

CRITERION FOR NONREGULAR FEEDBACK LINEARIZATION WITH APPLICATION

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Abstract: A new criterion for nonregular static state feedback linearization is presented for a class of affine nonlinear control systems. This criterion is applied to several classes of nonholonomic systems and discontinuous stabilizing control design is outlined based on linear system theory and the backstepping techniques.

Keywords: Nonregular feedback linearization, Nonlinear systems, Nonholonomic systems

1. INTRODUCTION

Feedback linearization is a standard technique for control of many nonlinear systems. Since the pioneering work of (Krener, 1973) which addressed linearization of nonlinear systems via state diffeomorphisms, the problem of linearization has been studied using increasingly more general transformations. The problem of regular static state feedback linearization was solved in (Brockett, 1978) and (Jakubczyk & Respondek, 1980). The problem of regular dynamic state feedback linearization was initiated in (Cheng, 1987) and addressed in many references, e.g., (Charlet, Levine & Marino, 1989; Charlet, Levine & Marino, 1991; Guay, McLellan & Bacon, 1997). Recently, the problem of nonregular static/dynamic state feedback linearization was studied in (Sun & Xia, 1997).

Nonregular state feedback linearization is a rigorous design theory/technique. Comparing with regular dynamic feedback linearization, this approach does not introduce additional dynamics, while it is applicable to a broad class of practi-

cal engineering systems, such as induction motors (Sun & Xia, 1996) and robots with flexible joints (Ge, Sun & Lee, 2001a). Moreover, associated nonregular feedback linearization with the backstepping design technique, the so-called nonregular backstepping design approach provides a Lyapunov-based recursive design mechanism (Sun, Ge & Lee, 2001). This approach is directly applicable to a class of complex mechanical systems in Euler-Lagrange form and does not involve any coordinate transformation, thus enable us to keep close insight into (and make full utilization of) the physical properties of the systems. This approach can also avoid undesired cancelation of beneficial nonlinearities and enhance robustness and softness through appropriate backstepping design of Lyapunov functions.

In this paper, we propose a nonsmooth formulation for the problem of nonregular state feedback linearization. One motivation for this extension stems from the existence of systems that cannot be asymptotically stabilized by a single continuous pure feedback controller (Brockett, 1983). The insight work of (Celikovsky & Nijmeijer, 1996; Clarke, et al., 1997) also inspire us to investigate nonsmooth state and feedback transformations.

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2. MAIN RESULT

Consider the affine nonlinear system given by

$$\dot{x} = f(x) + \sum_{i=1}^m u_i g_i(x) = f(x) + g(x)u \quad (1)$$

where $x \in \Omega \subseteq \mathfrak{R}^n$ is the state, $u \in \mathfrak{R}^m$ is the input, entries of $f(x)$ and $g(x)$ are analytic functions of x , and $\text{rank } g(x) = m, \forall x \in \Omega$. Without loss of generality, assume $f(0) = 0$ and Ω is a connected open set containing the origin.

Definition 1. Nonlinear control system (1) is said (nonsmooth) nonregular (static state) feedback linearizable, if there exist a state transformation

$$z = T(x), \quad z \in \mathfrak{R}^n \quad (2)$$

and a nonregular state feedback

$$u(t) = \alpha(x) + \beta(x)v(t), \quad v \in \mathfrak{R}^{m_0}, m_0 \leq m \quad (3)$$

where entries of $T(x)$, $\alpha(x)$ and $\beta(x)$ are defined and smooth on an open and dense subset Ω_0 of Ω , and map $T : \Omega_0 \rightarrow T(\Omega_0)$ is a diffeomorphism, such that the transformed system with state z and input v is a controllable linear system.

The following theorem establishes nonregular feedback linearizability for a class of nonlinear system.

Theorem 1. For a two-input affine nonlinear system

$$\dot{x} = f(x) + g_1(x)u_1 + g_2(x)u_2 \quad (4)$$

suppose there exist vector fields $p(x)$ and $q(x)$ with $\text{span}\{p(x), q(x)\} = \text{span}\{g_1, g_2\}$, and a sequence of integers $0 \leq \kappa_0 < \kappa_1 < \dots < \kappa_l \leq n-1$ with $l \geq 2$, such that the nested distributions defined by

$$\begin{aligned} G_0 &= \text{span}\{q\} \\ G_i &= G_{i-1} + \text{ad}_f G_{i-1}, \quad i \geq 1, i \neq \kappa_1, \dots, \kappa_l \\ G_{\kappa_j} &= G_{\kappa_j-1} + \text{ad}_{\text{ad}_f^{\kappa_0} p} G_{\kappa_j-1}, \quad j = 1, \dots, l-1 \\ G_{\kappa_l} &= G_{\kappa_l-1} + \text{span}\{p\} \end{aligned} \quad (5)$$

satisfy the following conditions

- (i) $\text{rank } G_{n-1} = n$;
- (ii) G_{κ_l-2} and G_{n-2} are involutive;
- (iii) $\text{ad}_f G_{\kappa_j-1} \subseteq G_{\kappa_j}$ for $j = 1, \dots, l$; and
- (iv) $\text{ad}_{\text{ad}_f^j p} G_{i-1} \subseteq G_i$ for $j = 0, \dots, \kappa_0$, and $i = 0, \dots, n-2$

then system (4) is nonsmooth nonregular feedback linearizable.

Proof. It follows from conditions (iii) and (iv) that

$$\begin{aligned} G_k &= \text{span}\{q, \dots, \text{ad}_f^k q\}, \quad k = 0, \dots, \kappa_1 - 1 \\ G_{\kappa_j+k} &= G_{\kappa_j-1} + \text{span}\{\chi_j, \dots, \text{ad}_f^k \chi_j\}, \\ &\quad j = 1, \dots, l-1, \quad k = 0, \dots, \kappa_{j+1} - \kappa_j - 1 \\ G_{\kappa_l+k} &= G_{\kappa_l-1} + \text{span}\{p, \dots, \text{ad}_f^k p\} \\ &\quad k = 0, 1, \dots \end{aligned} \quad (6)$$

where χ_j , $j = 1, \dots, l-1$ are given recursively by

$$\begin{aligned} \chi_1 &= \text{ad}_{\text{ad}_f^{\kappa_0} p} (\text{ad}_f^{\kappa_1-1} q) \\ \chi_j &= \text{ad}_{\text{ad}_f^{\kappa_0} p} (\text{ad}_f^{\kappa_j-\kappa_{j-1}-1} \chi_{j-1}), \quad j = 2, \dots, l-1 \end{aligned}$$

Accordingly, $\dim G_i \leq \dim G_{i-1} + 1$ for $i \geq 1$. It follows from condition (i) and $\dim G_0 = 1$ that

$$\dim G_i = i + 1, \quad i = 0, 1, \dots, n-1 \quad (7)$$

Because $\text{span}\{p, q\} = \text{span}\{g_1, g_2\}$, one can express vector fields p and q in terms of g_1 and g_2 as follows

$$\begin{aligned} p(x) &= \bar{\beta}_{1,1}(x)g_1 + \bar{\beta}_{1,2}(x)g_2(x), \\ q(x) &= \bar{\beta}_{2,1}(x)g_1 + \dots + \bar{\beta}_{2,2}(x)g_m(x) \end{aligned}$$

where $\bar{\beta}_{i,j}(x)$ are smooth real-valued functions and elements of the matrix

$$\bar{\beta}(x) = \begin{bmatrix} \bar{\beta}_{1,1}(x) & \bar{\beta}_{1,2}(x) \\ \bar{\beta}_{2,1}(x) & \bar{\beta}_{2,2}(x) \end{bmatrix}, \quad \text{rank } \bar{\beta}(x) = 2$$

Applying the input transformation

$$u = \bar{\beta}(x)v$$

to system (4) gives

$$\dot{x} = f(x) + p(x)v_0 + q(x)v_1 \quad (8)$$

where $v = [v_0, v_1]^T$ is the new input to be designed.

It is readily seen that, if the transformed system (8) is nonregular feedback linearizable, then the original system (4) is nonregular feedback linearizable too.

From the Frobenius Theorem, there exists a real-valued functions $\phi(x)$, such that

$$d\phi \perp G_{\kappa_l-2}, \quad d\phi \not\perp G_{\kappa_l-1} \quad (9)$$

As a matter of fact, $\phi(x)$ can be replaced by any of its non-zero constant multiplication $c\phi(x)$, $c \neq 0$ without violating (9). This flexibility in choosing ϕ will be utilized in the following derivations.

Let

$$v_0 = \phi(x) \quad (10)$$

System (8) can be rewritten as

$$\dot{x} = \bar{f}(x) + q(x)v_1 \quad (11)$$

where $\bar{f}(x) = f(x) + \phi(x)p(x)$.

In the sequel, we focus on system (11) and prove its linearizability.

Because G_{n-2} is involutive and of dimension $n-1$, by Frobenius' Theorem, there exists a real-valued function $h(x)$, such that

$$\text{span}\{dh\} = G_{n-2}^\perp \quad (12)$$

Note that

$$q \in G_0 \quad (13)$$

$$ad_{\bar{f}}q = ad_fq + \phi ad_pq \quad (14)$$

For convenience, for two sets S_1, S_2 and an element s , let us denote $s \in S_1 - S_2$ if $s \in S_1$ and $s \notin S_2$. It follows from (14) and condition (iv) that

$$ad_fq \in G_1 - G_0, \quad ad_pq \in G_1 \quad (15)$$

Suppose $ad_{\bar{f}}q \in G_0$, then it follows from (15) that $ad_pq \notin G_0$. In this case, $ad_fq + c\phi ad_pq \notin G_0$ for any constant $c \neq 1$. Accordingly, by appropriately choosing of ϕ , it can be always made that

$$ad_{\bar{f}}q \in G_1 - G_0 \quad (16)$$

Suppose for some $1 \leq i \leq n-2$, we have

$$ad_{\bar{f}}^j q \in G_j - G_{j-1}, \quad j = 1, \dots, i \quad (17)$$

Then, it can be proven that

$$ad_{\bar{f}}^{i+1} q \in G_{i+1} - G_i \quad (18)$$

To this end, compute

$$ad_{\bar{f}}^{i+1} q = ad_f(ad_{\bar{f}}^i q) + \phi ad_p(ad_{\bar{f}}^i q) + (L_{ad_{\bar{f}}^i q} \phi) p$$

For $i < \kappa_l - 1$ and $i \neq \kappa_j - 1, j = 1, \dots, l-2$, it follows from (17) that $L_{ad_{\bar{f}}^i q} \phi = 0$, and

$$ad_f(ad_{\bar{f}}^i q) \in G_{i+1} - G_i, \quad ad_p(ad_{\bar{f}}^i q) \in G_{i+1}$$

Therefore, up to a constant multiplication of ϕ , it has

$$ad_{\bar{f}}^{i+1} q = ad_f(ad_{\bar{f}}^i q) + \phi ad_p(ad_{\bar{f}}^i q) \in G_{i+1} - G_i$$

For $i = \kappa_j - 1$ with $1 \leq j \leq l-1$, we have

$$ad_f(ad_{\bar{f}}^i q) \in G_{i+1}, ad_p(ad_{\bar{f}}^i q) \in G_{i+1}, L_{ad_{\bar{f}}^i q} \phi = 0$$

Therefore,

$$ad_{\bar{f}}^{i+1} q = ad_f(ad_{\bar{f}}^i q) + \phi ad_p(ad_{\bar{f}}^i q) \in G_{i+1}$$

Suppose

$$ad_f(ad_{\bar{f}}^i q) \in G_i, \quad ad_p(ad_{\bar{f}}^i q) \in G_i \quad (19)$$

then by Jacobi identity (Isidori, 1989, pp.10)

$$\begin{aligned} ad_{ad_{\bar{f}}^i p}(ad_{\bar{f}}^i q) &= ad_f(ad_p(ad_{\bar{f}}^i q)) \\ &\quad - ad_p(ad_f(ad_{\bar{f}}^i q)) \in G_i \end{aligned}$$

and by recursion

$$\begin{aligned} ad_{ad_{\bar{f}}^j p}(ad_{\bar{f}}^j q) &= ad_f(ad_{ad_{\bar{f}}^{j-1} p}(ad_{\bar{f}}^j q)) \\ &\quad - ad_{ad_{\bar{f}}^{j-1} p}(ad_f(ad_{\bar{f}}^j q)) \in G_i, \quad j = 2, 3, \dots \end{aligned}$$

which contradicts (7). Consequently, relation (19) must not be hold, thus it has

$$ad_f(ad_{\bar{f}}^i q) \in G_{i+1} - G_i, \quad \text{or} \quad ad_p(ad_{\bar{f}}^i q) \in G_{i+1} - G_i$$

which imply that, up to a constant multiplication of ϕ , it has

$$ad_{\bar{f}}^{i+1} q \notin G_i$$

For $i = \kappa_l - 1$, we have

$$\begin{aligned} ad_f(ad_{\bar{f}}^i q) &\in G_{i+1}, \quad ad_p(ad_{\bar{f}}^i q) \in G_{i+1} \\ L_{ad_{\bar{f}}^i q} \phi &\neq 0, \quad p \in G_{i+1} - G_i \end{aligned}$$

Therefore, up to a constant multiplication of ϕ , it has

$$\begin{aligned} ad_{\bar{f}}^{i+1} q &= ad_f(ad_{\bar{f}}^i q) + \phi ad_p(ad_{\bar{f}}^i q) \\ &\quad + (L_{ad_{\bar{f}}^i q} \phi) p \in G_{i+1} - G_i \end{aligned}$$

For $\kappa_l \leq i \leq n-2$, we have

$$ad_f(ad_{\bar{f}}^i q) \in G_{i+1} - G_i, \quad ad_p(ad_{\bar{f}}^i q) \in G_{i+1}, \quad p \in G_i$$

Therefore, up to a constant multiplication of ϕ , it has

$$ad_{\bar{f}}^{i+1} q \in G_{i+1} - G_i$$

The above reasonings show that (17) implies (18) for $1 \leq i \leq n-2$. From the initial condition (16), it follows by induction that

$$\begin{aligned} ad_{\bar{f}}^k q &\in G_k, \quad k = 0, 1, \dots, n-2 \quad (20) \\ ad_{\bar{f}}^{n-1} q &\notin G_{n-2} \end{aligned}$$

Accordingly, we have

$$\begin{aligned} \langle dh, ad_{\bar{f}}^k q \rangle &= 0, \quad k = 0, 1, \dots, n-2 \\ \langle dh, ad_{\bar{f}}^{n-1} q \rangle &\neq 0 \end{aligned}$$

which, by (Isidori, 1989, Lemma 4.1.3), implies that

$$\begin{aligned} L_q L_{\bar{f}}^k h &= 0, \quad k = 0, 1, \dots, n-2 \\ L_q L_{\bar{f}}^{n-1} h &\neq 0 \end{aligned}$$

Define new coordinates z and new input w as

$$z = [h, L_{\bar{f}}h, \dots, L_{\bar{f}}^{n-1}h]^T \quad (21)$$

$$w = L_{\bar{f}}^n h + (L_q L_{\bar{f}}^{n-1} h)v_1 \quad (22)$$

The state space description of system (11) in z -coordinate is then given by

$$\dot{z} = [z_2, z_3, \dots, z_n, w]^T \quad (23)$$

which is exactly the single-input Brunovsky canonical system.

By Definition 1, system (11) is nonregular feedback linearizable, which implies that the original system (4) is also nonregular feedback linearizable. In addition, the corresponding linearizing state and input transformations for system (4) are equation (21) and

$$u = \bar{\beta}(x) \begin{bmatrix} \phi(x) \\ w - L_{\bar{f}}^n h \\ L_q L_{\bar{f}}^{n-1} h \end{bmatrix} \quad (24)$$

respectively. \diamond

Remark 1. Once the linearizing output $h(x)$ and the function $\phi(x)$ (called singular input function in the sequel) are determined, the linearizing state and input transformations (21) and (24) can be calculated routinely. The determination of $h(x)$ and $\phi(x)$ involve the integration of a set of completely integrable systems, or equivalent, the solution of some solvable partial differential equations, which may not be produced in a routine way. However, for many nonlinear systems with some particular structure, the integration of the integrable systems is available, thus $h(x)$ and $\phi(x)$ can be explicitly obtained accordingly.

3. APPLICATION TO NONHOLONOMIC SYSTEMS

3.1 Linearizable Nonholonomic Systems

The criterion provided in the previous section is very general, and several general conventional forms of nonholonomic systems are special cases under our framework.

Firstly, consider nonholonomic systems of the form

$$\begin{aligned} y_1^{(r_1)} &= u_1 \\ y_i^{(r_i)} &= \xi_i(\bar{y}^1, \dots, \bar{y}_i, y_{i+1})u_1, \quad i = 2, \dots, m-1 \\ y_m^{(r_m)} &= u_2 \end{aligned} \quad (25)$$

where $m \geq 3$, $r_i \geq 1$, $\bar{y}^i = [y_i, \dots, y_i^{(r_i-1)}]^T$, $i = 1, \dots, m$, and ξ_i , $i = 2, \dots, m-1$ are analytic functions vanishing at the origin with

$$\frac{\partial \xi_i}{\partial y_{i+1}} \neq 0, \quad i = 2, \dots, m-1$$

It can be verified that Theorem 1 holds and the linearizing output and the singular input function could be explicitly constructed, say

$$\begin{aligned} h &= y_1 \\ \phi &= \phi_1(y_1, \dots, y_1^{(r_1)}, y_2) \quad \text{with} \quad \frac{\partial \phi_1}{\partial y_2} \neq 0 \end{aligned} \quad (26)$$

Note that model (25) includes the chained form (Murray & Sastry, 1991) and the second-order chained form (Egeland & Berglund, 1994) as special cases.

Secondly, consider nonholonomic systems of the form

$$\dot{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ 1 \\ \xi_3(x_1) \\ \vdots \\ \xi_n(x_1) \end{bmatrix} u_2 \quad (27)$$

where $\xi_i(x)$, $i = 3, \dots, n$ are analytic functions vanishing at the origin with

$$\frac{\partial^{i-2} \xi_i}{\partial x_1^{i-2}} \neq 0, \quad i = 3, \dots, n$$

It can be verified that Theorem 1 holds with

$$\begin{aligned} h(x) &= x_1 \\ \phi(x) &= x_n - \sum_{i=2}^{n-1} \varphi_i(x_1)x_i \end{aligned} \quad (28)$$

where $\varphi_i(x_1)$, $i = 2, \dots, n-1$ satisfy

$$\begin{aligned} \begin{bmatrix} \varphi_3 \\ \vdots \\ \varphi_{n-1} \end{bmatrix} &= \begin{bmatrix} \frac{\partial \xi_3}{\partial x_1} & \dots & \frac{\partial \xi_{n-1}}{\partial x_1} \\ & \ddots & \\ \frac{\partial^{n-3} \xi_3}{\partial x_1^{n-3}} & \dots & \frac{\partial^{n-3} \xi_{n-1}}{\partial x_1^{n-3}} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial \xi_3}{\partial x_1} \\ \vdots \\ \frac{\partial^n \xi_n}{\partial x_1^{n-3}} \end{bmatrix} \\ \varphi_2 &= \xi_n - \sum_{j=3}^{n-1} \varphi_j(x)\xi_j(x) \end{aligned}$$

Note that the above model includes the power form (M'Closkey & Murray, 1992) as a special case.

Thirdly, consider nonholonomic systems of the form

$$\dot{x} = \begin{bmatrix} 1 \\ 0 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ 1 \\ -x_1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} u_2 \quad (29)$$

It can be verified that Theorem 1 holds with linearizing output

$$h = x_1$$

The singular input function can be determined recursively by

$$\begin{aligned}\phi_3(x) &= \varphi_3(x_1) + x_3 + x_1x_2 \\ \phi_i(x) &= \varphi_i(x_1) + x_i + \frac{1}{i-2}x_1\phi_{i-1}(x), i = 4, \dots, n \\ \phi(x) &= \phi_n(x)\end{aligned}\quad (30)$$

where $\varphi_i(x_1)$, $i = 3, \dots, n$ are any analytic functions of x_1 vanishing at the origin.

Note that this model includes the Brockett integrator (Brockett, 1983) as a special case.

Finally, consider dynamic nonholonomic systems of the form

$$\begin{cases} \dot{y} = g_1(y)v_1 + g_2(y)v_2 \\ v_1^{(r_1)} = u_1 \\ v_2^{(r_2)} = u_2 \end{cases}\quad (31)$$

where $r_i \geq 1$, $i = 1, 2$, and $\dot{y} = g_1(y)v_1 + g_2(y)v_2$ is either a (high order) chained system (25) or a generalized power system (27) when view y as the state and $v = [v_1, v_2]^T$ as the input.

It can be verified that system (31) is nonregular feedback linearizable. Moreover, any linearizing output and singular input functions for system

$$\dot{y} = g_1(y)v_1 + g_2(y)v_2$$

are also linearizing output and singular input function for system (31).

Note that the above model includes the extended power form (Kolmanovsky, Reyhanoglu & McClamroch, 1996) as a special case.

3.2 Controller Design

In this subsection, two stabilizing strategies are outlined for systems which are nonregular feedback linearizable. One is based on the classical linear design theory, the other is based on the backstepping design technique.

For a nonlinear system

$$\dot{x} = f(x) + g_1(x)u_1 + g_2(x)u_2 \quad (32)$$

satisfying Theorem 1 with $p(x) = g_1(x)$ and $q(x) = g_2(x)$, suppose $h(x)$ and $\phi(x)$ are a linearizing output and the a singular input function, respectively. Then, by the proof of Theorem 1, under the state transformation

$$z = \Phi(x) = [h(x), L_{\bar{f}}h(x), \dots, L_{\bar{f}}^{n-1}h(x)]^T \quad (33)$$

where $\bar{f}(x) = f(x) + \phi(x)g_1(x)$, and the input transformation

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \phi(x) \\ -L_{\bar{f}}^n h(x) \\ L_{g_2} L_{\bar{f}}^{n-1} h(x) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ L_{g_2} L_{\bar{f}}^{n-1} h(x) \end{bmatrix} w$$

where w is the new input, system (32) reads as a linear controllable system in state z and input w

$$\dot{z} = Az + bw \quad (34)$$

In general, the state and input transformations are well-defined and smooth in an open and dense subset Ω_0 of the state space, and the Jacobian matrix of $\Phi(x)$ is nonsingular in Ω_0 .

For linear system (34), the controller design is standard. Suppose

$$\omega(s) = s^n + a_0s^{n-1} + \dots + a_{n-2} + a_{n-1}$$

is a Hurwitz polynomial of s . Then the linear state feedback

$$w = -\sum_{i=1}^n a_{n-i} z_i \quad (35)$$

will render system (34) exponentially stable.

For nonlinear system (32), design a pure state feedback as

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \phi(x) \\ \frac{1}{L_{g_2} L_{\bar{f}}^{n-1} h(x)} (\Gamma \Phi(x) - L_{\bar{f}}^n h(x)) \end{bmatrix} \quad (36)$$

where $\Gamma = [a_{n-1}, \dots, a_0]$.

Let $z(t; t_0, z_0)$ denote the solution of the closed-loop system (34) and (35) with initial condition $z(t_0) = z_0$. Similarly, denote $x(t; t_0, x_0)$ the solution of the closed-loop system (32) and (36).

Define

$$\begin{aligned}\Omega_0^z &= \{z \in \mathbb{R}^n : \exists x \in \Omega_0, \text{ s.t. } z = \Phi(x)\} \\ \Omega_\omega(t_0) &= \{x \in \Omega : z(t; t_0, \Phi(x)) \in \Omega_0^z, \forall t \geq t_0\}\end{aligned}$$

It is readily seen that, if $x_0 \in \Omega_\omega(t_0)$, then $z(t; t_0, \Phi(x_0)) \in \Omega_0^z$, which implies that the $x(t; t_0, x_0)$ is well defined for all $t \geq t_0$ and converge to the origin asymptotically.

To make system (32) globally attractive, we only need to drive any initial configuration into the allowed initial set $\Omega_\omega(t_0)$ by an appropriate control input, see (Ge, Sun & Lee, 2001b) for a detailed case study.

Remark 2. Note that the above design procedure is totally different with the so-called σ -process (Astolfi, 1996). In (Astolfi, 1996), first divide the system into the ‘base’ subsystem and the ‘extended’ subsystem, and then design the controllers for each subsystem. Our approach, however, is based on nonregular feedback linearization of the whole system, thus it involves no system division. As the approaches are essentially different, the resulted controllers differ from each other accordingly.

Now let us turn to the backstepping-based design for a nonregular feedback linearizable system

$$\dot{x} = f(x) + g_1(x)u_1 + g_2(x)u_2 \quad (37)$$

Suppose it satisfies Theorem 1 with $p(x) = g_1(x)$ and $q(x) = g_2(x)$, and $\phi(x)$ is a singular input function. Let

$$u_1 = \phi(x) \quad (38)$$

then system (32) is given by

$$\dot{x} = f(x) + g_1(x)\phi(x) + g_2(x)u_2 \quad (39)$$

which is feedback linearizable. Hence there is a state transformation

$$z = \Phi_1(x)$$

such that system (39) in the new coordinate z processes a triangular structure to which the backstepping design procedure is applicable.

4. CONCLUSION

In this paper, a new criterion has been presented for nonregular feedback linearization of a class of affine nonlinear systems with two inputs. This criterion was then applied to several classes of nonholonomic systems and the design issues were briefly discussed.

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