbol{q}\mathrm{sat}(u_1) \tag{13}$$

If the saturation error for $|\mathrm{sat}(w) - u_1|$ can vanish as $t \to \infty$, then the asymptotic output tracking can be achieved while $\dot{V}_1 < 0$ by eq. (9).

Test: For the illustrated CCSTR model in the presence of unknown actuator dynamic ($\boldsymbol{q} = 1$) and wall temperature bounds ($\boldsymbol{w}_{\min}, \boldsymbol{w}_{\max}$)=(450K, 550K), while the control law can be restricted in the describing set such as eq. (11), Fig. 2(a) shows that using small parameter $\boldsymbol{e}$ in eq. (12) the good output tracking can be achieved, and the corresponding, unsaturated control action is depicted in Fig. 2(b).

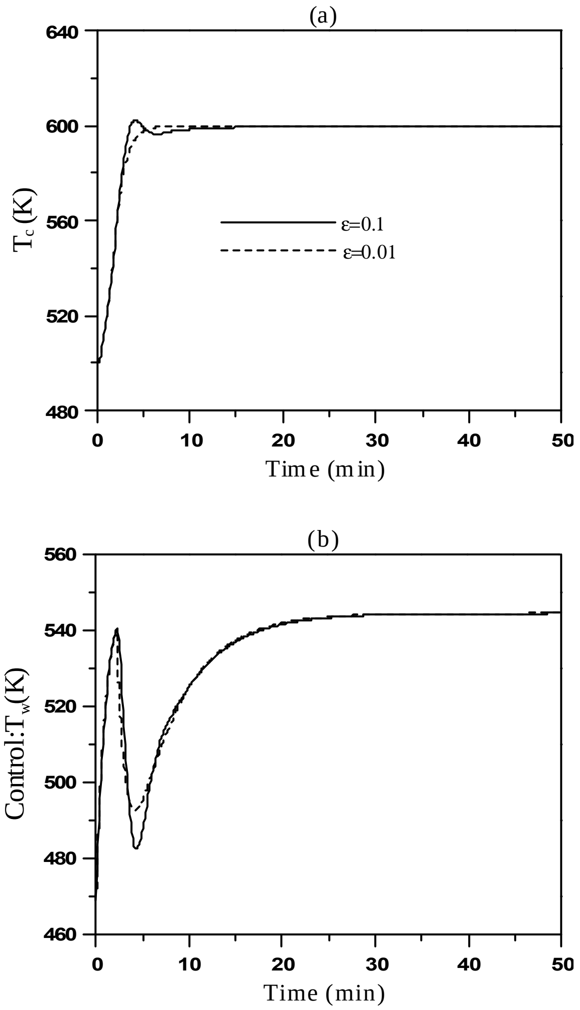

Fig. 2. Closed-loop state profiles by manipulating the input T_w on the basis of saturation-type parameterized state feedback

3.2 Two-input control scheme

In practice, the inlet flowrate of CCSTR system can be manipulated and its relevant inlet combination and temperature may be affected by external disturbances. Thus, the two-input constrained nonlinear system in the presence of unknown disturbances can be rewritten as

$$\begin{aligned} \dot{x} &= f(x) + g_1(x)\mathrm{sat}(\boldsymbol{w}) + g_2(x)u_2 + \boldsymbol{d}(u_2, d) \\ \dot{\boldsymbol{w}} &= -\boldsymbol{q}\boldsymbol{w} + \boldsymbol{q}u_1 \\ y &= h(x) \end{aligned} \tag{14}$$

where the adding input variable $u_2 = F$, and the unmeasurable disturbance d. Furthermore, we will develop the two-input control scheme for stabilizing the CCSTR system in the presence of disturbances and constraints. If the same tracking coordinates (e_1, e_2) are considered, and the similar state feedback law by eq. (7)

$$u_1 = (\varepsilon L_{g_1} L_f h)^{-1} \times \left(-\frac{e_2}{\varepsilon^2} - \varepsilon (L_f^2 h + L_f h) - \varepsilon y_{d2} L_{g_2} L_f h \right) \quad (15)$$

where y_{d2} is the adding setpoint of the inlet flowrate. Moreover, the system (14) can be transformed into

$$\begin{aligned} \varepsilon \dot{e}_1 &= -\varepsilon e_1 + e_2 + \varepsilon L_d h \\ \varepsilon \dot{e}_2 &= -\frac{e_2}{\varepsilon} + \varepsilon^2 (u_2 - y_{d2}) L_{g_2} L_f h \\ &\quad + \varepsilon^2 [(\mathrm{sat}(\omega) - u_1) L_{g_1} L_f h + L_d L_f h] \quad (16) \\ \dot{\omega} &= -\theta \omega + \theta u_1 \\ \dot{\eta} &= \tilde{q}(e, \eta, u_2, d) \end{aligned}$$

Basically, the stable internal dynamics $\dot{\eta}$ is required for bounded inputs u_2 and d. Although the high-gain feedback can suppress the effects of nonlinearities in eq. (16), the perfect control action cannot be realized. Since $L_d h$ is zero, we suggest that the adding control scheme can compensate the disturbance effects in the subsystem $\dot{e}_2$. Owing to non-minimum phase behavior in the integration of control designs, the following design procedure for u_2 is not based on conventional feedforward control algorithm (Daoutidis and Kravaris, 1989). The adaptive-like control scheme would be developed while the new Lyapunov function $V_2 = V_1 + (1/2)\gamma^{-1}(u_2 - y_{d2})^2$ for $\gamma > 0$ is considered. Moreover, the time derivative of V_2 can be written as

$$\begin{aligned} \dot{V}_2 = &-\left(\sqrt{\varepsilon} e_1 - \frac{1}{2\sqrt{\varepsilon}} e_2 \right)^2 - \frac{3}{4\varepsilon} e_2^2 \\ &+ \varepsilon^2 e_2 \left[(\mathrm{sat}(\omega) - u_1) L_{g_1} L_f h + L_d L_f h \right] \\ &+ (u_2 - y_{d2})(\gamma^{-1} \dot{u}_2 + \varepsilon^2 e_2 L_{g_2} L_f h) \quad (17) \end{aligned}$$

If the integrator is chosen as

$$\dot{u}_2 = -\hat{\gamma} e_2 L_{g_2} L_f h \quad (18)$$

where $\hat{\gamma} = \gamma \varepsilon^2$. If both bounds of $\left|(\mathrm{sat}(\omega) - u_1) L_{g_1} L_f h\right|$ and $\left|L_d L_f h\right|$ exist, and the integrator (18) is finite as $t \to \infty$, thus $\dot{V}_2 \le 0$ for a small ε is reachable.

***Proposition 2*:** Consider the two-input CCSTR system modeled as eq. (14), if the non-singular condition $L_{g_1} L_f h \neq 0$ and stable internal dynamics in eq. (16) can hold, two-input control scheme by eqs (15) and (18) with appropriate parameters ε and $\hat{\gamma}$ can ensure the stable, bounded output regulation.

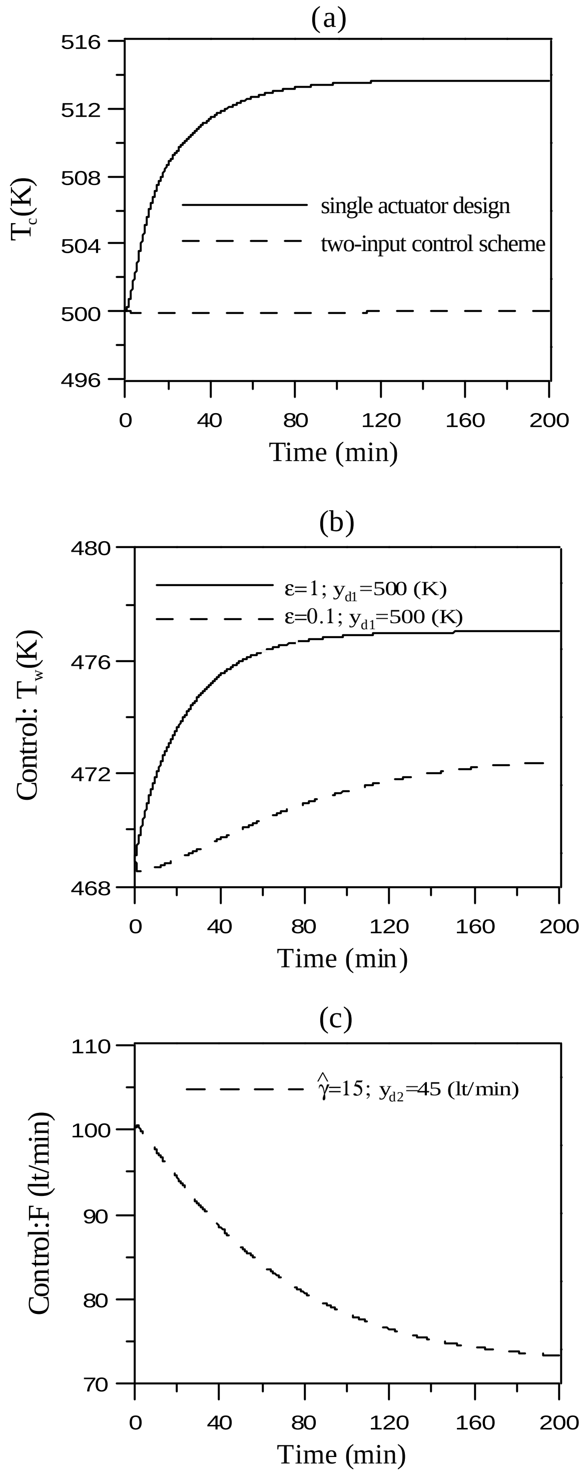

Fig. 3. Disturbance rejection in the presence of inlet temperature perturbation, showing comparison between single actuator design on the basis of saturation-type parameterized state feedback and two-input control framework

Remark 4. Following the previous Lyapunov-based linearization strategy, the two-input control scheme can be conditionally established. The original control law u_1 is restricted to the primary actuator action, and the second input u_2 can robustly attenuate the effects of unknown disturbances. Notably, the integrator design in u_2 can avoid the unstable inverse problem, so the universal control effort is divided into both controllers. If the term $L_d L_f h = 0$ can hold, similarly by Proposition 1 the tracking error e_1 would become zero as time approaches infinity, and $\lim_{t\to\infty} u_2 = y_{d2}$ also can be achieved.

Test: Compared to the previous single actuator design for the same CCSTR model in the presence of inlet temperature perturbation ($d = 20\%$), Fig. 3(a) shows that using the two-input control scheme the unknown disturbance can be almost attenuated and that it is extremely superior to the single actuator design, and Fig. 3(b) and 3(c) depict that the corresponding control actions are smooth and low. In these demonstrations all controller parameters have been stated in corresponding figures. Consequently, the two-input control scheme not only reduces single actuator overload, but also it provides the feasible, high-effective operating manner in the face of unknown disturbances.

4. CONCLUSION

In this work, two-input control design methodologies are proposed, which can ensure the robust stabilization of a CCSTR system in the presence of actuator constraints and unknown disturbances. The Lyapunov-based linearization technique and adaptive-like design procedure is the main ingredient in the formulation of a class of nonlinear control schemes. It is shown that the saturation-type high-gain feedback can enforce the satisfactory performance recovery, and the integrated two-input control framework aims to attenuate the effect of unknown perturbations on the output and also reduce undesirable controller saturation.

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