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WIENER MODELS OF DIRECTION-DEPENDENT DYNAMIC SYSTEMS

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Abstract: Direction-dependent dynamic systems are defined, and Wiener models for them are described. For first-order systems with pseudo-random binary inputs, optimising the model parameters by cross-correlation function matching methods based on analysis gives excellent results. For first-order systems with inverse-repeat pseudo-random binary inputs, optimisation by discrete Fourier transform matching and output matching methods also give excellent results. These optimisation methods may be extended to direction-dependent dynamic systems that cannot be analysed, as illustrated by a first-order system with a pseudo-random ternary input, and a second-order system with an inverse-repeat pseudo-random binary input.

Keywords: Discrete Fourier transforms; First-order systems; Identification; Modelling; Nonlinear models; Nonlinear systems; Optimisation; Pseudo-random sequences.

1. INTRODUCTION

In direction-dependent dynamic systems, the system dynamics when the system output is increasing are different from those when the output is decreasing. Such systems are not readily amenable to analysis, and theoretical solutions are known only for cases in which the system dynamics in each direction are first-order linear and the system inputs are binary. The paucity of analytical results for these systems contrasts sharply with their abundance in industry, where examples include steam-raising plants (Godfrey and Briggs, 1972), gas turbines, chemical processes and nuclear reactors (Godfrey and Moore, 1974), distillation columns (Turner *et al.*, 1996), and automotive suspensions and tyres (Tan and Godfrey, 2001).

In the absence of methods for dealing with these systems analytically, interest turns naturally to modelling them in forms known to be suitable for analysis. For nonlinear systems of this kind, models based on the Volterra functional series (Volterra, 1930) have traditionally been deemed suitable. This series is known to converge for block-oriented models, where the blocks represent subsystems that either have linear dynamics or are static nonlinearities (Brilliant, 1958), and models of this kind are therefore used here. From among the many different kinds available, the Wiener model in which dynamic linear subsystems precede static power law nonlinearities is chosen because it is particularly suitable for use with binary inputs (Barker *et al.*, 2000a), and these are the only inputs for which analytical results are available for the outputs and cross-correlation functions of first-order direction-dependent systems (Tan and Godfrey, 2001).

The approach adopted here is to commence with the most general form of a direction-dependent system capable of being analysed, which is a first-order system in which the values of both the gain and the time constant are direction-dependent. A Wiener model is then developed for the system, using analytical results obtained when both the system and the model have a pseudo-random binary input. The most effective modelling methods are then applied to examples of direction-dependent systems for which no theoretical methods are available. These are a first-order system with a pseudo-random ternary input, and a second-order system with an inverse-repeat pseudo-random binary input.

2. DIRECTION-DEPENDENT SYSTEMS

Recent work by Tan and Godfrey (2001) has led to new and important results for direction-dependent systems. In particular, analytical expressions have been obtained for the outputs and cross-correlation functions of first-order systems with direction-dependent gains and time constants when the inputs are either maximum-length pseudo-random binary signals or inverse-repeat pseudo-random binary signals. Here the transfer functions are taken to be $\dfrac{K_\mathrm{U}}{1+\mathrm{s}T_\mathrm{U}}$ when the direction of the system output $y(t)$ is positive, and $\dfrac{K_\mathrm{D}}{1+\mathrm{s}T_\mathrm{D}}$ when it is negative. A system for which $K_\mathrm{U}=4$, $K_\mathrm{D}=1$, $T_\mathrm{U}=3T$, $T_\mathrm{D}=12T$ and $T=1$ is used for illustration in this paper.

The system input $u(t)$ is taken to be either a maximum-length pseudo-random binary signal with period NT generated using a constant clock-pulse interval of T from a maximum-length binary sequence $s(i)$ with period N and characteristic polynomial $f(\mathrm{D})$, or the inverse-repeat maximum length pseudo-random binary signal with period $2NT$ derived from it by inverting alternate values. In both cases, the zero elements of $s(i)$ are converted to –1 and the unity elements of $s(i)$ are converted to +1. A pseudo-random binary signal for which N = 127 and $f(\mathrm{D})=1\oplus_2\mathrm{D}\oplus_2\mathrm{D}^4\oplus_2\mathrm{D}^6\oplus_2\mathrm{D}^7$ is used for illustration in this paper.

Of particular importance is the discrete periodic cross-correlation function $\phi_{uy}(iT)$ between the system input $u(t)$ and the steady-state system output $y(t)$, given by

$$\phi_{uy}(iT)=\frac{1}{N}\sum_{j=0}^{N-1}y(jT)u((j-i)T) \qquad (1)$$

The cross-correlation function is the sum of an infinite number of components, for which the analytical expressions involve four parameters and an impulse response function:

- $a=(\exp(-T/T_\mathrm{U})+\exp(-T/T_\mathrm{D}))/2$
- $b=(\exp(-T/T_\mathrm{U})-\exp(-T/T_\mathrm{D}))/2$
- $c=(1-\exp(-T/T_\mathrm{U}))(1-\exp(-T/T_\mathrm{D}))$
- $A=(K_\mathrm{U}+K_\mathrm{D})/2$
- $w_\mathrm{C}(iT)=\dfrac{Ac}{1-a}a^i$ for $i\ge 0$ and $=0$ for $i<0$

The significant components of the system cross-correlation function are as follows.

Constant component

$$\phi_{uy0}=-\frac{1}{N}K_\mathrm{D} \qquad (2)$$

This component is simply a bias.

Linear component

$$\phi_{uy1}(iT)=\frac{N+1}{N}w_\mathrm{C}(iT) \qquad (3)$$

This component, which is the most significant, is the impulse response $w_\mathrm{C}(iT)$ scaled by $\dfrac{N+1}{N}$. The equivalent transfer function has gain $K_\mathrm{C}=\dfrac{Ac}{(1-a)^2}$ and time constant $T_\mathrm{C}=-T/\ln(a)$.

Quadratic component

$$\phi_{uy2}(iT)=-\frac{N+1}{N}b\sum_{r=1}^{N-1}a^{r-1}w_\mathrm{C}((i-i_r)T) \qquad (4)$$

where i_r is defined through $s_{i-i_r}=s_i\oplus_2 s_{i-r}$ and is determined by dividing the polynomial $1\oplus_2\mathrm{D}^r$ by the characteristic polynomial $f(\mathrm{D})$ until the single-term remainder D^{i_r} is obtained. This component is the sum of scaled and shifted replicas of the impulse response, with the more significant replicas obtained when $r=1,2,3,4,\ldots$. For the characteristic polynomial used here, the corresponding shifts are $i_r=89,51,21,102,\ldots$.

Cubic component

$$\phi_{uy3}(iT)=\frac{N+1}{N}b^2\sum_{q=1}^{N-2}\sum_{r=1}^{N-1-q}a^{q+r-2}w_\mathrm{C}((i-i_{q,r})T) \quad (5)$$

where $i_{q,r}$ is defined through $s_{i-i_{q,r}}=s_i\oplus_2 s_{i-q}\oplus_2 s_{i-q-r}$ and is determined by dividing the polynomial $1\oplus_2\mathrm{D}^q\oplus_2\mathrm{D}^{q+r}$ by the characteristic polynomial $f(\mathrm{D})$ until the single-term remainder $\mathrm{D}^{i_{q,r}}$ is obtained. This component is also the sum of scaled and shifted replicas of the linear impulse response. For the characteristic polynomial used here, the more significant replicas are at shifts $i_{1,1}$ = 59, $i_{1,2}$ = 85, $i_{2,1}$ = 113, $i_{1,3}$ = 107, $i_{2,2}$ = 118, and $i_{3,1}$ = 95.

If the system input is the inverse-repeat maximum-length pseudo-random binary signal, then the odd-order components in the new cross-correlation function are identical to those above and the even-order components are zero, except for a very small zero-order term that is an oscillatory bias (Tan and Godfrey, 2001).

3. WIENER MODEL

The Wiener model of the system is shown in Fig. 1. As the highest-order component of any significance in the system cross-correlation function is cubic, the model paths are restricted to those of order 0 to 3. The discrete periodic cross-correlation function $\phi_{uv}(iT)$ between the model input $u(t)$ and the steady-state model output $v(t)$ is the sum of four components.

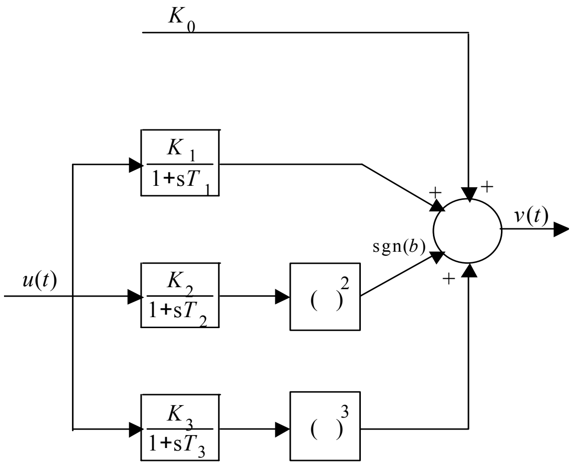

Fig. 1. Wiener model of first-order direction-dependent system

The analytical expression for the component corresponding to the *j*-th order path involves one parameter and two impulse response functions (Barker and Obidegwu, 1973):

- $a_j = \exp(-T/T_j)$
- $w_j(iT) = K_j(1-a_j)a_j^i$ for $i \geq 0$ and $= 0$ for $i < 0$
- $w_j'(iT) = K_j^j(1-a_j^j)a_j^{ji}$ for $i \geq 0$ and $= 0$ for $i < 0$

The components of the model cross-correlation function are as follows.

Constant component

$$\phi_{uv0} = \frac{K_0}{N} \tag{6}$$

This component is simply a bias.

Linear component

$$\phi_{uv1}(iT) = \frac{N+1}{N}w_1(iT) - \frac{K_1}{N} \tag{7}$$

This component is the impulse response $w_1(iT)$ scaled by $\frac{N+1}{N}$, with a bias.

Quadratic component

$$\phi_{uv2}(iT) = -[\mathrm{sgn}(b)]\frac{N+1}{N}2\frac{1-a_2}{1+a_2}\sum_{r=1}^{N-1}a_2^r w_2'((i-i_r)T) + [\mathrm{sgn}(b)]\frac{K_2^2}{N} \tag{8}$$

where i_r is the same as in Section 2. This component is the sum of scaled and shifted replicas of the impulse response $w_2'(iT)$, with a bias.

Cubic component

$$\phi_{uv3}(iT) = \frac{N+1}{N}6\frac{(1-a_3)^3}{1-a_3^3}\sum_{q=1}^{N-2}\sum_{r=1}^{N-1-q}a_3^{2q+r}w_3'((i-i_{q,r})T) + \frac{N+1}{N}3K_3^2\frac{1-a_3}{1+a_3}w_3(iT) - \frac{N+1}{N}2\frac{(1-a_3)^3}{1-a_3^3}w_3'(iT) - \frac{K_3^3}{N} \tag{9}$$

where $i_{q,r}$ is the same as in Section 2. This component is the sum of scaled and shifted replicas of the impulse response $w_3'(iT)$, together with a scaled replica of the impulse response $w_3(iT)$ and a bias.

If the system input is the inverse-repeat maximum-length pseudo-random binary signal, then the odd-order components in the new cross-correlation function are identical to those above with the biases removed and the even-order components are zero, except for a very small zero-order term that is an oscillatory bias (Barker and Obidegwu, 1973).

4. MODEL OPTIMISATION WITH PSEUDO-RANDOM BINARY INPUTS

The Wiener model that best represents the direction-dependent system is obtained when its parameters are set to their optimal values. As the system and model cross-correlation functions have similar components, the cross-correlation component (CCC) matching method may be used (Barker *et al.*, 2000b; Barker *et al.*, 2001).

The constant components are simply chosen to equate the biases. The linear components are the most significant, and these are matched when $K_1 = K_C$ and $T_1 = T_C$. The quadratic components are the next most significant, and the most consistent methods for matching them involves minimising the sum of either the squares or the moduli of the differences between them. These both give similar results, so that only the former will be used here. For this, the MATLAB Optimization Toolbox (Coleman *et al.*, 1999) can be used to obtain the optimal values of K_2 and a_2 that give

$$\underset{K_2,a_2}{\text{minimum}}\sum_{r=1}^{N-1}\sum_{j=1}^{N}\left[\frac{Ac|b|}{1-a}a^{r+j-2} - 2K_2^2(1-a_2)^2a_2^{r+2j-2}\right]^2$$

The cubic components, which are the least significant, can be matched in the same way, by obtaining the optimal values of K_3 and a_3 that give

$$\underset{K_3,a_3}{\text{minimum}}\sum_{q=1}^{N-2}\sum_{r=1}^{N-1-q}\sum_{j=1}^{N}\left[\frac{Acb^2}{1-a}a^{q+r+j-3} - 6K_3^3(1-a_3)^3a_3^{2q+r+3j-3}\right]^2$$

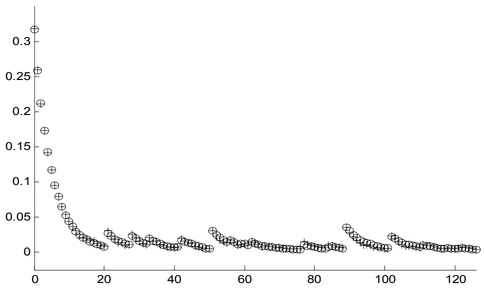

Fig. 2. Cross-correlation functions of first-order direction-dependent system (o) and Wiener model (+) with pseudo-random binary input

This method represents the limit of the analytical approach, but the results obtained are excellent. Fig. 2 shows the cross-correlation functions of the system. The correspondence between the linear components is striking, as is that between the quadratic components in which the more significant replicas at shifts of 89*T*, 51*T*, 21*T*, 102*T*, etc., are easily seen. The cubic components are less obvious.

The optimal model parameters obtained by this method are given in Table 1.

Table 1 Optimal parameters for Wiener model of first-order direction-dependent system with pseudo-random binary input

Matching Method	K_0	K_1	T_1	K_2	T_2	K_3	T_3
CCC	2.31	1.72	4.99	1.00	7.03	0.84	9.04
CCF	2.40	1.59	4.76	1.00	6.90	0.94	8.92
OUT	2.97	1.60	4.78	0.94	6.44	0.82	7.96

In contrast to the completely analytical approach above, two partly analytical methods may also be used to obtain the optimal model parameters. Both methods use the results obtained by Tan and Godfrey (2001) to compute the system output values at the clock-pulse epochs. For a given set of model parameters, each of the model path output values at the same epochs is also computed and summed to obtain the model output values.

In the cross-correlation function (CCF) matching method the cross-correlation functions of the system and the model are computed and matched using the MATLAB Optimization Toolbox to obtain the optimal values of K_0, K_1, T_1, K_2, T_2, K_3 and T_3 that give

$$\underset{K_0, K_1, T_1, K_2, T_2, K_3, T_3}{\text{mimimum}} \sum_{i=0}^{N-1} [\phi_{uy}(iT) - \phi_{uv}(iT)]^2$$

As seen from Table 1, the optimal values obtained by this method are similar to those obtained by the component matching method.

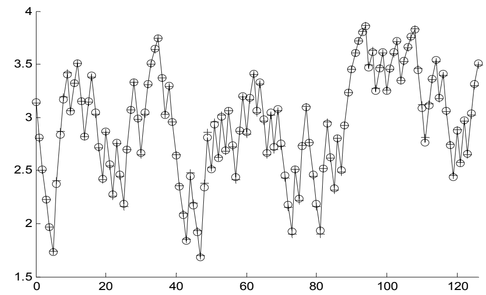

Fig. 3. Outputs of first-order direction-dependent system (o) and Wiener model (+) with pseudo-random binary input

In the output (OUT) matching method, the system and model output values are matched by obtaining the optimal values of K_0, K_1, T_1, K_2, T_2, K_3 and T_3 that give

$$\underset{K_0, K_1, T_1, K_2, T_2, K_3, T_3}{\text{mimimum}} \sum_{i=0}^{N-1} [y(iT) - v(iT)]^2$$

Fig. 3 shows the very close match between the system and model outputs obtained by this method. As seen from Table 1, the optimal values obtained are similar to those obtained by the cross-correlation function methods.

5. MODEL OPTIMISATION WITH INVERSE-REPEAT PSEUDO-RANDOM BINARY INPUTS

If the input is now taken to be the inverse-repeat maximum-length pseudo-random binary signal with period 2*NT*, then matching the odd-order components of the system and model cross-correlation functions by the methods in Section 4 gives optimal values for K_1, T_1, K_3 and T_3 similar to those in Table 1. However, the even-order components are zero except for the very small zero-order term, so no meaningful information is available for the optimal values of K_0, K_2, and T_2. Fortunately, the use of an inverse-repeat input confers properties on the output that can be exploited to give two improved matching methods.

The system and model outputs may be separated into odd and even components by subtracting or adding members separated by a half-period. In the output (OUT) matching method, the odd components of the system and model outputs are matched to obtain the optimal values of K_1, T_1, K_3 and T_3 that give

$$\underset{K_1, T_1, K_3, T_3}{\text{minimum}} \sum_{k=0}^{N-1} [y(i) - y(i+N) - v(i) + v(i+N)]^2$$

and the even components are matched to obtain the optimal values of K_0, K_2 and T_2 that give

$$\underset{K_0, K_2, T_2}{\text{minimum}} \sum_{i=0}^{N-1} [y(i) + y(i+N) - v(i) - v(i+N)]^2$$

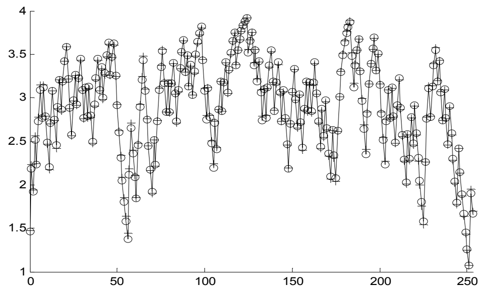

Fig. 4. Outputs of first-order direction-dependent system (o) and Wiener model (+) with inverse-repeat pseudo-random binary input

Fig. 4 shows the very close match between the system and model outputs that is obtained by this method. As seen from Tables 1 and 2, the optimal parameter values obtained are similar to those obtained with a pseudo-random binary input.

Table 2 Optimal parameters for Wiener model of first-order direction-dependent system with inverse-repeat pseudo-random binary input

Matching method	K_0	K_1	T_1	K_2	T_2	K_3	T_3
CCF	-	1.64	4.90	-	-	0.81	8.91
OUT	2.97	1.64	4.88	1.01	7.22	0.81	8.90
DFT	2.97	1.64	4.87	1.01	7.22	0.84	9.60

Odd and even harmonics in the system output are attributable to the odd-order and even-order nonlinearities of the system respectively. In the discrete Fourier transform (DFT) matching method, the odd-order and even-order nonlinearities are matched independently through the odd and even members of the discrete Fourier transform of a period of the output (Barker and Godfrey, 1999).

The system output transform $Y(k)$ is given by

$$Y(k) = \sum_{i=0}^{2N-1} y(iT)\exp(-\mathrm{j}\pi ki/N) \tag{10}$$

and the model output transform $V(k)$ is given by

$$V(k) = \sum_{i=0}^{2N-1} v(iT)\exp(-\mathrm{j}\pi ki/N) \tag{11}$$

Matching the odd members of the system and model transforms yields the optimal values of K_1, T_1, K_3 and T_3 that give

$$\underset{K_1, T_1, K_3, T_3}{\text{minimum}} \sum_{k=0}^{N-1} \left| Y(2k+1) - V(2k+1) \right|^2$$

and matching the even members yields the optimal values of K_0, K_2 and T_2 that give

$$\underset{K_0, K_2, T_2}{\text{minimum}} \sum_{k=0}^{N-1} \left| Y(2k) - V(2k) \right|^2$$

As seen from Tables 1 and 2, the optimal values obtained by this method are similar to those obtained using the CCC, CCF and OUT methods with a pseudo-random binary input.

6. MULTI-LEVEL INPUTS AND HIGHER-ORDER SYSTEMS

No analytical solutions are known for direction-dependent systems that either have inputs with more than two levels or are of order greater than one, so for those systems cross-correlation component matching cannot be used and cross-correlation function matching becomes considerably less attractive. Output matching and output transform matching with inverse-repeat pseudo-random inputs are therefore preferred and the methods described in Section 5 can be applied in both cases.

As an example of a system with a multi-level input, the same first-order system as before is considered, but with an inverse-repeat maximum-length pseudo-random ternary input with levels –1, 0 and +1, and period $N = 242$.

Fig. 5 shows that a close match between the system and model outputs can be obtained by the output matching method. Table 3 shows that the optimal parameter values obtained by the output matching and output transform matching methods are remarkably consistent, and that the cubic component in the system and the model is very small.

Table 3. Optimal parameters for Wiener model of first-order direction-dependent system with inverse-repeat pseudo-random ternary input

Matching method	K_0	K_1	T_1	K_2	T_2	K_3	T_3
OUT	2.40	2.41	6.14	0.67	8.66	0.00	8.31
DFT	2.40	2.43	6.23	0.67	8.66	0.00	9.00

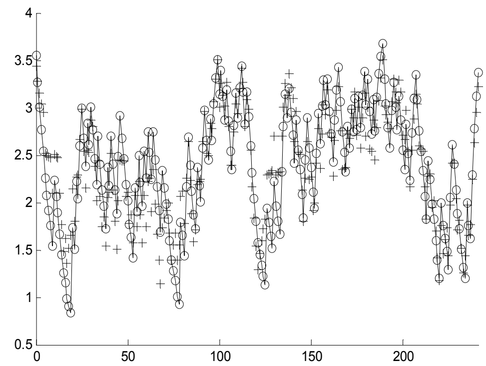

Fig. 5. Outputs of first-order direction-dependent system (o) and Wiener model (+) with inverse-repeat pseudo-random ternary input

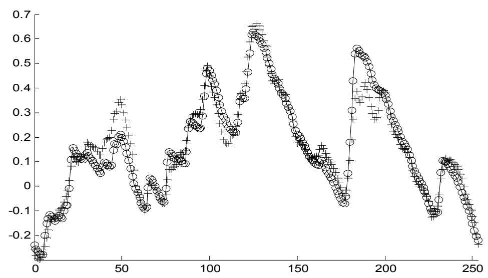

Fig. 6. Outputs of second-order direction-dependent system (equal gains) (o) and Wiener model (+) with inverse-repeat pseudo-random binary input

As an example of a higher-order system, the transfer function of the system dynamics is taken to be $\frac{1}{(1+sT_U)^2}$ with $T_U = 3T$ when the direction of the system output is positive, and $\frac{1}{(1+sT_D)^2}$ with $T_D = 12T$ when it is negative. In the Wiener model, the transfer function in the j-th order path is taken to be $\frac{K_j}{(1+sT_j)^2}$. The choice of critical damping for the linear dynamics in the Wiener model paths reflects the nature of the system dynamics, and reduces the number of parameters to be optimised, but is not necessarily optimal.

Fig. 6 shows that a close match between the system and model outputs can be obtained by the output matching method. Table 4 shows that the optimal parameter values obtained by the output matching and output transform matching methods are reasonably consistent.

Table 4. Optimal parameters for Wiener model of second-order direction-dependent system with inverse-repeat pseudo-random binary input

Matching Method	Model Parameters K_0	K_1	T_1	K_2	T_2	K_3	T_3
OUT	0.46	2.22	6.94	0.94	3.63	0.01	7.03
DFT	0.46	2.16	7.41	0.94	3.63	0.74	2.01

7. CONCLUSIONS

As direction-dependent dynamic systems can be analysed in only simple cases, it is useful to model them in forms that are amenable to analysis, and Wiener models with paths of order 0 to 3 have been shown to be suitable for this purpose. Methods for obtaining optimal values of the model parameters have been developed for those cases that can be analysed, which are first-order direction-dependent systems with binary inputs. The methods involve matching appropriate functions or characteristics of the systems and models with either pseudo-random inputs or inverse-repeat pseudo-random inputs. Cross-correlation function matching methods give excellent results with pseudo-random inputs, but cannot give complete results with inverse-repeat pseudo-random inputs. Output matching and discrete Fourier transform matching methods give excellent results with inverse-repeat pseudo-random inputs, and are to be preferred in this application. These methods have also been used with good results in two cases that cannot be analysed, which are a first-order system with a pseudo-random ternary input and a second-order system with an inverse-repeat binary input.

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