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ROBUST STABILIZATION OF AN R/C HELICOPTER

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Abstract: This paper presents robust stabilization for an R/C helicopter whose degree of freedom is reduced by fixing at a (joint) point. In our previous paper, we have achieved stable control for the R/C helicopter using fuzzy model-based nonlinear control. However, after simplifying the nonlinear dynamics, we have replaced the simplified nonlinear dynamics with a Takagi-Sugeno fuzzy model. In this paper, we design a robust fuzzy controller so as to compensate the modeling error for the simplification. A robust stability condition achieveing good speed of response is represented in terms of linear matrix inequalities (LMIs). By simultaneously solving the condition and input constraint condition, we design a robust fuzzy controller that achieves good speed of response with small control effort. The simulation and experimental results illustrate the utility of this approach.

Keywords: Fuzzy control, Fuzzy models, Helicopter control, Model-based control, Modeling errors, Nonlinear control

1. INTRODUCTION

A lot of theoretical research on fuzzy model-based nonlinear control has been reported. However, there are a few studies of practical applications (Tanaka *et al.*, 1998a; Tanaka *et al.*, 1999) for the control. We have reported stable control for an R/C helicopter whose degree of freedom is reduced by fixing at a (joint) point (Tanaka and Ohtake, 2001) to discuss the applicability of fuzzy model-based nonlinear control. On the other hand, there are several excellent works on fuzzy control of unmanned helicopter by Sugeno and his group (Sugeno *et al.*, 1995; Sugeno, 1999). However, these papers do not address guarantee of the stability of the control system. In (Tanaka and Ohtake, 2001), we have designed a fuzzy controller guaranteeing not only stability but also both decay rate and constraints on each control input for the R/C helicopter. However, after simplifying the nonlinear dynamics, we have replaced the simplified nonlinear dynamics with a Takagi-Sugeno fuzzy model in the previous papers (Tanaka and Ohtake, 2001).

In this paper, we design a robust fuzzy controller so as to compensate the modeling error for the simplification. A robust stability condition with good speed of response is represented in terms of linear matrix inequalities (LMIs). By simultane-

Fig. 1. R/C Helicopter.

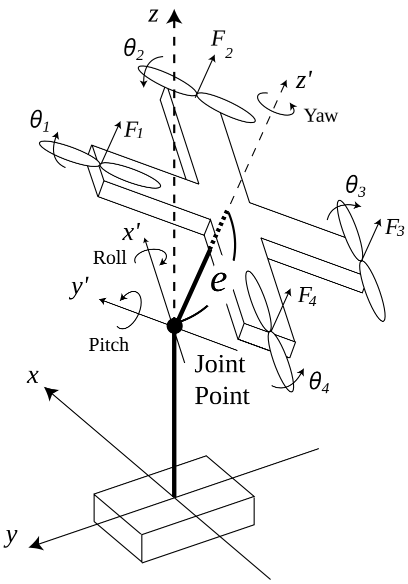

Fig. 2. Helicopter model fixed at a joint point.

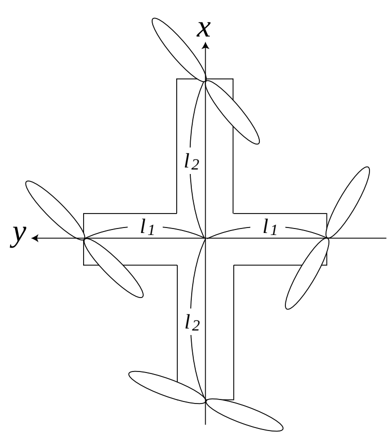

Fig. 3. Helicopter with four propellers.

ously solving the condition and input constraint condition, we design a robust fuzzy controller that achieves good speed of response with small control effort. The simulation and experimental results illustrate the utility of this approach.

2. DYNAMICS OF R/C HELICOPTER

Figure 1 shows an R/C helicopter whose degree of freedom is reduced by fixing at a (joint) point. Figures 2 and 3 show the R/C helicopter model. The equations of motion (Tanaka and Ohtake, 2001) for the R/C helicopter which is fixed at a joint point are

$$
\begin{aligned}
&Me^2\ddot{\gamma}(t) + I_\gamma\ddot{\gamma}(t) - Mge\sin\gamma(t) \\
&\quad = F_1(t)\sqrt{l_1^2+e^2}\cos\left(\tan^{-1}\frac{e}{l_1}\right) \\
&\qquad - F_3(t)\sqrt{l_1^2+e^2}\cos\left(\tan^{-1}\frac{e}{l_1}\right), \qquad (1)
\end{aligned}
$$

$$
\begin{aligned}
&Me^2\ddot{\beta}(t) + I_\beta\ddot{\beta}(t) - Mge\sin\beta(t) \\
&\quad = -F_2(t)\sqrt{l_2^2+e^2}\cos\left(\tan^{-1}\frac{e}{l_2}\right) \\
&\qquad + F_4(t)\sqrt{l_2^2+e^2}\cos\left(\tan^{-1}\frac{e}{l_2}\right), \qquad (2)
\end{aligned}
$$

$$
\begin{aligned}
&(I_\alpha + 4I_1)\,\ddot{\alpha}(t) \\
&\quad = I_1\left(\ddot{\theta}_1(t) - \ddot{\theta}_2(t) + \ddot{\theta}_3(t) - \ddot{\theta}_4(t)\right), \qquad (3)
\end{aligned}
$$

where $\gamma(t)$, $\beta(t)$ and $\alpha(t)$ are the angles of roll, pitch and yaw, respectively. M is the mass of the helicopter. I_γ, I_β and I_α are the moments of inertia around x, y and z-axes with respect to the gravity point of the helicopter, respectively. I_1 is the moment of inertia of a propeller. g is the gravity constant and e, l_1 and l_2 are lengths shown in Figures 2 and 3. F_i is the lift force generated by the i th propeller. It is described as

$$F_i(t) = \frac{1}{3}C_L\rho S l_w^2\dot{\theta}_i(t)^2, \qquad (4)$$

where C_L is the lift coefficient, ρ is the air density, S is the area of a wing of each propeller, l_w is the length of a wing and $\dot{\theta}_i(t)$ is the i th propeller's angular velocity. We assume from the property of the motors (of the propellers) that $\dot{\theta}_i(t) \geq 0$ for all i. Consider that $\dot{\Theta}_0$ is an equilibrium point of the angular velocity and $\Delta\dot{\theta}_i(t)$ is the change of the i th propeller's angular velocity around $\dot{\Theta}_0$. The relation among $\dot{\Theta}_0$, $\Delta\dot{\theta}_i(t)$ and $\dot{\theta}_i(t)$ is given as $\dot{\theta}_i(t) = \dot{\Theta}_0 + \Delta\dot{\theta}_i(t)$. By considering some assumptions (Tanaka and Ohtake, 2001), the following matrix representation is obtained.

$$
\dot{\boldsymbol{x}}(t) = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 \\
\dfrac{C_r\sin x_1(t)}{x_1(t)} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & \dfrac{C_p\sin x_3(t)}{x_3(t)} & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}\boldsymbol{x}(t)
$$

$$+\begin{bmatrix} 0 & 0 & 0 & 0 \\ \underset{\sim\sim\sim}{C_{ur}(2\dot{\Theta}_0 + u_2(t))} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \underset{\sim\sim\sim}{C_{up}(2\dot{\Theta}_0 + u_4(t))} & 0 \\ 0 & C_{uy} & 0 & -C_{uy} \end{bmatrix} \boldsymbol{u}(t), \tag{5}$$

where

$$\boldsymbol{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \end{bmatrix} = \begin{bmatrix} \gamma(t) \\ \dot{\gamma}(t) \\ \beta(t) \\ \dot{\beta}(t) \\ \alpha(t) \end{bmatrix},$$

$$\boldsymbol{u}(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \\ u_4(t) \end{bmatrix} = \begin{bmatrix} \Delta\dot{\theta}_1(t) - \Delta\dot{\theta}_3(t) \\ \Delta\dot{\theta}_1(t) + \Delta\dot{\theta}_3(t) \\ -\Delta\dot{\theta}_2(t) + \Delta\dot{\theta}_4(t) \\ \Delta\dot{\theta}_2(t) + \Delta\dot{\theta}_4(t) \end{bmatrix}.$$

C_r, C_p, C_{ur}, C_{up} and C_{uy} are model constants. In (Tanaka and Ohtake, 2001), we considered the following assumptions.

$$2\dot{\Theta}_0 + u_2(t) \simeq 2\dot{\Theta}_0, \quad 2\dot{\Theta}_0 + u_4(t) \simeq 2\dot{\Theta}_0. \tag{6}$$

By simplifying the elements of the wavy lines in (5) with (6), a Takagi-Sugeno fuzzy model was constructed. In this paper, we design a robust fuzzy controller so as to compensate the modeling error for the simplification.

3. FUZZY MODEL WITH MODEL UNCERTAINTY

Consider the following fuzzy model with uncertain blocks.

$$\dot{\boldsymbol{x}}(t) = \sum_{i=1}^{r} h_i(\boldsymbol{z}(t)) \{(\boldsymbol{A}_i + \boldsymbol{D}_{ai}\boldsymbol{\Delta}_{ai}(t)\boldsymbol{E}_{ai})\boldsymbol{x}(t) + (\boldsymbol{B}_i + \boldsymbol{D}_{bi}\boldsymbol{\Delta}_{bi}(t)\boldsymbol{E}_{bi})\boldsymbol{u}(t)\}, \tag{7}$$

where $\|\boldsymbol{\Delta}_{ai}(t)\| \leq \frac{1}{\rho_{ai}}$, $\|\boldsymbol{\Delta}_{bi}(t)\| \leq \frac{1}{\rho_{bi}}$. $\boldsymbol{\Delta}_{ai}(t)$ and $\boldsymbol{\Delta}_{bi}(t)$ are unknown uncertain blocks. We assume that the upper bounds of these uncertain blocks are known, i.e., ρ_{ai} and ρ_{bi} are known. $\boldsymbol{D}_{ai}$, $\boldsymbol{E}_{ai}$, $\boldsymbol{D}_{bi}$ and $\boldsymbol{E}_{bi}$ are known matrices to provide the uncertain elements.

We simplify the elements of the wavy lines as well as in (Tanaka and Ohtake, 2001). The uncertain blocks are constructed so as to cover the modeling errors for the simplification. A fuzzy model with uncertain blocks is constructed as follows:

$$\boldsymbol{A}_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ C_r & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & C_p & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\boldsymbol{A}_2 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ C_r & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{2C_p}{\pi} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\boldsymbol{A}_3 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ \frac{2C_r}{\pi} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & C_p & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\boldsymbol{A}_4 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ \frac{2C_r}{\pi} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{2C_p}{\pi} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\boldsymbol{B}_i = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 2C_{ur}\dot{\Theta}_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2C_{up}\dot{\Theta}_0 & 0 \\ 0 & C_{uy} & 0 & -C_{uy} \end{bmatrix},$$

$$\boldsymbol{\Delta}_{ai} = \boldsymbol{D}_{ai} = \boldsymbol{E}_{ai} = 0,$$

$$\boldsymbol{\Delta}_{bi} = \begin{bmatrix} u_2(t) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & u_4(t) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\boldsymbol{D}_{bi} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ C_{ur} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & C_{up} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\boldsymbol{E}_{bi} = \boldsymbol{I}_4,$$

$$\frac{1}{\rho_{ai}} = 0, \ \frac{1}{\rho_{bi}} = \sqrt{u_{2max}^2 + u_{4max}^2},$$

$$i = 1, 2, 3, 4.$$

The membership functions are described as

$$h_1(\boldsymbol{z}(t)) = h_{r1}(x_1(t)) \times h_{p1}(x_3(t)),$$
$$h_2(\boldsymbol{z}(t)) = h_{r1}(x_1(t)) \times h_{p2}(x_3(t)),$$
$$h_3(\boldsymbol{z}(t)) = h_{r2}(x_1(t)) \times h_{p1}(x_3(t)),$$
$$h_4(\boldsymbol{z}(t)) = h_{r2}(x_1(t)) \times h_{p2}(x_3(t)),$$

where

$$h_{r1}(x_1(t)) = \begin{cases} \dfrac{\sin x_1(t) - \frac{2}{\pi}x_1(t)}{x_1(t) - \frac{2}{\pi}x_1(t)}, & x_1(t) \neq 0, \\ 1, & \text{otherwise}. \end{cases}$$

$$h_{r2}(x_1(t)) = \begin{cases} \dfrac{x_1(t) - \sin x_1(t)}{x_1(t) - \frac{2}{\pi}x_1(t)}, & x_1(t) \neq 0, \\ 0, & \text{otherwise}. \end{cases}$$

$$h_{p1}\left(x_3(t)\right) = \begin{cases} \dfrac{\sin x_3(t) - \frac{2}{\pi}x_3(t)}{x_3(t) - \frac{2}{\pi}x_3(t)}, & x_3(t) \neq 0, \\ 1, & \text{otherwise.} \end{cases}$$

$$h_{p2}\left(x_3(t)\right) = \begin{cases} \dfrac{x_3(t) - \sin x_3(t)}{x_3(t) - \frac{2}{\pi}x_3(t)}, & x_3(t) \neq 0, \\ 0, & \text{otherwise.} \end{cases}$$

u_{2max} and u_{4max} denote the maximum values of $|u_2(t)|$ and $|u_4(t)|$, respectively. These values correspond to the saturations of actuators (,i.e., motors of the fans).

4. ROBUST FUZZY CONTROLLER DESIGN

To design a robust fuzzy controller for the fuzzy model with uncertain blocks (7), the so-called parallel distributed compensation (PDC) (Tanaka and Wang, 2001) is employed.

Control Rule i

$$\text{If } z_1(t) \text{ is } M_{i1} \text{ and} \cdots \text{and } z_p(t) \text{ is } M_{ip}$$
$$\text{then } \boldsymbol{u}(t) = -\boldsymbol{F}_i \boldsymbol{x}(t), \qquad (8)$$

where $i = 1, 2, \cdots, r$ and r is the number of rules. The overall fuzzy controller is represented by

$$\boldsymbol{u}(t) = -\sum_{i=1}^{r} h_i(\boldsymbol{z}(t))\boldsymbol{F}_i \boldsymbol{x}(t). \qquad (9)$$

The PDC fuzzy controller design is to determine the local feedback gains $\boldsymbol{F}_i$ in the consequent parts. The feedback gains $\boldsymbol{F}_i$ are determined by solving the decay rate conditions guaranteeing robust stability (Theorem 1) and constraints on each control input (Theorem 2). Since they are represented in terms of LMIs, the feedback gains satisfying both of them can be numerically obtained. That is, the design reduces to a numerically feasibility problem.

Theorem 1. The PDC controller that simultaneously satisfies both the robust stability condition and the decay rate condition can be designed by solving the following LMIs.

$$\underset{\substack{\boldsymbol{X}, \boldsymbol{M}_1, \cdots, \boldsymbol{M}_r, \boldsymbol{Y}_0, \\ d_{a1}, \cdots, d_{ar}, d_{b1}, \cdots, d_{br}}}{\text{maximize}} \quad \alpha$$

subject to

$$\boldsymbol{X} > \boldsymbol{0}, \quad \boldsymbol{Y}_0 \geq \boldsymbol{0},$$
$$d_{ai} > 0, \quad d_{bi} > 0$$
$$\hat{\boldsymbol{S}}_{ii} + (s-1)\boldsymbol{Y}_1 < \boldsymbol{0} \qquad (10)$$
$$\hat{\boldsymbol{T}}_{ij} - 2\boldsymbol{Y}_2 < \boldsymbol{0}, \qquad (11)$$
$$i < j \;\; s.t. \;\; h_i \cap h_j \neq \phi,$$

where

$$\hat{\boldsymbol{S}}_{ii} = \begin{bmatrix} \begin{pmatrix} \mathcal{L}(\boldsymbol{A}_i, \boldsymbol{B}_i, \boldsymbol{X}, \boldsymbol{M}_i) \\ +2\alpha\boldsymbol{X} + d_{ai}\boldsymbol{D}_{ai}\boldsymbol{D}_{ai}^T \\ +d_{bi}\boldsymbol{D}_{bi}\boldsymbol{D}_{bi}^T \end{pmatrix} & * & * \\ \boldsymbol{E}_{ai}\boldsymbol{X} & -d_{ai}\rho_{ai}^2\boldsymbol{I} & \boldsymbol{0} \\ -\boldsymbol{E}_{bi}\boldsymbol{M}_i & \boldsymbol{0} & -d_{bi}\rho_{bi}^2\boldsymbol{I} \end{bmatrix},$$

$$\hat{\boldsymbol{T}}_{ij} = \begin{bmatrix} \begin{pmatrix} \mathcal{L}(\boldsymbol{A}_i, \boldsymbol{B}_i, \boldsymbol{X}, \boldsymbol{M}_j) \\ +\mathcal{L}(\boldsymbol{A}_j, \boldsymbol{B}_j, \boldsymbol{X}, \boldsymbol{M}_i) \\ +4\alpha\boldsymbol{X} + d_{ai}\boldsymbol{D}_{ai}\boldsymbol{D}_{ai}^T \\ +d_{bi}\boldsymbol{D}_{bi}\boldsymbol{D}_{bi}^T \\ +d_{aj}\boldsymbol{D}_{aj}\boldsymbol{D}_{aj}^T \\ +d_{bj}\boldsymbol{D}_{bj}\boldsymbol{D}_{bj}^T \end{pmatrix} & * & * & * & * \\ \boldsymbol{E}_{ai}\boldsymbol{X} & -d_{ai}\rho_{ai}^2\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ -\boldsymbol{E}_{bi}\boldsymbol{M}_j & \boldsymbol{0} & -d_{bi}\rho_{bi}^2\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{E}_{aj}\boldsymbol{X} & \boldsymbol{0} & \boldsymbol{0} & -d_{aj}\rho_{aj}^2\boldsymbol{I} & \boldsymbol{0} \\ -\boldsymbol{E}_{bj}\boldsymbol{M}_i & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & -d_{bj}\rho_{bj}^2\boldsymbol{I} \end{bmatrix},$$

$$\boldsymbol{Y}_1 = block - diag(\; \boldsymbol{Y}_0 \;\; \boldsymbol{0} \;\; \boldsymbol{0} \;),$$
$$\boldsymbol{Y}_2 = block - diag(\; \boldsymbol{Y}_0 \;\; \boldsymbol{0} \;\; \boldsymbol{0} \;\; \boldsymbol{0} \;\; \boldsymbol{0} \;),$$

$$\mathcal{L}(\boldsymbol{A}_i, \boldsymbol{B}_i, \boldsymbol{X}, \boldsymbol{M}_j) = \boldsymbol{X}\boldsymbol{A}_i^T + \boldsymbol{A}_i\boldsymbol{X} - \boldsymbol{B}_i\boldsymbol{M}_j - \boldsymbol{M}_j^T\boldsymbol{B}_i^T.$$

The symbol $*$ denotes the transposed elements (matrices) for symmetric positions. s is the maximum number of fuzzy rules that fire simultaneously, where $1 < s \leq r$. The feedback gains are obtained by $\boldsymbol{F}_i = \boldsymbol{M}_i\boldsymbol{X}^{-1}$.

Remark 1 In the fuzzy model for the R/C helicopter, $\boldsymbol{\Delta}_{ai} = \boldsymbol{D}_{ai} = \boldsymbol{E}_{ai} = 0$, i.e., the elements are zero. For this case, the size of the LMIs (10) and (11) can be reduced. Therefore, for this case, Theorem 1 can be simplified as follows:

$$\underset{\substack{\boldsymbol{X}, \boldsymbol{M}_1, \cdots, \boldsymbol{M}_r, \\ \boldsymbol{Y}_0, d_{b1}, \cdots, d_{br}}}{\text{maximize}} \quad \alpha$$

subject to

$$\boldsymbol{X} > \boldsymbol{0}, \quad \boldsymbol{Y}_0 \geq \boldsymbol{0}, \quad d_{bi} > 0,$$
$$\hat{\boldsymbol{S}}_{ii} + (s-1)\boldsymbol{Y}_1 < \boldsymbol{0}$$
$$\hat{\boldsymbol{T}}_{ij} - 2\boldsymbol{Y}_2 < \boldsymbol{0},$$
$$i < j \;\; s.t. \;\; h_i \cap h_j \neq \phi,$$

where

$$\hat{\boldsymbol{S}}_{ii} = \begin{bmatrix} \begin{pmatrix} \mathcal{L}(\boldsymbol{A}_i, \boldsymbol{B}_i, \boldsymbol{X}, \boldsymbol{M}_i) \\ +2\alpha \boldsymbol{X} + d_{bi} \boldsymbol{D}_{bi} \boldsymbol{D}_{bi}^T \end{pmatrix} & * \\ -\boldsymbol{E}_{bi} \boldsymbol{M}_i & -d_{bi}\rho_{bi}^2 \boldsymbol{I} \end{bmatrix},$$

$$\hat{\boldsymbol{T}}_{ij} = \begin{bmatrix} \begin{pmatrix} \mathcal{L}(\boldsymbol{A}_i, \boldsymbol{B}_i, \boldsymbol{X}, \boldsymbol{M}_j) \\ +\mathcal{L}(\boldsymbol{A}_j, \boldsymbol{B}_j, \boldsymbol{X}, \boldsymbol{M}_i) \\ +4\alpha \boldsymbol{X} + d_{bi} \boldsymbol{D}_{bi} \boldsymbol{D}_{bi}^T \\ +d_{bj} \boldsymbol{D}_{bj} \boldsymbol{D}_{bj}^T \end{pmatrix} & * & * \\ -\boldsymbol{E}_{bi} \boldsymbol{M}_j & -d_{bi}\rho_{bi}^2 \boldsymbol{I} & \boldsymbol{0} \\ -\boldsymbol{E}_{bj} \boldsymbol{M}_i & \boldsymbol{0} & -d_{bj}\rho_{bj}^2 \boldsymbol{I} \end{bmatrix},$$

$$\boldsymbol{Y}_1 = block - diag(\, \boldsymbol{Y}_0 \;\; \boldsymbol{0} \,),$$
$$\boldsymbol{Y}_2 = block - diag(\, \boldsymbol{Y}_0 \;\; \boldsymbol{0} \;\; \boldsymbol{0} \,).$$

Remark 2 The fuzzy model for the R/C helicopter also has common $\boldsymbol{B}$, ρ_b, $\boldsymbol{D}_b$ and $\boldsymbol{E}_b$. Therefore, for this case, Remark 1 can be simplified as follows:

$$\underset{\boldsymbol{X}, \boldsymbol{M}_1, \cdots, \boldsymbol{M}_r, d_b}{\text{maximize}} \quad \alpha$$

subject to

$$\boldsymbol{X} > \boldsymbol{0}, \quad d_b > 0, \quad \hat{\boldsymbol{S}}_{ii} < \boldsymbol{0}, \quad \forall i,$$

where

$$\hat{\boldsymbol{S}}_{ii} = \begin{bmatrix} \begin{pmatrix} \mathcal{L}(\boldsymbol{A}_i, \boldsymbol{B}_i, \boldsymbol{X}, \boldsymbol{M}_i) \\ +2\alpha \boldsymbol{X} + d_b \boldsymbol{D}_b \boldsymbol{D}_b^T \end{pmatrix} & * \\ -\boldsymbol{E}_b \boldsymbol{M}_i & -d_b\rho_b^2 \boldsymbol{I} \end{bmatrix}.$$

Theorem 2. (Tanaka and Wang, 2001) Assume that the initial state $\boldsymbol{x}(0)$ is known. The constraint $\|u_j(t)\|_2 \le \mu_j$ is enforced at all times $t \ge 0$ if the LMIs

$$\begin{bmatrix} 1 & \boldsymbol{x}^T(0) \\ \boldsymbol{x}(0) & \boldsymbol{X} \end{bmatrix} \ge \boldsymbol{0}, \tag{12}$$

$$\begin{bmatrix} \boldsymbol{X} & \boldsymbol{M}_i^T \boldsymbol{E}_j^T \\ \boldsymbol{E}_j \boldsymbol{M}_i & \mu_j^2 \boldsymbol{I} \end{bmatrix} \ge \boldsymbol{0} \tag{13}$$

hold, where $\boldsymbol{X} = \boldsymbol{P}^{-1}$, $\boldsymbol{M}_i = \boldsymbol{F}_i \boldsymbol{X}$. $\boldsymbol{E}_j$ is the vector to determine which each input is constrained. That is, $u_j(t) = \boldsymbol{E}_j \boldsymbol{u}(t)$, where

$$\boldsymbol{E}_j = [\, \overset{1}{0} \cdots \overset{j-1}{0} \;\; \overset{j}{1} \;\; \overset{j+1}{0} \cdots \overset{m}{0} \,],$$

for $\boldsymbol{u}(t) \in R^m$.

5. SIMULATION

The model constants C_r, C_p, C_{ur}, C_{up} and C_{uy} in the fuzzy model (7) are set as follows:

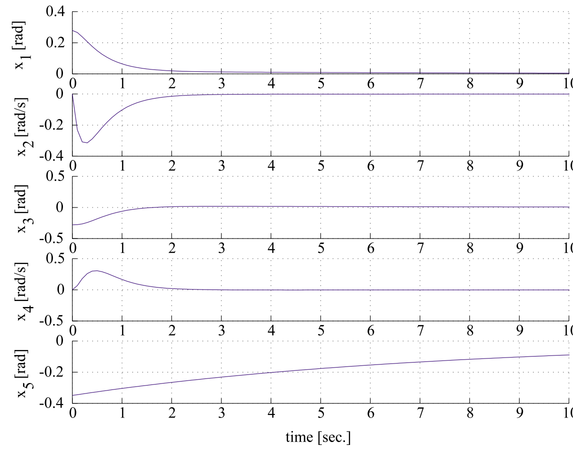

Fig. 4. Robust stability & Decay rate controller.

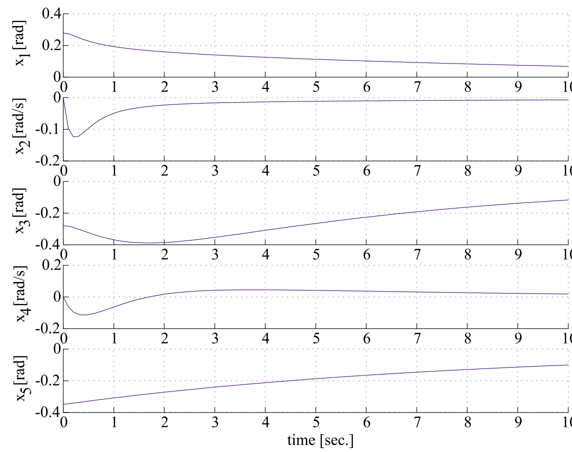

Fig. 5. Decay rate controller.

$$\begin{aligned} C_r &= 47.102 \text{ [1/s}^2], \; C_p = 36.191 \text{ [1/s}^2] \\ C_{ur} &= 5.011 \times 10^{-4}, \; C_{up} = 5.126 \times 10^{-4} \\ C_{uy} &= 1.155 \times 10^{-3}. \end{aligned}$$

$\dot{\theta}_{i\max} = 2\pi f_0$ [rad/sec], $\dot{\theta}_{i\min} = 0$ [rad/sec] and $\dot{\Theta}_0 = \pi f_0$ [rad/sec], where $f_0 = 30$[Hz] which is the maximum frequency of propellers. The parameters on constraints on each control input are set as follows:

$$\begin{aligned} \boldsymbol{E}_1 &= [1 \;\; 0 \;\; 0 \;\; 0], \\ \boldsymbol{E}_2 &= [0 \;\; 1 \;\; 0 \;\; 0], \\ \boldsymbol{E}_3 &= [0 \;\; 0 \;\; 1 \;\; 0], \\ \boldsymbol{E}_4 &= [0 \;\; 0 \;\; 0 \;\; 1], \\ \mu_1 &= \mu_3 = 0.9 \times 2\pi f_0, \\ \mu_2 &= \mu_4 = 0.1 \times 2\pi f_0. \end{aligned}$$

Initial state is selected as

$$\boldsymbol{x}(0) = [0.279 \quad 0 \quad -0.279 \quad 0 \quad -0.349]^T.$$

The robust fuzzy controller is designed by simultaneously solving both the decay rate condition guaranteeing robust stability (Theorem 1) and constraints on each control input (Theorem 2).

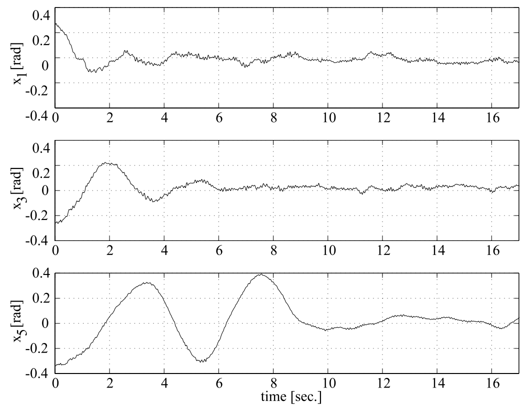

Fig. 6. Experimental result (time response).

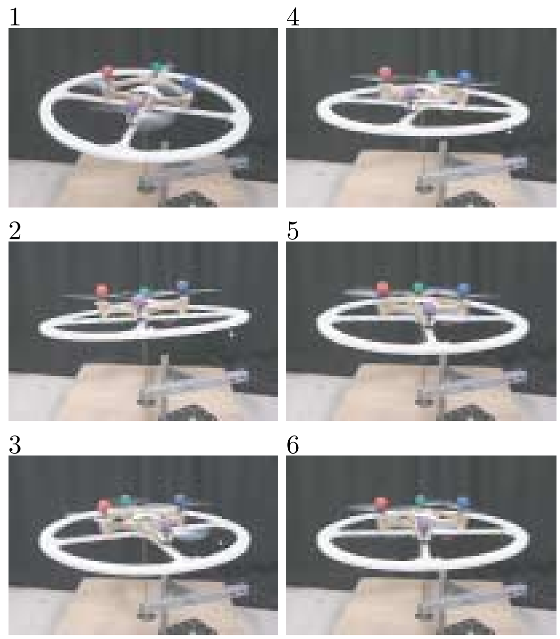

Fig. 7. Experimental result (photographs).

Figure 4 shows the simulation result by the controller that satisfies the decay rate condition guaranteeing robust stability and constraints on each control input. Figure 5 shows the simulation result by the controller that satisfies the decay rate condition and constraints on each control input. The decay rate controller without the robust stability condition does not realize good speed of response due to the modeling errors.

6. EXPERIMENTAL RESULT

Figures 6 and 7 show the experimental result for the real R/C helicopter. The designed controller stabilizes the real R/C helicopter.

7. CONCLUSIONS

This paper has presented robust stabilization for the R/C helicopter using fuzzy model-based nonlinear control. We have designed the robust fuzzy controller guaranteeing robust stability and satisfying the decay rate condition and constraints on each control input. The simulation and experimental result have illustrated the utility of this approach. A future work is to achieve stable flight for the R/C helicopter without fixing at a joint point.

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