PATH TRACKING FOR MOBILE ROBOTS WITH A TRAILER

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Abstract: In this paper it is proposed the extension of an efficient method employed for non-holonomic mobile robots to the path tracking of tractors with a trailer. The main advantage of this pure geometric technique is that no kinematic model of the vehicle is needed, which is an interesting property when working in irregular terrain or with tracks instead of wheels. The differences of applying this method to the combined system appear when avoiding collisions of the vehicle with the trailer and while moving in reverse direction. This technique has been successfully implemented and tested in the outdoor mobile robot Auriga- . *Copyright © 2002 IFAC*

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1. INTRODUCTION

The hitching of a trailer to a mobile robot expands the capabilities of the latter by providing it with free space for load haulage or material removal (Lane and King, 1994), as well as for equipment operation such as lawn mowing (Larsson, *et al.*, 1994) or road paving. However, the path planning and motion execution for the entire plant is more complex than for the single vehicle because the complete system becomes under-actuated (DeSantis, 1998).

The parameters of the kinematic chain for this kind of robotic system can be observed in Fig. 1. L_1 and L_2 are both positive or null constants (but not at the same time), whereas   is the variable that represents the relative angle between the tractor and the trailer. The vehicle's position in the plane is described by its *x-y* coordinates and its orientation   with respect to fixed global axes.

When $L_1=0$, which is called the on-axle problem, the joint hitching the trailer to the vehicle is located at the guide point of the tractor (Altafini, 1999). On the other hand, $L_2=0$ means that the joint is placed at the trailer's reference system (see Fig. 1).

In the path tracking problem the objective is to follow a previously recorded or planned path in spite of external perturbations by manipulating the tractor's speed and steering. Explicit path tracking is based on fast position estimation produced by the mobile robot's sensorial system. The method proposed in this paper for tractor-trailer systems is a generalization of a simple technique employed for autonomous vehicles.

This paper is organized as follows. Next section reviews the path tracking strategy. Collision avoidance between the vehicle and the trailer is addressed in section 3. Special considerations for backward movement are described in section 4. Then, the mobile robot Auriga- along with its trailer and experimental results are explained in sections 5 and 6 respectively. Last paragraphs are dedicated to conclusions, acknowledgments and references.
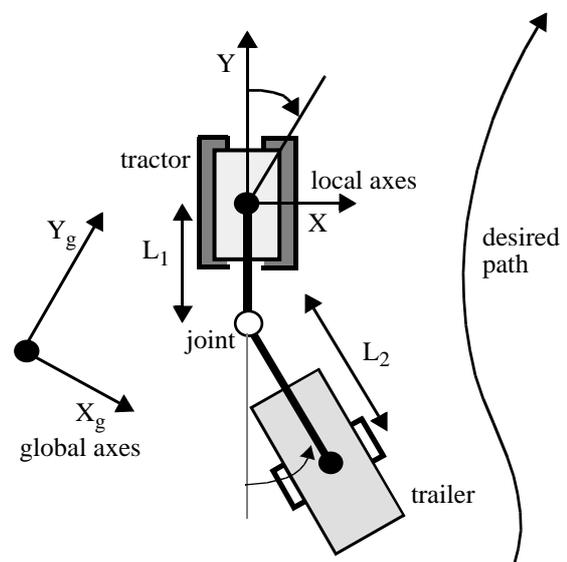

Fig. 1. Parameters of the kinematic chain.

2. PATH TRACKING STRATEGY

The truck changes its curvature by repeatedly fitting circular arcs to the goal point in a pure pursuit strategy (Amidi, 1990). Thus, the required curvature γ_r is computed by:

$$\gamma_r = -2\,\frac{x}{D^2} \qquad (1)$$

where D is the distance that separates the guide point of the vehicle from the goal point, and x the coordinate X of the goal point in the vehicle's reference system (see Fig. 2).
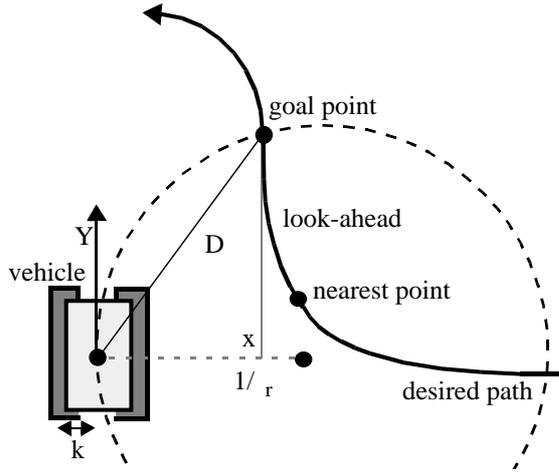

Fig. 2. Pure pursuit strategy.

Since non-holonomic vehicles cannot correct the errors with respect to the nearest point in the path being followed, the goal point is chosen some distance ahead from the nearest point. This distance is measured over the desired path in order to make the search easier (see Fig. 2).

A bigger look-ahead implies smoother control but it also means worse tracking (Ollero, *et al.*, 1994). A smaller look-ahead can reduce tracking errors, but control actions are increased and can even make the movement unstable.

The use of this pure geometric technique, or others like fuzzy direct path tracking (Ollero, *et al.*, 1997), avoids problems in applying control laws derived from complex kinematics and dynamic models, particularly if vehicle-terrain interaction is considered or tracks are employed instead of wheels.

The longitudinal speed of the vehicle is usually considered piece-wise constant with the desired positive values for different path segments. However, when using differential steering, velocity should be modified continually according to the curvature requested:

$$v_r \le \frac{v_m}{1+k\,|\gamma_r|} \qquad (2)$$

where k is half the distance between the tractor's contact points (that can be outside the vehicle when employing tracks instead of wheels) and v_m the maximum speed of the locomotion system.

Changes of speed and curvature are not immediate and depend on the vehicle's low-level control, but they are usually implemented without oscillations.

3. AVOIDING COLLISION BETWEEN TRACTOR AND TRAILER

While applying a path tracking control law to the truck, the trailer is pulled by without direct control over its position. If the internal dynamics of this subsystem is at least stable, the overall behavior of the system can be acceptable (Slotine and Li, 1991).

Under the assumption of small velocities and accelerations as well as light loads on board (Lamiraux, *et al.*, 1999), the kinematic model of the trailer is:

$$\frac{d\beta}{dt} = -v\left(\gamma + \frac{\sin(\beta)}{L_2} + \frac{L_1\,\gamma\,\cos(\beta)}{L_2}\right) \qquad (3)$$

This equation can be specified as a function of the distance travelled s instead of the elapsed time t:

$$\frac{d\beta}{ds} = -\gamma - \frac{\sin(\beta)}{L_2} - \frac{L_1\,\gamma\,\cos(\beta)}{L_2} \qquad (4)$$

Thus, the trailer is represented by a first-order and non-linear subsystem, where β is the state variable and γ the input.

With constant speed and curvature, tractor and trailer tend to follow concentric circumferences (see Fig. 3). The steady value for angle β_s depends on the sign of the vehicle's speed:

a) If $v>0$ then

$$\beta_s = -atan(L_1\,\gamma) - atan\frac{L_2\,\gamma}{\sqrt{1+\gamma^2\,(L_1^2 - L_2^2)}} \qquad (5)$$

b) If $v<0$ then

$$\beta_s = \pi - atan(L_1\,\gamma) + atan\frac{L_2\,\gamma}{\sqrt{1+\gamma^2\,(L_1^2 - L_2^2)}} \qquad (6)$$
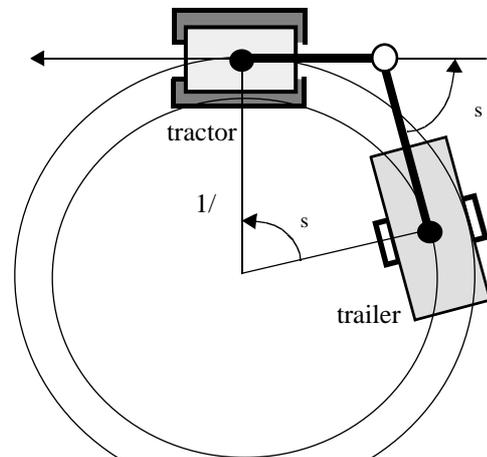

Fig. 3. Stationary response with constant actuations.

The transitory response of the trailer's angle to steps in demanded curvature is always over-damped and with a space-constant proportional to L_2. Thus, if $L_2=0$ the subsystem's model (4) becomes static and a direct relationship between and appears:

$$\beta = -\mathrm{atan}(L_1 \kappa) \qquad (7)$$

Equations (5) and (6) have always solution with the exception of:

$$L_1 < L_2 \qquad |\kappa| > \frac{1}{\sqrt{L_2^2 - L_1^2}} = \kappa_m \qquad (8)$$

In this case no equilibrium point exists and the subsystem becomes unstable. In order to solve this problem during forward path tracking, the requested curvature should be always maintained in absolute value below the limit value κ_m of expression (8).

However, it is still possible to reach a physical limit β_m related to the collision between track and trailer or to the breaking of their link (see Fig. 4). Again, the solution is to limit κ_m even more with the value obtained by solving κ in expression (5) with the peak values $\pm \beta_m$ for the relative angle.
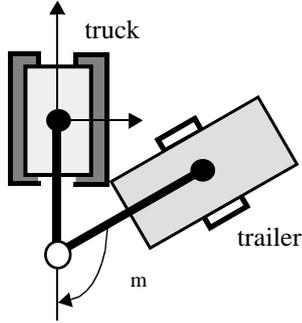

Fig. 4. Physical limit of turning angle β.

4. BACKWARD MOVEMENT

When path tracking is implemented in reverse, the trailer pointing to the rear is considered the virtual vehicle (see Fig. 5). Collision avoidance between trailer and tractor can be prevented limiting the virtual curvature by using the same methodology shown in the previous section, but taking into account that the distances L_1 and L_2 swap their roles.
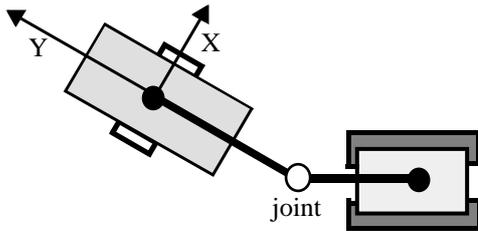

Fig. 5. Virtual vehicle.

The proposed method for backward motion contains the followings steps for each control period:

1. To compute the position and the orientation of the virtual vehicle in the plane based on measured mobile robot's situation and β angle:

$$x_v = x + \sin(\theta) L_1 + \sin(\theta + \beta) L_2$$
$$y_v = y - \cos(\theta) L_1 - \cos(\theta + \beta) L_2 \qquad (9)$$
$$\theta_v = \theta + \beta + \pi$$

2. To calculate the desired curvature κ_{vr} for the virtual vehicle moving forward applying the path tracking control law (1).

3. To transform virtual actuations into actual ones. Two cases must be considered:

a) If $L_1>0$ the control law is

$$\kappa_r = \frac{L_2 \cos(\beta) \kappa_{vr} - \sin(\beta)}{\sin(\beta) L_1 L_2 \kappa_{vr} + L_1 \cos(\beta)} \qquad (10)$$

The relationship between virtual and actual speed is

$$v_r = -v_{vr} (\sin(\beta) L_2 \kappa_{vr} + \cos(\beta)) \qquad (11)$$

However, conversion (10) introduces a time-varying singular position given by:

$$\tan(\beta) = \frac{-1}{L_2 \kappa_{vr}} \qquad (12)$$

In fact, these singularities come from the transformation of the non-holonomic restriction of the tractor. For example, if $\beta=90°$ and the virtual vehicle has to move in a straight line, then the tractor must turn with zero radius.

Moreover, singularity (12) introduces a forbidden region related to the change of sign of the requested speed v_r given by conversion (11). For example, Fig. 6 shows the mapping between virtual and actual curvatures for $\beta = 45°$.

b) If $L_1=0$ then

$$\kappa_r \approx \frac{-\sin(\mathrm{atan}(L_2 \kappa_{vr}))}{L_2} + \frac{\beta - \mathrm{atan}(L_2 \kappa_{vr})}{s} \qquad (13)$$

where s is the allowed distance for the change of β, that acts like a proportional gain of the error, and

$$v_r = \frac{-v_{vr}}{\cos(\beta)} \qquad (14)$$

In this case, only two fixed singular points can be obtained from equation (14): $\beta = \pm 90°$ that correspond to the jackknife phenomenon (Tanaka and Kosaki, 1997). They should be avoided since they require that tractor's speed grows without any limit.
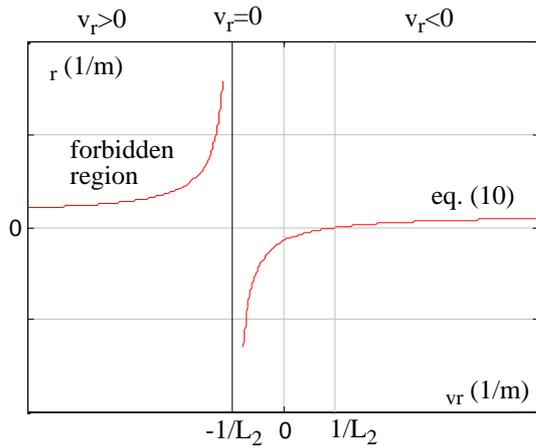
Fig. 6. Transformation for $\varphi = 45°$.

5. IMPLANTATION IN AURIGA-α

The autonomous vehicle Auriga-α has been designed and built by the System Engineering and Automation Department of the University of Málaga (Spain) for outdoor applications (see Fig. 7).

Auriga-α has compact dimensions: 1.24x0.75x0.84 m. A petrol-fed electric generator provides energy to the vehicle. But laboratory tests can also be taken by plugging it into the conventional electrical network.

The vehicle's movement is obtained by the independent performance of two DC motors, with a gear-reduction of 1:12 and incremental encoders. Differential steering is employed to move the vehicles's tracks at a maximum speed $v_m=1$ m/s.

In the mobile robot Auriga-α two control systems can be distinguished:

1. The low level control is responsible for the interface between the actuators and sensors on board the vehicle. Low level control is governed by a DSP and can be managed manually with a joystick, employing elemental commands that are small changes in curvature and speed. The vehicle has two emergency buttons that force the machine to stop when pressed.

2. The navigation system processes the sensorial information and commands the vehicle's speed and curvature every 50 ms. An industrial PC based on a Pentium microprocessor controls all the robot navigation via the real time operating system Lynx. Vehicle's location is obtained by continuously combining differential GPS data with odometry.

The dimensions of the wheeled trailer are similar to the vehicle: 1.17x0.58x0.55 m. Constant values are the following: $L_1=0.7$ m, $L_2=1$ m and $\psi_m=70°$. The car-like joint allows an easy attachment of the trailer to the tractor (see Fig. 8).
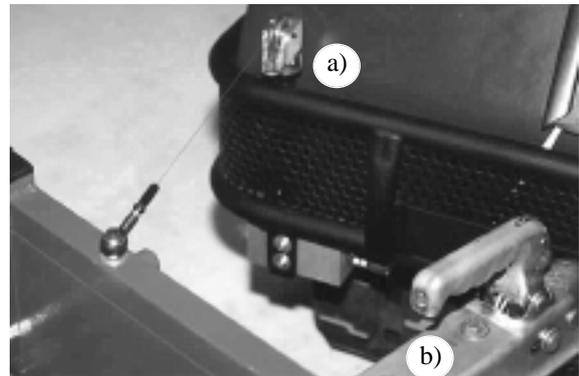

Fig. 8. Wire sensor a) and the link b).

The angle ψ is measured by a displacement sensor with a rotary head, whose wire is also simple to detach (see Fig. 8). The following relationship is fulfilled:

$$\psi = 1.51 - a\cos\frac{0.217 - d^2}{0.214} \qquad (15)$$

where d is the sensed distance in meters (see Fig. 9).
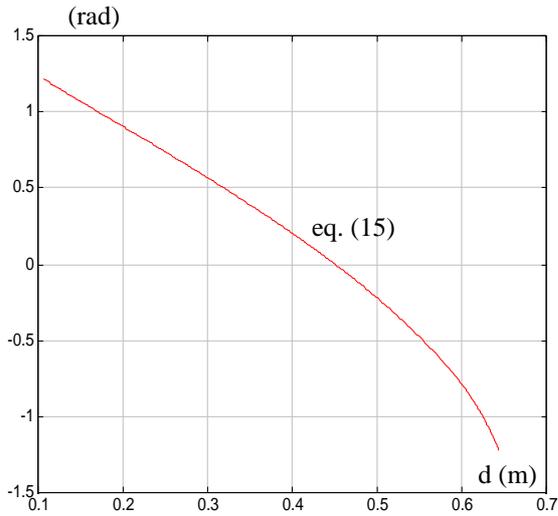

Fig. 7. The mobile robot Auriga-α and its trailer.

Fig. 9. Relationship between *d* and γ.

6. EXPERIMENTS

For forward movement of Auriga-β, due to $L_1 < L_2$, $\kappa_m = 1.4$ m^{-1}, but γ_m imposes that $\kappa_m = 0.758$ m^{-1} (see Fig. 10).
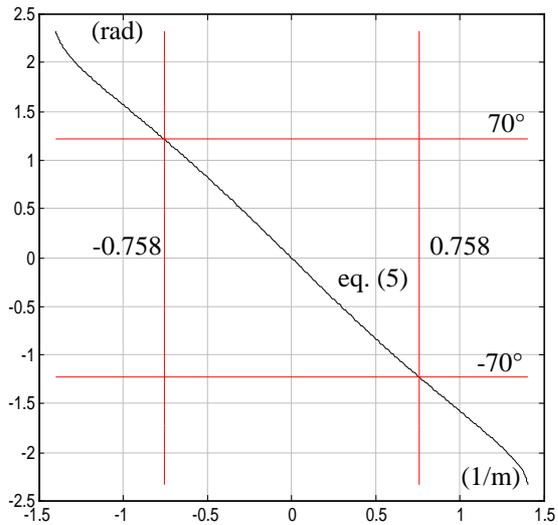

Fig. 10. Limits in requested curvature.

Fig. 11 shows the tracking of a straight line at the maximum speed when beginning far from the path with $\alpha = \beta = 0°$ and employing a small look-ahead distance of 0.3 m. If no limit in curvature is imposed a stroke between tractor and trailer appears (see Fig. 12).

For backward movement, $\kappa_{vm} = 0.664$ m^{-1} prevents collisions between tractor and trailer. It also avoids forbidden regions of transformation to real actuations (see Fig. 13).

In Fig. 14 it can be observed the backward tracking of a straight path when $\alpha = 0°$ and $\beta = 180°$ at the beginning.
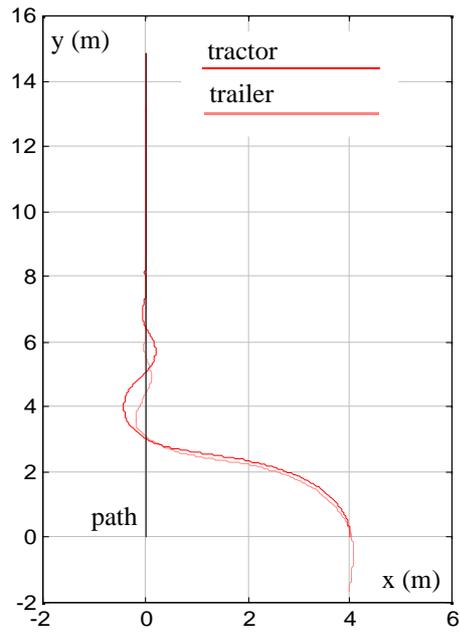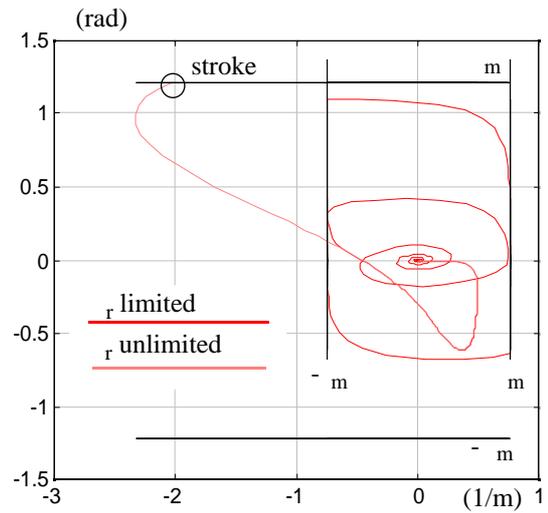
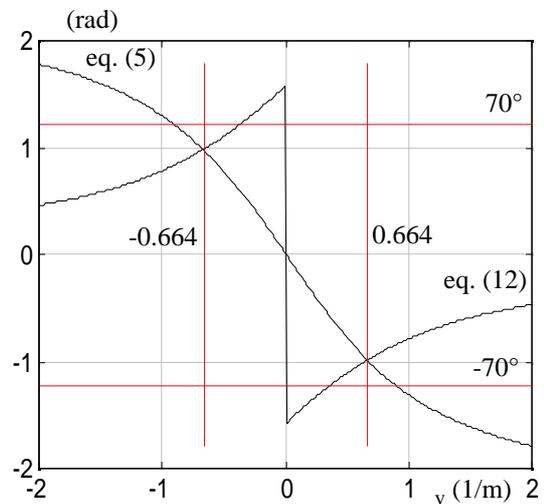

Fig. 11. Forward path tracking.

Fig. 12. State plane $\kappa - \gamma$.

Fig. 13. Limits in virtual curvature.

Fig. 14. Backward path tracking.

7. CONCLUSIONS

Along this paper, it has been described the generalization of an efficient method employed for non-holonomic mobile robots to the path tracking of articulated vehicles. The new issues introduced (avoiding collision truck-trailer and backward movement) have been addressed successfully by simply limiting the demanded curvature.

Thus, the autonomous vehicle Auriga- together with its trailer are able to carry items from one place to another following a mobile object (Martínez, *et al.*, 1998), pre-planned paths or recorded paths while being guided by the joystick.

Future work includes the use of the proposed method to the related problem of path planning for mobile robots with a trailer. It is also interesting to relax the limitations in curvature while the relative angle is distant from the limit $_m$. Moreover, it will be necessary to introduce dynamic effects in the trailer's model when working with heavy loads.

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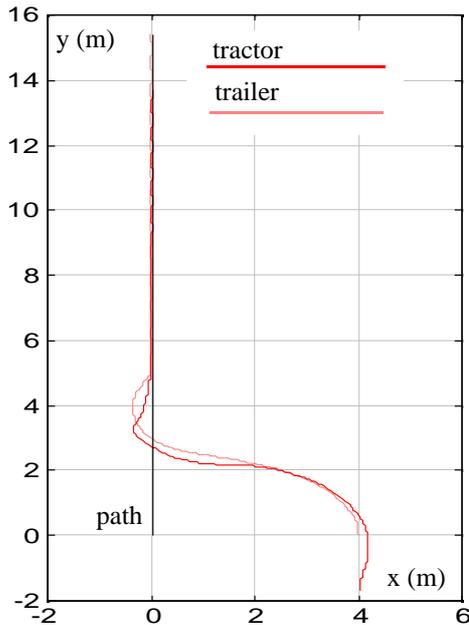Tanaka, K. and T. Kosaki (1997). Design of a stable fuzzy controller for an articulated vehicle. *IEEE Trans. on SMC*, Part B, Vol. 27, No. 3, pp. 552-558.