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DYNAMICAL ESTIMATION OF AN INPUT IN NONLINEAR DIFFERENTIAL SYSTEMS [1]

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Abstract: A problem of dynamical estimation of inputs in nonlinear differential systems is considered. Solution dynamical algorithms based on the methods of the theory of positional control are suggested. The algorithms operate in “real time” mode. They are stable with respect to informational noises and computational errors.

Keywords: Estimation, Identification methods, Nonlinear systems.

1. INTRODUCTION

Problems of reconstruction of input disturbances, determining the motion of a dynamical system, through measurements of a part of phase coordinates are embedded in the theory of inverse problems of dynamics of control systems. This theory is under intensive development at the present time. Inputs, as a rule, are the factors which uniquely determine the motion of a system. Any available information on the process may be considered as an output. Usually, such information is given by a signal on a part of current system’s state. One of approaches to solving similar problems based on the methods of the theory of positional control (Krasovskii and Subbotin, 1988) was suggested in (Kryazhimskii and Osipov, 1983) and then developed in (Osipov and Kryazhimskii, 1985, 1995; Maksimov, 1995, 1999, 2000). In the present paper, following researches in this field, for systems described by ordinary differential equations or differential-functional equations, a number of algorithms of dynamical reconstruction of inputs is suggested. These algorithms are dynamical and operate in “real time” mode. They are stable with respect to informational noises and computational errors.

Briefly, the essence of the problems under consideration may be formulated in the following way. There is a dynamical system Σ functioning on a time interval $T = [0, \vartheta]$. Its trajectory

$$x(t) = x(t; x_0, u_r(\cdot)) \in \mathbb{R}^q, \quad t \in T,$$

depends on an unknown time-varying input $u_r(\cdot) \in P(\cdot)$. Here $P(\cdot) \subset L_2(T; \mathbb{R}^N)$ is a set of admissible controls. On the interval T, a uniform net $\Delta = \{\tau_k\}_{k=0}^n$ with a step δ is taken, $\tau_0 = 0$, $\tau_{k+1} = \tau_k + \delta$, $\tau_n = \vartheta$. An output

$$y(t) = Cx(t)$$

is measured at the moments τ_k (C is an $r \times q$-dimensionalß matrix). Sometimes it is assumed (for the sake of simplicity) that the output $y(t)$ is measured at all time moments $t \in T$. Results of inaccurate measurements are vectors $\xi_k \in \mathbb{R}^r$ satisfying the inequalities

$$||\xi_k - y(\tau_k)|| \le h, \quad k \in [0 : n-1],$$

where h is the value of the level of informational noise, symbol $||x||$ denotes the Euclidean norm of vector x. It is required to indicate an algorithm that allows to reconstruct an input $u_*^h(\cdot)$ being an approximation to some input $u_*(\cdot)$ generating the output $y(\cdot)$.

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2. SOLUTION SCHEME

The approach developed in (Kryazhimskii and Osipov, 1983; Osipov and Kryazhimskii, 1985, 1995; Maksimov, 1995, 1999, 2000) is applied in the present paper. Following this approach, the problem of approximate calculation of control $u_*(\cdot)$ is substituted by the problem of control of an auxiliary system M (a model), which is formed according to the feedback principle. The phase trajectory of the model is denoted by $y^h(\cdot)$ and the control action is denoted by $u^h(\cdot)$. The process of control of the model is organized in conformity to the rule identified with some map $u^h(\cdot) = u^h(\cdot;\xi(\cdot),y^h(\cdot))$. The process is realized so that under appropriate conditions of concordance of some parameters the control $u^h(\cdot)$ is "close" to the control $u_*(\cdot)$.

Let us describe the scheme of the work of the algorithm in details. An auxiliary dynamical system M (a model) is introduced. This model functioning on the time interval T has unknown input (control) $u^h(t)$ and output $y^h(t)$. The process of synchronous feedback control of the systems Σ and M is organized on the interval T. This process is decomposed into $(n-1)$ identical steps. At the k-th step carried out during time interval $\delta_k = [\tau_k, \tau_{k+1})$ the following actions are fulfilled. First, at the time moment τ_i the control

$$u^h(t) = u^h(\tau_k, \xi_k, y^h(\tau_k)), \quad t \in [\tau_k, \tau_{k+1})$$

is calculated according to the chosen rule u^h. Then (till the moment τ_{k+1}) the control $u^h = u^h(t)$, $\tau_k \le t < \tau_{k+1}$, is fed onto the input of the system M. The value $y^h(\tau_{k+1})$ is the result of the work of the algorithm at the k-th step. Thus, all complexity of solving the problem under consideration is reduced to the appropriate choice of the model M and the function u^h.

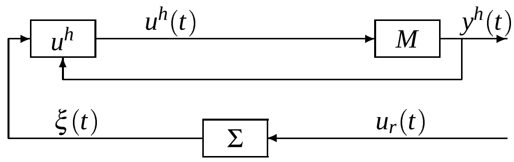
The procedure for solving the problem of reconstruction is, in essence, equivalent to the procedure for solving the following two problems:

a) the problem of choice of the model M and

b) the problem of choice of the rule u^h for forming a control in the model.

Note that the next two aspects play an important role in the process of solving problems a) and b). The first one is a priori information on the structure of the system Σ (the form of equation, the properties of its solution and so on), and the second one is the structure of the set of admissible controls $P(\cdot)$.

The solution flowchart for the problem of reconstruction is shown below.



Here symbol u_r stands for the real control acting upon the system Σ.

3. ORDINARY DIFFERENTIAL EQUATIONS

Consider the following system Σ of equations

$$\begin{cases} \dot{x}_1(t) = A_1 x_{1t} + C_1 x_2(t), \quad t \in T, \\ \dot{x}_2(t) = A_2 x_{2t} + E(x_1(t)) + B(x_1(t))u(t) \end{cases} \tag{1}$$

with the initial state

$$x_1(0) = x_{10}, \quad x_2(0) = x_{20}.$$

Here $x_1(t) \in \mathbb{R}^N$; $x_2(t) \in \mathbb{R}^q$; $u(t) \in \mathbb{R}^m$; A_1, A_2, C_1 are constant matrices of dimensions $N \times N$, $q \times q$ and $N \times q$ respectively; $E(\cdot) : \mathbb{R}^N \to \mathbb{R}^q$ and $B(\cdot) : \mathbb{R}^m \to \mathbb{R}^q$ are matrix functions satisfying the global Lipschitz conditions, i. e.

$$\|E(x) - E(y)\| \le L_E \|x - y\|, \quad x, y \in \mathbb{R}^N,$$
$$\|B(x) - B(y)\| \le L_B \|x - y\|, \quad x, y \in \mathbb{R}^m.$$

Let $P \subset \mathbb{R}^m$ be a convex, closed and bounded set. Fix time moments $\tau_k^n = k\vartheta/n$, $1 \le k \le n-1$.

Let the first component of the state $x_1(\tau_k^n)$ be observed at every time moment τ_k^n. The observation results are represented by vectors $\xi_k \in \mathbb{R}^N$ such that

$$\|x_1(\tau_k^n) - \xi_k\| \le h_n,$$

i. e. in this case $y = x_1$ and matrix C has the form

$$C = \begin{bmatrix} I \\ 0 \end{bmatrix}.$$

Our goal is to reconstruct the real control (input) $u_r(\cdot) = u_r(\cdot; x_1(\cdot))$ compatible with the output $x_1(\cdot)$.

In this case, the model M is described by the system

$$\begin{cases} \dot{y}_1^{h_n}(t) = A_1 y_{1t}^{h_n} + C_1 u^{1,h_n}(t), \quad t \in [\tau_k^n, \tau_{k+1}^n) \\ \dot{y}_2^{h_n}(t) = A_2 y_{2t}^{h_n} + E(\xi_k) + B(\xi_k) u^{h_n}(t) \end{cases}$$

$$y_1^{h_n}(0) = x_{10}, \; y_2^{h_n}(0) = x_{20}, \; k = 0, 1, \ldots, n-1$$

with controls $\{u^{1,h_n}(t), u^{h_n}(t)\} \in \mathbb{R}^q \times \mathbb{R}^m$.

Let the control in the model be defined by the rule

$$u^{1,h_n}(t) = \arg\min\{L_1(\alpha_n, v, s_k^0) : v \in S(d_1)\} \qquad (2)$$

$$u^{h_n}(t) = \arg\min\{L_2(\beta_n, v, s_k) : v \in P\}.$$

Here $t \in [\tau_k^n, \tau_{k+1}^n)$.

$$L_1(\alpha, v, s_k^0) = \alpha_n \|v\|^2 + 2(s_k^0, C_1 v),$$

$$L_2(\beta, v, s_k^*) = \beta_n \|v\|^2 + 2(s_k^*, B(\xi_k)v),$$

$$s_k^0 = (y_1^{h_n}(\tau_k^n) - \xi_k)\exp(-2\omega_1 \tau_{k+1}^n),$$

$$s_k = (y_2^{h_n}(\tau_k^n) - v^{1,h_n}(\tau_k^n))\exp(-2\omega_2 \tau_{k+1}^n),$$

$$d_1 = \sup\{\|x_2(t; x_0, u(\cdot))\| : u(\cdot) \in P(\cdot),\ t \in T\},$$

$$S(d_1) = \{v \in \mathbb{R}^q : \|v\| \le d_1\}, \quad x_0 = \{x_{10}, x_{20}\},$$

$$\omega_j = 1/2 + |A_j|, \quad j = 1, 2,$$

$\{\alpha_n\}_{n=1}^\infty$ and $\{\beta_n\}_{n=1}^\infty$ are sequences of positive numbers, symbol (x, y) denotes the scalar product of vectors x, y and symbol $|A|$ denotes the Euclidean norm of matrix A.

It is assumed that the following relationships between the parameters are valid:

$$\alpha_n \to 0, \quad \beta_n \to 0, \quad \{(h_n + n^{-1/2} + \alpha_n)^{1/2}$$
$$+ (h_n + n^{-1/2})\alpha_n^{-1}\}\beta_n^{-1} \to 0 \quad \text{as} \quad n \to \infty.$$

For example, one can set $h_n = n^{-1/2}$, $\alpha_n = h_n^{1/2}$, $\beta_n = h_n^\mu$, $\mu = \text{const} \in (0, 1/4)$. Let $s_j(\cdot)$ be a unique solution on T of the matrix equation

$$\frac{ds_j(t)}{dt} = A_j s_j(t) \text{ for a. a. } t \in T$$

with the initial state $s_j(t) = I$, $t \le 0$, i. e. $s_j(t)$ is the fundamental matrix of the system $\dot{x} = A_j x$; symbol $V(T; \mathbb{R}^q)$ denotes the space of all functions $t \to x(t) \in \mathbb{R}^q$ with the bounded variation.

Theorem 1. Let a control $v^{h_n}(\cdot)$ be determined by (2). Let $q \le N$ and the following conditions be fulfilled:

(1) there exist a number $d_2 > 0$ and a q-th order minor of matrix $s_1(t)C_1$, such that $q \times q$-matrix $\overline{s_1(t)C_1}$ corresponding to this minor satisfies the inequality

$$\inf_{t \in T} \|\overline{s_1(t)C_1} v\| \ge d_2 \|v\| \quad \forall v \in \mathbb{R}^q,$$

(2) for any solution $x_2(\cdot)$ of system (1) the inclusion $\{\overline{s_1(\vartheta - t)C_1}\}^{-1} x_2(t) \in V(T; \mathbb{R}^q)$ is true.

Then the convergence $u^{h_n}(\cdot) \to u_r(\cdot; x_1(\cdot)))$ in $L_2(T; \mathbb{R}^m)$ takes place.

Let the following conditions be also fulfilled:

(3) there exist a number $d_3 > 0$ and an m-th order minor of matrix $s_2(\vartheta - t)B(x(t))$, such that $m \times m$-matrix $\{\overline{s_2(\vartheta - t)B(x(t))}\}$ corresponding to this minor satisfies the inequality

$$\inf_{t \in T} |\{\overline{s_2(\vartheta - t)B(x(t))}\}| \ge d_3 \|v\| \quad \forall v \in \mathbb{R}^m.$$

If $\{\overline{s_2(\vartheta - t)Bx_1(t)}\}^{-1} u_r(t, x_1(\cdot)) \in V(T; R)$ then the following estimate of convergence rate of the algorithm takes place

$$|u^{h_n}(\cdot) - u_r(\cdot; x_1(\cdot))|_{L_2(T; \mathbf{R}^m)}$$
$$\le c\{\mu_n + \mu_n \beta_n^{-1}\}. \qquad (3)$$

Here

$$\mu_n = (h_n + n^{-1} + \alpha_n)^{1/2} + ((h_n + n^{-1/2})\alpha_n^{-1})^{1/2}.$$

Proof outline. In the beginning, we estimate variation of the value

$$\varepsilon_1(t) = \exp(-2\omega_1 t)\|y_1^{h_n}(t) - x_2(t)\|^2$$
$$+ \alpha_n \int_0^t \{\|u^{1,h_n}(\tau)\|^2 - \|x_2(\tau)\|^2\}\, d\tau.$$

Taking into account (for $t \in [\tau_k^n, \tau_{k+1}^n]$) the equalities

$$y_1^{h_n}(t) = s_1(t - \tau_k^n) y_1^{h_n}(\tau_k^n) + \int_{\tau_k^n}^t s_1(t - \tau) C_1 u^{1,h_n}(\tau)\, d\tau,$$

$$x_2(t) = s_1(t - \tau_k^n) x_2(\tau_k^n) + \int_{\tau_k^n}^t s_1(t - \tau) C_1 x_2(\tau)\, d\tau,$$

one can derive the estimate

$$\varepsilon_1(\tau_{k+1}^n) \le \exp(-2\omega_1 \tau_k^n)\|y_1^{h_n}(\tau_k^n) - x_2(\tau_k^n)\|^2$$
$$+ \lambda_k + \mu_k + \alpha_n \int_{\tau_k^n}^{\tau_{k+1}^n} \{\|u^{1,h_n}(\tau)\|^2 - \|x_2(\tau)\|^2\}\, d\tau,$$

where $s_1(t)$ is the fundamental matrix of the system $\dot{x} = A_1 x$,

$$\lambda_k = 2(s_k^0, \int_{\tau_k^n}^{\tau_{k+1}^n} s_1(\tau_{k+1}^n - \tau)$$
$$\times C_1\{u^{1,h_n}(\tau) - x_2(\tau)\}\, d\tau) + k_1 \vartheta h_n / n,$$

$$\mu_k = (\vartheta / n)\exp(-2\omega_1 \tau_k^n)$$
$$\times \int_{\tau_k^n}^{\tau_{k+1}^n} \|C_1\{u^{1,h_n}(\tau) - x_2(\tau)\}\|^2\, d\tau.$$

Note that $\mu_k \le k_2(1/n)^2$. In addition, in virtue of (2) the following inequalities

$$\lambda_k + \alpha_n \int_{\tau_k^n}^{\tau_{k+1}^n} \{\|u^{1,h_n}(\tau)\|^2 - \|x_2(\tau)\|^2\}\,d\tau$$
$$\le k_3(h_n + 1/n)/n$$

are true. Hence the following estimate takes place:

$$\varepsilon_1(\tau_{k+1}^n) \le \varepsilon_1(\tau_k^n) + d_0(h_n + 1/n)/n. \tag{4}$$

By means of estimate (4) we obtain

$$\|y^{h_n}(\tau_k^n) - x_2(\tau_k^n)\|^2 \tag{5}$$
$$\le k_4\{h_n + \alpha_n + 1/n\},$$

$$\int_0^\vartheta \|u^{1,h_n}(\tau)\|^2\,d\tau \le \int_0^\vartheta \|x_2(\tau)\|^2\,d\tau \tag{6}$$
$$+ k_5\{h_n + 1/n\}/\alpha_n.$$

From (5), using the Cauchy formula for representation of solution of a linear differential equation, we derive

$$\max_{t\in[0,\vartheta]} \|\int_0^t S(\tau)C_1\{u^{1.h_n}(\tau) - x_2(\tau)\}\,d\tau\| \tag{7}$$
$$\le k_6\{h_n + \alpha_n + 1/n\},$$

where $S(\tau) = \overline{s_1(\vartheta - \tau)C_1}$. Then, using (6) we deduce that

$$\int_0^\vartheta \|u^{1,h_n}(\tau) - x_2(\tau)\|^2\,d\tau \le 2\int_0^\vartheta (S(\tau)(x_2(\tau) \tag{8}$$
$$- u^{1,h_n}(\tau)), S^{-1}(\tau)x_2(\tau))\,d\tau + k_7\{h_n + 1/n\}/\alpha_n.$$

Let us apply the following lemma.

Lemma 2. (Osipov and Kryazhimskii, 1995) Let $u(\cdot) \in L_\infty(T;\mathbb{R}^q)$, $v(\cdot) \in V(T;\mathbb{R}^q)$, $\|\int_0^t u(\tau)\,d\tau\| \le \varepsilon$, $\|v(t)\| \le K\ \forall t \in T$. Then

$$\|\int_0^\vartheta (u(\tau), v(\tau))\,d\tau\| \le \varepsilon(K + \mathrm{var}(T;v(\cdot))).$$

Here the symbol $\mathrm{var}(T;v(\cdot))$ denotes the full variation of $v(\cdot)$ in T. From (7), (8) and this lemma we obtain

$$\int_0^\vartheta \|u^{1,h_n}(\tau) - x_2(\tau)\|^2\,d\tau \le \nu(n)$$
$$= d_1\{h_n + 1/n + \alpha_n)^{1/2} + (h_n + 1/n)\alpha_n^{-1}\}.$$

Now, by the standard scheme, we establish the convergence $u^{h_n}(\cdot)$ to $u_r(\cdot;x_1(\cdot))$. Let condition 3 be also fulfilled. Let us estimate the variation of the value

$$\varepsilon_2(t) = \exp(-2\omega_2 t)\|u^{h_n}(t) - u_r(t;x_2(\cdot))\|^2$$
$$+ \beta_n \int_0^t \{\|u^{h_n}(\tau)\|^2 - \|u_r(t;x_2(\cdot))\|^2\}\,d\tau.$$

Analogously to (Krasovskii and Subbotin, 1988; Maksimov, 2000), we deduce

$$\varepsilon_2(\tau_{k+1}^n) \le d_1\mu^{(0)}(n)\beta_n^{-1},$$

where

$$\mu^{(0)}(n) = h_n + 1/n + \nu^{1/2}(n).$$

This implies (3).

4. DIFFERENTIAL EQUATIONS WITH TIME DELAY

Let a system Σ be described by the nonlinear differential equation with time delay

$$\dot{x}(t) = f(x(t), x(t-\tau)) + Bu(t), \tag{9}$$
$$t \in T = [0,\vartheta], \quad x(s) = x_0(s), \quad s \in [-\tau, 0],$$

where $x \in \mathbb{R}^q$, f is a nonlinear Lipschitz transformation from $\mathbb{R}^q \times \mathbb{R}^q$ to $\mathbb{R}^q$, $u \in \mathbb{R}^N$, B is $q \times N$-matrix, $\tau = \mathrm{const} > 0$, $x_0(s) \in C([-\tau,0];R^g)$. Consider the case of measurement of all coordinates, i. e. the case when

$$C = I.$$

Denote by $u_*(\cdot)$ a unique input from $L_2(T;\mathbb{R}^N)$ of minimal norm which produces the same evolution $x(\cdot)$ as $u_r(\cdot)$. This is the input that can be approximated. Fix time moments $\tau_k^n = k\vartheta/n$, $1 \le k \le n-1$.

Let the following condition be fulfilled.

Condition 1. $P(\cdot) = L_2(T;\mathbb{R}^N)$.

In this case, the model M is described by the linear differential equation

$$\dot{y}^{h_n}(t) = f(\xi_k, \xi_{i-k_\tau}) + Bu^{h_n}(t) + u^{1,h_n}(t), \tag{10}$$
$$t \in [\tau_k^n, \tau_{k+1}^n)$$

with the initial condition $y^{h_n}(0) = \xi_0$. For simplicity assume that $k_\tau = \tau/m$. The control in the model is defined by the rule

$$u^{h_n}(t) = \frac{1}{\alpha_n}B'[\xi_k - y^{h_n}(\tau_n^n)], \ t \in [\tau_k^n, \tau_{k+1}^n), \tag{11}$$

$$u^{1,h_n}(t) = c\frac{\vartheta}{n\alpha}B'[\xi_k - y^{h_n}(\tau_k^n)].$$

Let sequences $\{h_n\}$, $\{\alpha_n\}$ be taken with the following properties:

$$h_n \to 0,\ \ \alpha_n \to 0,\ \ (h_n + n^{-1})\alpha_n^{-1} \to 0. \qquad (12)$$

Theorem 3. Let condition (12) be fulfilled. Then the sequence of functions $\{u^{h_n}(\cdot)\}$ defined by (11) converges to $u_*(\cdot)$ in $L_2(T;\mathbb{R}^N)$ as $n \to \infty$.

Proof outline. The functional

$$\mu(t) = \|x(t) - y^{h_n}(t)\|^2$$
$$+\alpha\int_0^t [\,|u^{h_n}(s)|^2 - |u_r(s)|^2]\,ds$$

is associated with the reconstruction process. The values of $\mu(t)$ cannot be computed since they depend on unknown values of $u_r(\cdot)$ and y^{h_n}. The control $u^h(\cdot)$ is chosen in such a way (see (11)) that $\mu(\tau_i)$ satisfies the difference equation of the following form:

$$\mu(\tau_{i+1}) \le \mu(\tau_i) + k_1 h_n \int_{\tau_i}^{\tau_{i+1}} \Big(\|u_r(\tau)\|\,d\tau \qquad (13)$$
$$+\,k_2 1/n \int_{\tau_i}^{\tau_{i+1}} \|u_r(\tau)\|^2 d\tau$$
$$+\,k_3(h_n^2 + h/n + 1/n^2 + \alpha_n^2 h_n + \alpha_n^2/n).$$

Namely, $u^{(1)h}(\cdot)$ is chosen so that

$$u^h_{|_{[\tau_i,\tau_{i+1})}}(\cdot)$$
$$= \arg\min\{\int_{\tau_i}^{\tau_{i+1}} \{2(y^{h_n}(\tau_i) - \xi_k, Bv^{(1)}(s))$$
$$+\,\alpha|v^{(1)}(s)|^2\}\,ds : v^{(1)}(\cdot) \in L_2([\tau_i,\tau_{i+1}];\mathbb{R}^n)\}.$$

Further arguments also follow the standard scheme.

From the results of (Osipov and Kryazhimskii, 1995; Maksimov, 2000), it follows

Theorem 4. Let the function $u_r(\cdot) = u_r(\cdot;x(\cdot))$ be a function of bounded variation. Then the following estimate of the rate of algorithm convergence holds:

$$\|u^h(\cdot) - u_r(\cdot)\|^2_{L^2(0,T)}$$
$$\le c_1\alpha_n^{-1}(h_n + 1/n) + c_2(\alpha_n + 1/n + h_n)^{1/2}.$$

Here c_1 and c_2 are some constants, which may be written in an explicit form.

5. EXAMPLE

In this section we present few numerical simulations. We are not going to present a numerical discussion of the method. Our goal is to illustrate the dependence of algorithm's output on the value h. The following system of the second order ($x_1, x_2 \in \mathbb{R}$) was considered:

$$\begin{cases} \dot{x}_1(t) \ = \ x_1(t) + a\sin(x_2(t-\tau)) + u_1(t) \\ \dot{x}_2(t) \ = \ b\cos(x_1(t-\tau)) + x_2(t) + u_2(t) \end{cases}$$

on time interval $T = [0,2]$. It was assumed that the initial state had the form $x_1(t) = 1 + t$, $x_2(t) = -2\cos(t)$, for $t \in [-\tau, 0]$. The input was computed by the following formulas $u_1(t) = t^2$, $u_2(t) = 5\sin(2t)$. In the numerical experiment, we assume for definiteness that $\xi_{i1} = x_1(\tau_i) + h\sin(M\tau_i)$, $\xi_{i2} = x_2(\tau_i) + h\cos(M_1\tau_i)$. The following system was taken as the model:

$$\begin{cases} \dot{y}_1^h(t) \ = \ \xi_{i1} + a\sin(\xi_{i-j2}) + v^{11}(\tau_i) + v^{21}(\tau_i) \\ \dot{y}_2(t) \ = \ b\cos(\xi_{i-j1}) + \xi_{i2} + v^{12}(\tau_i) + v^{22}(\tau_i), \end{cases}$$

$$t \in \delta_i = [\tau_i, \tau_{i+1})$$

with initial state $w_1(0) = 1 + h$, $w_2(0) = -2 - h$. Here $j = [\tau/\delta]$ denotes the integral part of the number $[\tau/\delta]$. Controls v^h at moments τ_i were calculated as follows (see (11))

$$v^{11}(\tau_i) \ = \ -(y_1^h(\tau_i) - \xi_{i1})/\alpha,$$
$$v^{12}(\tau_i) \ = \ -(y_2^h(\tau_i) - \xi_{i2})/\alpha,$$
$$v^{21}(\tau_i) \ = \ C\delta(y_1^h(\tau_i) - \xi_{i1})/\alpha,$$
$$v^{22}(\tau_i) \ = \ C\delta(y_2^h(\tau_i) - \xi_{i2})/\alpha.$$

In figures 1–3, the results of calculations are presented for the case when $a = 5$, $b = 3$, $\tau = 1$, $\alpha = 0.01$, $C = 1$, $M = 10$, $M_1 = 50$. Fig. 1 corresponds to the case when $h = 0.001$, $\delta = 0.001$, Fig. 2— $h = 0.001$, $\delta = 0.005$, Fig. 3— $h = 0.1$, $\delta = 0.005$. In figures 1–3, solid (dash) lines represent control $u(t)$ (model controls $v^h(t)$). As it is seen from figures, the larger is the value of system's phase trajectory measurement error h, the "worse" is algorithm's output.

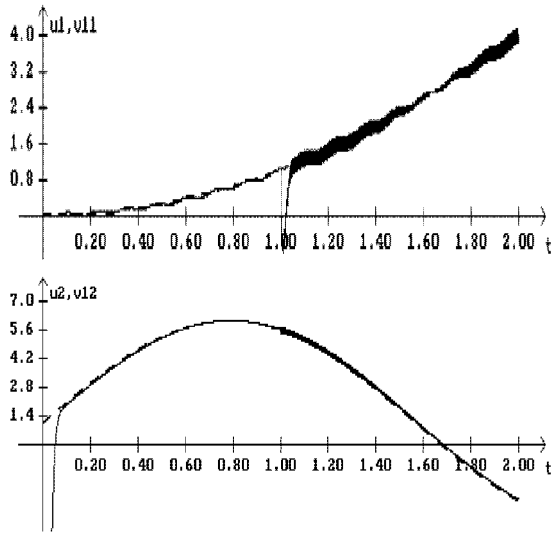


Fig. 1. $h = 0.001$, $\delta = 0.001$.

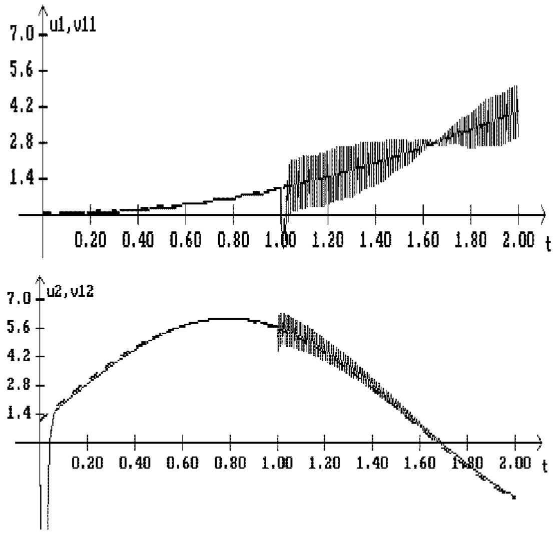


Fig. 3. $h = 0.001$, $\delta = 0.005$.

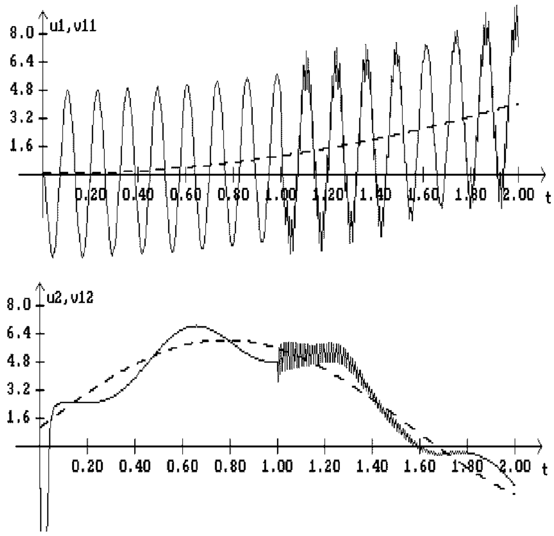


Fig. 5. $h = 0.1$, $\delta = 0.005$.

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