15th Triennial World Congress, Barcelona, Spain

# LOCAL ANTI-WINDUP COMPENSATION USING OUTPUT LIMITING

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Abstract: In order to circumvent difficulties associated with synthesising anti-windup compensators for systems containing exponentially unstable modes, we propose an alternative strategy for such systems. By elaborating on some recent results, we cast the anti-windup problem as a problem of stabilising a specially constructed outer loop, which is "wrapped around" the nominal system. One of the central ideas to this strategy is the transference of input constraints to output constraints, which, at least for simple systems, appears to be feasible. The success of this approach is demonstrated via a simple example.

Keywords: Anti-windup, stability, linear systems

## 1. INTRODUCTION

Some constrained input linear systems are either difficult, or impossible, to globally stabilise. This is certainly true of those containing at least one exponentially unstable mode, and even those which contain poles on the imaginary axis are often problematic. Hence one is forced to consider local stabilisation.

The problem one is then faced with is that there are few strategies available which can claim to handle, satisfactorily, local anti-windup synthesis. In fact, there are really two problems: the literature which purports to handle such systems is often ad hoc and has no accompanying stability guarantees; or else the literature is not appropriate. For example there are many anti-windup schemes which concentrate on stable (or globally asymptotically null controllable) systems, for example Miyamoto and Vinnicombe (1996), Crawshaw and Vinnicombe (2000) Mulder *et al.* (2001),Grimm *et al.* (2001) to name but a few. Conversely, the techniques of (Hanus *et al.*, 1987) or the conventional high-gain technique may be applied to unstable systems, but without any guarantee of success.

In our opinion one of the few anti-windup schemes which *does* tackle local anti-windup synthesis for exponentially unstable systems is that reported by Teel (1999). Teel gives a constructive technique for synthesising compensators which solve a local anti-windup problem and also gives an example where his proposal works well. However, on the negative side, his construction is quite expensive in terms of compuation, as the compensator is of order equal to that of the plant, and also the compensator requires measurement of the exponentially unstable modes, which may not always be possible. Another mild criticism is that the model-based structure imposed might not necessarily be optimal in any way.

In this paper we expand slightly on some results derived in Turner and Postlethwaite (2001), which report the development of a technique to handle output constraints in linear systems. The method is similar to, but more general and rigorous than, override control systems (Glattfelder and Schaufelberger, 1988). Essentially one models the constrained output of a system with a saturation function, as shown in Figure 1. Then one places, in a feedback loop around the nominal system, a compensator $\Phi(s)$ which becomes active when the difference $\tilde{y} = y_l - sat(y_l)$ becomes non-zero. In many respects this is like anti-windup compensation, but directed towards output constraints;

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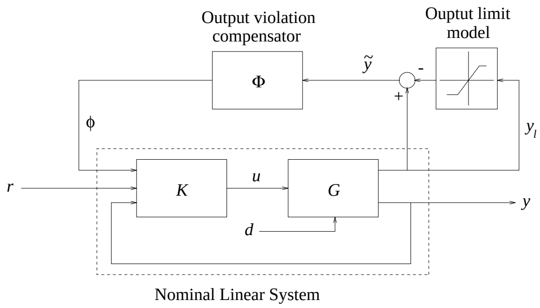


Fig. 1. Anti-windup via output violation compensation

in particular, linear operation is unhindered except if an output constraint is violated.

The main idea is, instead of tackling the control constraints at the plant input directly, to transfer these constraints to a suitable ouput variable, which we call $y_l$. Then we assume that violation of a control constraint is equivalent to violation of an output constraint. Hence, if an output violation occurs, the output violation compensator then becomes active to suppress $y_l$ below its limit, which in turn, by the pseudo-linearity, of the system also regulates the control signal back below its threshold. Of course this can only be achieved for certain sizes of exogenous inputs and initial states, but quite often, it seems to work well.

Obviously one is prompted to wonder why it is worth doing this, as it seems to be tackling the problem in a somewhat artificial fashion. The answer is that by tackling the problem at the output, under some very weak assumptions, one can easily synthesise locally stabilising anti-windup compensators. The main problem with many methods, particularly those based on LMI's, is that they contain a term $A_p'P + PA_p$ which must be negative definite for some $P > 0$, where $A_p$ is the A-matrix of the plant. As $A_p$ has some exponentially unstable eigenvalues there does not exist a $P > 0$ such that $A_p'P + PA_p < 0$, which renders many of these techniques useless. The advantage of our formulation is that, by tackling the problem at the output we consider a closed loop $A$-matrix, which is always assumed stable and thus we do not encounter any difficulties with our LMI's. It is important to remark that we have not removed the problem completely: it is still impossible to obtain global stabilisation, but we *have* removed a technical difficulty which can hinder the synthesis procedure. The precise details of this will become clearer as the paper progresses.

Notation is standard throughout, with $\|x\| = \sqrt{x'x}$ denoting the Euclidean norm, $\|x\|_p$ denoting the $\mathcal{L}_p$ norm of a vector $x(t)$. The induced $\mathcal{L}_p$ norm is $\|H(.)\|_{i,p} = \sup_{x \in \mathcal{L}_p \neq 0} \frac{\|H(x)\|_p}{\|x\|_p}$. The distance is given by $dist(x, \mathcal{X}) := \inf_{w \in \mathcal{X}} \|x - w\|$. The space of real rational, $i \times j$ transfer function matrices is denoted $\mathcal{R}^{i \times j}$, the subset which are analytic in the closed right-half complex plane, with supremum on the imaginary axis is denoted $\mathcal{RH}^{\infty}$.

## 2. OUTPUT VIOLATION COMPENSATION

We provide a brief overview of the output violation compensator problem description and a method for its solution. This is treated more fully in Turner and Postlethwaite (2001).

### 2.1 *System description*

We consider the configuration in Figure 1, where the plant is described by the state-space equations

$$G(s) \sim \begin{cases} \dot{x}_p = A_p x_p + B_p u + B_{pd} d \\ y = C_p x_p + D_p u + D_{pd} d \\ y_l = C_{pl} x_p + D_{pl} u + D_{pdl} d \end{cases} \quad (1)$$

where $x_p \in \mathbb{R}^{n_p}$ is the plant state, $u \in \mathbb{R}^m$ is the control input, $y \in \mathbb{R}^{n_y}$ is the output, which is fed back to the controller, and $d \in \mathbb{R}^{n_d}$ is a disturbance acting on the plant. $y_l \in \mathbb{R}^q$ is the output on which limits are imposed and may often be a subset of the the output $y$, viz $y_l = \bar{S}y$, where $\bar{S}$ selects the channels on which limits are imposed. We make no assumption on the location of the poles of $G(s)$.

We assume that the following stabilising linear controller has been designed

$$K(s) \sim \begin{cases} \dot{x}_c = A_c x_c + B_c y + B_{cr} r \\ u = C_c x_c + D_c y + D_{cr} r \end{cases} \quad (2)$$

where $x_c \in \mathbb{R}^{n_c}$ is the controller state and $r \in \mathbb{R}^{n_r}$ represents a disturbance on the controller, normally the reference input. The interconnection of these equations defines what we call the *nominal closed loop system*, which we assume to be well-defined and stable. However, the presence of the output violation compensator, $\Phi(s)$, inputs two extra signals to the controller, modifying its representation to

$$K(s) \sim \begin{cases} \dot{x}_c = A_c x_c + B_c y + B_{cr} r + \phi_1 \\ u = C_c x_c + D_c y + D_{cr} r + \phi_2 \end{cases} \quad (3)$$

where $\phi = [\phi_1' \quad \phi_2']' \in \mathbb{R}^{n_c+m}$. $\phi$ is the signal produced by the output violation compensator $\Phi(s)$ which is in turn driven by $y_l - sat(y_l)$, where $sat(y_l)$ is the standard saturation nolinearity

$$sat(y_l) = \begin{bmatrix} sat(y_{l,1}) \\ sat(y_{l,2}) \\ \vdots \\ sat(y_{l,q}) \end{bmatrix} \quad (4)$$

and $sat(y_{l,i}) = \text{sign}(y_{l,i}) \times \min\{|y_{l,i}|, \bar{y}_{l,i}\}, \quad \bar{y}_{l,i} > 0, \quad \forall \ i \in \{1, \ldots, q\}$. Note that $\bar{y}_{l,i}$ denotes the output limit in the $i$'th channel of $y_l$. We describe the combination of $G(s)$ and this modifed $K(s)$ as $G_{cl}(s)$, which has the following state-space realisation

$$G_{cl}(s) \sim \begin{cases} \dot{x} = Ax + B_0 w + \bar{B}\phi \\ y = C_y x + D_{y0} w + \bar{D}_y \phi \\ y_l = Cx + D_0 w + \bar{D}\phi \end{cases} \quad (5)$$

where $w = [r' \quad d']'$. A full description of the remainder of the matrices is not given, but they are easy to calculate from the realisations of $G(s)$ and $K(s)$.

From Figure 1 observe that $\Phi(s)$ operates on the difference between $y_l$ and $sat(y_l)$, which models the output constraints. So when an output constraint is not violated $\tilde{y} = y_l - sat(y_l) = 0$ and the output violation compensator is inactive. When $\tilde{y} \neq 0$ the output violation compensator is active and, if designed correctly, will attempt to regulate $y_l$ below its limit. Note that the deadzone operator is given by the identity minus the saturation operator, so another way of expressing $\tilde{y}$ is $\tilde{y} = Dz(y_l)$.

If there is a severe output disturbance, $d_o$, on $y_l$, the method described here may not be appropriate. Although in the linear region of the saturation function the effect of the disturbance on $\tilde{y}$ will not be noticeable as,

$$\tilde{y} = y_l + d_o - sat(y_l + d_o) = y_l - sat(y_l) = 0$$

Unfortunately, as soon as $sat(y_l + d_o)$ enters its saturated regime, erroneous results will be obtained as $Dz(y_l) \neq Dz(y_l + d_o)$. One could perhaps overcome this deficiency with appropriate filtering or prediction, but we do not discuss this here and for the remainder of the paper it is assumed that any output disturbance on $y_l$ is negligible.

### 2.2 *Problem formulation*

Before discussing the design of $\Phi(s)$, it is useful to define the problem we would like to solve. As $\tilde{y}$ is a measure of the deviation of the constrained output $y_l$ from its constraints $sat(y_l)$, it is obviously desirable to keep this small in some appropriate metric. Additionally, $\phi$ is the signal which dictates the type of alteration the system undergoes in order to respect the output constraints. A large $\phi$ indicates that the original linear objectives are incompatible with the output constraints and will be modified quite drastically. Thus it might be desirable to keep this modifcation small in some sense, and hence it would be desirable to keep $\phi$ small in some metric. With this in mind it would seem ideal if we could synthesise $\Phi(s)$ as

$$\Phi^\star(s) = \arg \inf_{stab.\Phi(s)} \left\{ \sup_{w \in \mathcal{L}_p} \frac{\left\| \begin{bmatrix} W_1^{\frac{1}{2}} \tilde{y} \\ W_2^{\frac{1}{2}} \phi \end{bmatrix} \right\|_p}{\|w\|_p} \right\} \quad (6)$$

for some integer $p$, although this is a hard (and non-smooth) optimisation problem, so instead we will be content to ensure that

$$\left\| \begin{bmatrix} W_1^{\frac{1}{2}} \tilde{y} \\ W_2^{\frac{1}{2}} \phi \end{bmatrix} \right\|_p < \gamma \|w\|_p \quad (7)$$

for some integer $p \in [1, \infty)$ and some suitably small $\gamma > 0$. We now formally define the problem we seek to address in the remainder of the paper.

**Definition**

$\Phi(s) \in \mathcal{RH}^\infty$ is said to solve the output violation compensation problem if the closed loop is well-posed and if

(1) $dist(y_l, \mathcal{Y}) = 0 \quad \forall t \geq 0$, then $\phi = 0 \quad \forall t$ (assuming zero intial conditions for $\Phi(s)$).
(2) $dist(y_l, \mathcal{Y}) \in \mathcal{L}_p$, for some integer $p \in [1, \infty)$, then $\phi \in \mathcal{L}_p$

where $\mathcal{Y} = [-\bar{y}_{l,1}, \bar{y}_{l,1}] \times \ldots \times [-\bar{y}_{l,q}, \bar{y}_{l,q}]$. $\Phi(s)$ is said to solve strongly the output violation compensation problem if, in addition,

(3) Equation (7) is satisfied for some integer $p \in [1, \infty)$, some $\gamma > 0$ and some matrices $W_1, W_2 > 0$.

$\circ\circ$

**Remark 1:**

- Condition 1 ensures linear behaviour if $y_l(t)$ never violates its limits.
- Condition 2 ensures that if $y_l(t)$ exceeds its limits for some finite time, thus exciting $\Phi(s)$, then after $y_l(t)$ falls below its threshold, linear behaviour will eventually resume. This is reminiscent of the anti-windup literature where the local structure of the controller is preserved unless saturation occurs. This property makes our work a special case of the general local-global framework introduced in Teel and Kapoor (1997)
- Condition 3 ensures a finite $\mathcal{L}_p$ gain which roughly captures the performance of the system as discussed earlier.
- Note that $\Phi(s) = 0$ solves the output violation compensation problem! $\Phi(s) = 0$ does not, however, solve the problem strongly. Essentially the weaker version of the problem is there to ensure compensators of a certain structure (i.e. ones which do not restrict local performance) and ones which guarantee stability (the closed loop system is stable) are admitted. The stronger version of the problem concentrates on performance.
- Note condition 3 implies condition 2. □

It is easy to prove that there always exists a compensator $\Phi(s)$ which strongly solves the output violation compensation problem (see Turner and Postlethwaite (2001)).

Although we have taken $\mathcal{Y}$ to be symmetric, which implies the saturation function is also symmetric, this is not strictly necessary. In fact, the upper and lower limits of the saturation function can be chosen to be non-symmetric, as this does not alter the sector to which the saturation function, or the deadzone, belong. For convenience, we follow convention and only consider symmetric saturation functions.

### 2.3 *A class of admissible compensators*

The work of Turner and Postlethwaite (2001) discusses this problem more completely - indeed it is the central theme of that paper. Here we merely give a certain class of compensators which strongly solve the output violation compensation problem, and which can also be used to minimise the $\mathcal{L}_2$ gain, $\gamma$, in inequality (7). The class of compensators we consider are those which are static, which seems to be acceptable in many situations.

*Theorem 1.* There exists a static compensator $\Phi \in \mathbb{R}^{(n_c+m)\times q}$ which solves strongly the output violation compensation problem if there exist matrices $Q > 0$, $U = diag(\nu_1, \ldots, \nu_q)$, $L \in \mathbb{R}^{(n_c+m)\times q}$ and a positive real scalar $\mu$ such that the following LMI is satisfied

$$\begin{bmatrix} QA' + AQ & \bar{B}L + QC' & B_0 & 0 & 0 \\ \star & -2U + \bar{D}L + L'\bar{D} & D_0 & U & L' \\ \star & \star & -\mu I & 0 & 0 \\ \star & \star & \star & -W_1^{-1} & 0 \\ \star & \star & \star & \star & -W_2^{-1} \end{bmatrix} < 0 \quad (8)$$

Furthermore a compensator satisfying an $\mathcal{L}_2$ gain bound of $\gamma = +\sqrt{\mu}$ is given by $\Phi = LU^{-1}$.

**Proof:** The proof is given in Turner and Postlethwaite (2001) and is not given here due to space restrictions. Essentially, it follows by noting that $\phi = \Phi\tilde{y}$, $Dz(.) \in \text{Sector}[0, I]$ and using a dissipation type argument to derive an $\mathcal{L}_2$ gain inequality. This can then be cast as a nonlinear matrix inequality, which after using common tricks from the LMI literature, can be transformed into inequality (8). □□

It is also possible to derive sub-optimal dynamic compensators in a similar way, for which we also refer the reader to Turner and Postlethwaite (2001). Dynamic compensators can lead to more appropriate dynamic responses, particularly in terms of frequency content, but are more expensive, in terms of computational requirements, to design and implement.

## 3. SOLVING A LOCAL ANTI-WINDUP PROBLEM

In order to pose an anti-windup problem as an output violation problem, we must transform the input constraints into output constraints, as the “anti-windup” compensator will be $\Phi$. We consider Figure 2, where the input into the plant is now given by $u_m = sat(u)$, which destroys the linearity of $G_{cl}(s)$. However denoting the saturation limits as $\bar{u} \in \mathbb{R}^m$ and defining the set

$$\mathcal{U} = \{u : |u| \preceq \bar{u}\} \quad (9)$$

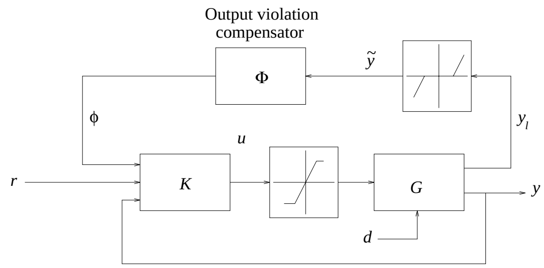


Fig. 2. Anti-windup via output violation compensation

where $|.|$ denotes componentwise magnitude and $\preceq (.)$ denotes componentwise inequality, we can see that for all $u \in \mathcal{U}$, we have $sat(u) = u$; in other words linearity of $G_{cl}(s)$ is maintained for all $u \in \mathcal{U}$. Now, if we can translate the set $\mathcal{U}$ to the output, that is if we can determine a set

$$\mathcal{Y} = \{y_l : |y_l| \preceq \bar{y}_l\} \quad (10)$$

such that $y_l \in \mathcal{Y} \Rightarrow u \in \mathcal{U}$ and if $\Phi(s)$ solves the output violation compensation problem, we will know that, for a certain set of intitial conditions $x(0)$ and exogenous inputs $w$, the system will be stable. For simplicity we define the anti-windup problem to be the problem of ensuring that the system in Figure 2 is stable. We now formalise this notion with the following result.

*Proposition 2.* Let $\Phi(s)$ solve strongly the output violation compensation problem and be such that $y_l \in \mathcal{Y}$ for all $(w, x) \in \mathcal{W} \times \mathcal{X}$, when the saturation element is replaced by the identity, in Figure 2. Furthermore let the set $\mathcal{X}$ be postively invariant. Then the system in Figure 2 is locally stable for all $(w, x(0)) \in \mathcal{W} \times \mathcal{X}$.

**Proof:** As $\Phi(s)$ solves the output violation compensation problem, we know that the system in Figure 2 is stable if the saturation element is replaced by the identity. As $\mathcal{X}$ is positively invariant, $x(t) \in \mathcal{X} \quad \forall t \geq 0$ if $x(0) \in \mathcal{X}$. By virtue of $(w, x(0)) \in \mathcal{W} \times \mathcal{X}$ and positive invariance of $\mathcal{X}$ we have that $y_l \in \mathcal{Y} \quad \forall t \geq 0$ and consequently that $u \in \mathcal{U} \quad \forall t \geq 0$. Hence for all $(w, x(0)) \in \mathcal{W} \times \mathcal{X}$ the system in Figure 2 is such that $sat(u) = u$ and hence the system is stable for all $(w, x(0)) \in \mathcal{W} \times \mathcal{X}$. □□

**Remark 2:** Note that the determination of the sets $\mathcal{Y}$ and $\mathcal{W}$ can vary in difficulty, depending on the systems in question. In a single input single output system the determination might be slightly easier as here $\mathcal{U} = [-\bar{u}, \bar{u}] \subset \mathbb{R}^1$ and it would be quite easy to see that $\pm\bar{u}$ corresponds to $\pm\bar{y}_l$ for a certain $\bar{y}_l$. Then it would also be quite easy to determine a set $\mathcal{W}$ as well. However, for multivariable systems, it is not quite as easy to determine $\mathcal{Y}$ or $\mathcal{W}$ from $\mathcal{U}$. However, assuming some sort of diagonal dominance and using some intuition it is conceivable for one to approximate $\mathcal{Y}$ and $\mathcal{W}$ and these approximations might be good enough for use in practice. □

**Remark 3:** One convenient way of determining $\mathcal{Y}$ from $\mathcal{U}$ would be in the instance that the control signal was available for measurement. In this case it would seem that we could simply set $C_{pl} = 0, D_{pdl} = 0, D_{pl} = I$, which would mean that $y_l = u$. In other words we have transferred the input constraints directly to the output. However, there is a subtlety which must not be ignored: Proposition 2 is based on $u$ never saturating (to ensure linearity of $G_{cl}(s)$), which means that $\mathcal{Y}$ must be chosen as a subset of $\mathcal{U}$ to ensure $u$ never saturates. As before, just how much smaller $\mathcal{Y}$ would have to be compared to $\mathcal{U}$ is not precisely clear, although this would seem an easier problem to solve than the general case. □

**Remark 4:** Unlike Teel (1999), we do not require measurement of the unstable modes, but, effectively we require an observability assumption. This manifests itself in the proposition by the assumption that $\forall y_l \in \mathcal{Y}$ means that $u \in \mathcal{U}$. In other words, we require some indication of $u$ becoming large to be observable by $y_l$ becoming large. □

We think that this connection might be quite useful in practice, because, particularly where LMI's are concerned, the synthesis of anti-windup compensators for exponentially unstable systems is often infeasible. Thus solving an output violation problem can be a way around infeasibility of LMI's in many anti-windup problems.

## 4. AN EXPONENTIALLY UNSTABLE ANTI-WINDUP EXAMPLE

We consider the following plant having two unstable eigenvalues at $0.005 \pm j0.0087$:

$$A_p = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & 0 \\ 1 & 10 & -100 \end{bmatrix}, \; B_p = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \; B_{pd} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (11)$$

$$C_p = [1 \quad 10 \quad 0], \quad D_p = D_{pd} = 0 \quad (12)$$

$$C_{pl} = [100 \quad 1000 \quad -10000], \quad D_{pl} = D_{pdl} = 0 \quad (13)$$

A 5th order $\mathcal{H}^\infty$ loopshaping controller using shaping functions $\tilde{W}_1 = \frac{s+10}{s}, \tilde{W}_2 = 1$ was designed for this plant using the `ncfsyn` command in the Matlab $\mu$-analysis and sythesis toolbox. This yielded a good nominal response.

The control limits were then set at $\pm 0.25$ (i.e. $\mathcal{U} = [-0.25, 0.25]$) and the uncompensated system was simulated. Figure 3 shows (dotted line) the response of the system with these control limits; clearly the response has degraded when compared to the linear response. A glance at Figure 4 shows why this is the case: the control signal of the uncompensated plant has become highly oscillatory in the presence of saturation.

To compensate for this saturation we use the results of Section 3 where the input constraints are transferred, somehow, to the output and hence the control limit

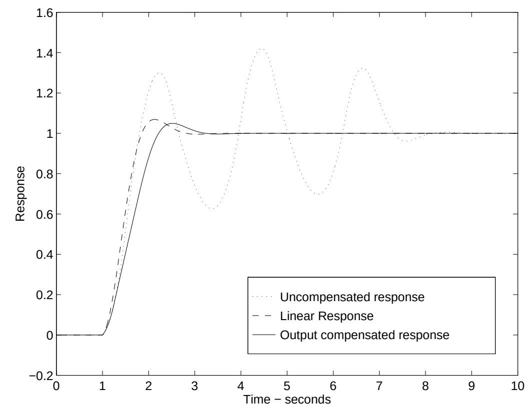


Fig. 3. Primary response of exponentially unstable example

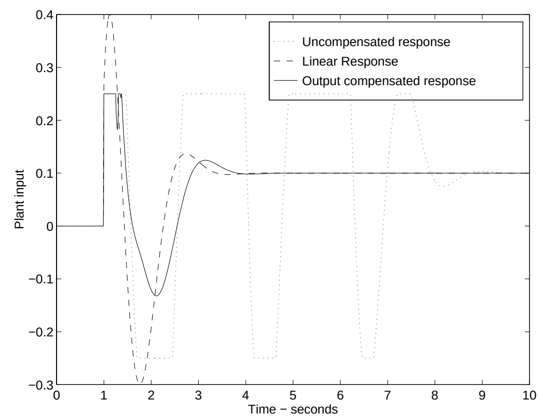


Fig. 4. Control response of exponentially unstable example

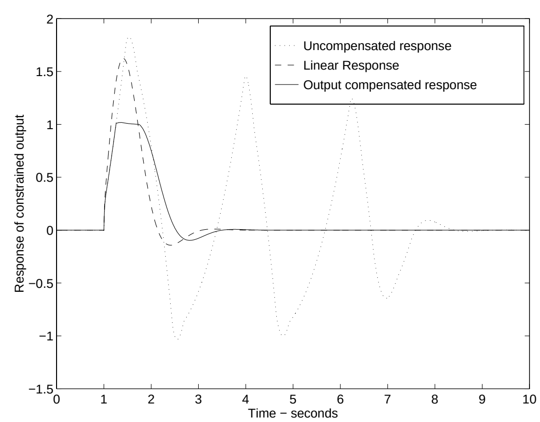


Fig. 5. Constrained response of exponentially unstable example

compensation is taken care of using the output violation compensator. For this example, we assumed that the control signal was not available for direct measurement. However, the output $y_l$ is, in this case, approximately the rate of $y$, viz $y_l \approx \dot{y}$ and it was observed that a high rate corresponds to a large peak in the control signal. Therefore it was deemed an appropriate measurement to use to suppress undesirable control activity. It was observed that setting the output limits on $y_l$ to about unity (i.e. $\mathcal{Y} = [-1, 1]$) ensured that the control signal saturated only slightly, which seemed good enough for demonstration purposes. Theorem 1 was used to sythesise a static output violation compensator using the matrices $W_1 = 100, W_2 = 0.05 \times I_{5+1}$. The resulting $\Phi$ was

$$\Phi = \begin{bmatrix} -13.1207 \\ -2.9257 \\ -0.0000 \\ 21.4543 \\ -25.5816 \\ -26.5094 \end{bmatrix}, \quad \gamma \approx 1.44 \qquad (14)$$

The solid line in Figure 3 shows the system's response using the output violation compensator: clearly it is better than the uncompensated system and, in fact, is not far from the system's nominal linear response. Figure 4 reveals that the control magnitude is reduced compared to the nominal linear response, and does not exhibit the oscillatory response of the uncompensated control signal. A slight amount of saturation does occur, but it was thought that this was not excessive, and it clearly does not compromise stability. Figure 5 shows the response of the constrained output, $y_l$. Using the output violation compensator, $y_l$ is constrained to lie below unity for most of the time - there is a small excursion to above unity as $\tilde{y} \neq 0$ to drive the compensator $\Phi$.

We should emphasize that the anti-windup schemes of Mulder *et al.* (2001), Miyamoto and Vinnicombe (1996) or Grimm *et al.* (2001) could not be used for such a system as the plant is exponentially unstable. Most schemes which offer some sort of stability guarantees assume stable plants. There are other schemes available for anti-windup compensation of unstable plants, such as Wurmthaler and Hippe (1994), but one of the few which gives stability guarantees is that of Teel (1999). Other possibilities are given by Gomes da Silva and Tarbouriech (2000) but this technique is not, strictly speaking, anti-windup: it restricts the local structure of the controller to a specific form. Hence we think that the results of Section 3 can offer an attractive, and reasonably intuitive, solution to the problem of anti-windup for unstable systems, although we do accept that the choosing of $\mathcal{Y}$ based on $\mathcal{U}$ may not always be obvious.

## 5. CONCLUSION

We have suggested tackling the anti-windup problem by transferring input constraints to the output and then designing an output violation compensator to account for any violation of these limits. This allows one to solve a local anti-windup problem which avoids some of the technical difficulties associated with many anti-windup schemes (particularly LMI based methods). A simple example has shown the effectiveness of this technique, although we concede that the determination of $\mathcal{Y}$, and hence $\mathcal{W}$, based on $\mathcal{U}$, was fairly easy in this case. It remains to be seen, and is a topic of future research, whether the determination of $\mathcal{W}$ and $\mathcal{Y}$ will be as easy for more complex, multivariable systems. Some results which may help this research, are those found in Reinelt (2000), which predict the maximum amplitude output of a (linear) system under magnitude and rate bounds on the input.

**Acknowledgements.** The authors would like to thank the reviewers for several useful comments and for making them aware of the work of Reinelt (2000).

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