15th Triennial World Congress, Barcelona, Spain

Observer-based Stability Controller Design for Linear Systems with Delayed State[1]

Feng Jun'e   Cheng Zhaolin

Department of Mathematics Shandong University, China, 250100
E-mail: thefengs@163.com

Abstract: An observer-based stability controller is established for linear systems with delayed state. The controller not only realizes the separation of characteristic roots for the closed-loop system, but also makes the closed-loop transfer function equal to the transfer function of closed-loop constructed directly by state feedback. Using the separation of characteristic roots, the stability condition is presented. The controller can be obtained by solving two linear matrix inequalities(LMIs). Two illustrative examples are given to show the applicability of the proposed approaches. *Copyright ©2002 IFAC*

Keywords: observer, closed-loop, controller, time-delay systems, characteristic roots

1.INTRODUCTION

Time delay is commonly encountered in various engineering systems. The theory for non-time-delay systems has been developed(Doyle, et al., 1984; Isidori, et al., 1989). The research for time-delay system has received a considerable amount of attention in recent years. There are many publications on solving the stabilization problem of system with: 1) constant delay (Joon, et al., 1994; Mahmoud, et al., 1999); 2) time-varying delay (Jeung, et al., 1998; Ikeda and Ashida, 1979); 3) constant delay and parameter uncertainty (Choi, et al., 1996; Mahmoud and Al-Muthairi, 1994; Shen, et al., 1991) and 4) time-varying delay and parameter uncertainty(Choi, et al., 1995; Jeung, et al., 1996; Kim, et al., 1996). The stability of the closed-loop system in literature(Joon, et al., 1994; Mahmoud, et al., 1999; Shen, et al., 1991) is independent of time delay, but one in literature (Jeung, et al., 1998; Choi, et al., 1995; Jeung, et al., 1996) is dependent on only the maximum value of the time derivative of time-varying delay. Literature(Joon, et al., 1994; Mahmoud, et al., 1999) gave state feedback controller for state delayed systems. However, it is not easy to measure the state, so it is difficult to realize state feedback. Thus, output feedback for state delayed systems have attracted out great attention. Some authors proposed dynamic output feedback controller(Mahmoud, et al., 1999; Jian-Hua, et al., 1996; Choi, et al., 1997; Su, et al., 1999; Choi, et al., 1996). However, it is needed to regulate some parameters to obtain the controller. It is not convenient in practical situation.

In this paper, using LMI Method, an observer-based controller is given for linear system with constant delay in state. This controller realizes the separation of characteristic roots, and makes the closed-loop transfer function equal to the transfer function of closed-loop constructed directly by state feedback. Moreover, using the separation theorem, a stability condition is presented. The design can be directly obtained by solving two LMIs. It isn't needed to regulate any parameter to obtain the controller. Additional, two small examples are given to illustrate the validity of the proposed design method.

[1] This work was supported by the project 973 of China, the University Doctaral Fundation of the Education Ministry of China, and the Natural Science Fundation of Shandong Province under great G1980020300, 98042212, and Q99A07 respectively.

2. MAIN RESULT

Consider the following system

$$\begin{cases} \dot{x}(t) = Ax(t) + A_h x(t-h) + Bu(t) \\ y(t) = Cx(t) \\ x(t) = 0, \quad t \in [-h, 0] \end{cases} \tag{1}$$

where, $x \in R^n$ is the state, $u \in R^m$ is the control, $y \in R^r$ is the measured output, and A, A_h, B and C are constant matrices with appropriate dimensions. $h \geq 0$ is the delay constant.

In this paper, (A, B, C) is stabilizable and detectable. Additional, the following lemma is necessary to the main result.

Lemma 1. (Joon, et al., 1994) If there exist positive-definite matrices $P > 0$, $Q > 0$ satisfying the following inequality:

$$PA + A^T P + PA_h Q^{-1} A_h^T P + Q < 0$$

the system $\dot{x}(t) = Ax(t) + A_h x(t-h)$ is zero-solution asymptotically stable.

Using the above lemma, an observer can be obtained for system (1).

Lemma 2. If there exist matrix L and positive-definite matrices $P_1 > 0$, $Q_1 > 0$ satisfying the following inequality:

$$\begin{aligned} M_1 \triangleq\ & (A - LC)^T P_1 + P_1(A - LC) + Q_1 \\ & + P_1 A_h Q_1^{-1} A_h^T P_1 \\ & < 0 \end{aligned} \tag{2}$$

then the following system

$$\begin{cases} \dot{\xi}(t) = A\xi(t) + A_h \xi(t-h) + Bu(t) \\ \qquad + L(y(t) - C\xi(t)) \\ \xi(t) = 0, \quad t \in [-h, 0] \end{cases} \tag{3}$$

is an observer of system (1).

Proof. Let $e(t) = x(t) - \xi(t)$, then

$$\dot{e}(t) = \dot{x}(t) - \dot{\xi}(t) = (A - LC)e(t) + A_h e(t-h)$$

From the above lemma 1 and the condition (2), the system $\dot{e}(t) = (A - LC)e(t) + A_h e(t-h)$ is zero-solution asymptotically stable.

In inequality (2), if let $\bar{L} = P_1 L$, get another description for condition (2):

Remark 1. There exist matrix L and positive-definite matrices $P_1 > 0$, $Q_1 > 0$ satisfying (2) if and only if there exist matrix $\bar{L}$ and positive-definite matrices $P_1 > 0$, $Q_1 > 0$ satisfying the following inequality

$$\begin{aligned} M_2 \triangleq\ & A^T P_1 + P_1 A - C^T \bar{L}^T - \bar{L}C + Q_1 \\ & + P_1 A_h Q_1^{-1} A_h^T P_1 \\ & < 0 \end{aligned} \tag{4}$$

From Schur complement, inequality (4) holds iff the following LMI holds.

$$\begin{bmatrix} \tilde{A} & P_1 A_h \\ A_h^T P_1 & -Q_1 \end{bmatrix} < 0 \tag{5}$$

where, $\tilde{A} = A^T P_1 + P_1 A - C^T \bar{L}^T - \bar{L}C + Q_1$

Example 1. Consider the state delayed linear (1) with the following date,

$$A = \begin{bmatrix} 0.2 & 0.1 \\ -0.1 & -0.2 \end{bmatrix}, \quad C = \begin{bmatrix} 0.2 & 0.1 \end{bmatrix}$$

$$A_h = \begin{bmatrix} 0.1 & 0.1 \\ 0.2 & 0.1 \end{bmatrix}, \quad B = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}$$

Using LMI toolbox in Matlab, by solving (5), get

$$P_1 = \begin{bmatrix} 40.5340 & -17.5276 \\ -17.5276 & 66.2187 \end{bmatrix}$$

$$Q_1 = \begin{bmatrix} 32.3359 & 10.6799 \\ 10.6799 & 15.8655 \end{bmatrix}, \quad \bar{L} = \begin{bmatrix} 198.3315 \\ 5.8640 \end{bmatrix}$$

Hence, obtain the observer (3) of system (1), here, the output gain matrix is

$$L = \begin{bmatrix} 5.5686 \\ 1.5625 \end{bmatrix}.$$

Suppose there exists system (3) satisfying the condition of lemma 2, then design the following dynamic output feedback controller.

$$\begin{cases} \dot{\xi}(t) = A\xi(t) + A_h \xi(t-h) + Bu(t) \\ \qquad + L(y(t) - C\xi(t)) \\ u(t) = F\xi(t) + v(t) \\ \xi(t) = 0, \quad t \in [-h, 0] \end{cases} \tag{6}$$

here, matrix L is the output gain, F is the state gain. Introducing the variables

$$e(t) = x(t) - \xi(t), \quad x_c^T(t) = (x^T(t), e^T(t)) \tag{7}$$

then the closed-loop system corresponding to (1), (6), (7) is given by state model

$$\begin{cases} \dot{x}_c(t) = A_c x_c(t) + A_{ch} x_c(t-h) + B_c v(t) \\ y(t) = C_c x_c(t) \end{cases} \tag{8}$$

where,

$$A_c = \begin{bmatrix} A + BF & -BF \\ 0 & A - LC \end{bmatrix}, \quad B_c = \begin{bmatrix} B \\ 0 \end{bmatrix},$$

$$A_{ch} = \begin{bmatrix} A_h & 0 \\ 0 & A_h \end{bmatrix}, \quad C_c = \begin{bmatrix} C & 0 \end{bmatrix}$$

Now, give the main result of this paper.

Theorem 1. The closed-loop system (8) realizes the separation of characteristic roots and the transfer function of closed-loop (8) is equal to the transfer function of closed-loop constructed directly by state feedback.

Proof. The characteristic roots set of closed-loop is

$$\{s|det(sI - A_c - A_{ch}e^{-sh}) = 0\}$$

Substituting A_c, A_{ch} into the above equality,

$$\begin{aligned} &\{s|det(sI - A_c - A_{ch}e^{-sh}) = 0\} \\ =&\{s|det(sI - (A + BF) - A_h e^{-sh}) = 0\} \\ &\cup\{s|det(sI - (A - LC) - A_h e^{-sh}) = 0\} \end{aligned}$$

Thus, the separation of closed-loop characteristic roofs is realized.

The transfer function of closed-loop (8) is

$$\begin{aligned} G_c(s) &= C_c(sI - A_c - A_{ch}e^{-sh})^{-1}B_c \\ &= (C\ 0)\begin{bmatrix} s(A_F) & BF \\ 0 & s(A_L) \end{bmatrix}^{-1}\begin{bmatrix} B \\ 0 \end{bmatrix} \\ &= C(s(A_F))^{-1}B \end{aligned}$$

where,

$$s(A_F) = sI - (A + BF) - A_h e^{-sh},$$

$$s(A_L) = sI - (A - LC) - A_h e^{-sh}.$$

It is obvious that $G_c(s)$ is equal to the transfer function of closed-loop corresponding to state feedback $u(t) = Fx(t) + v(t)$.

It is well known, when $v(t) = 0$, closed-loop (8) is zero-solution asymptotically stable if and only if all the characteristic roots have negative real part. Therefore, from above theorem, when $v(t) = 0$, the closed-loop (6) is zero-solution asymptotically stable if and only if the two following systems;

$$\dot{z}_1(t) = (A - LC)z_1(t) + A_h z_1(t - h),$$

$$\dot{z}_2(t) = (A + BF)z_2(t) + A_h z_2(t - h)$$

are zero-solution asymptotically stable. Hence, using the Lemma 1, get a stability condition for system(1).

Theorem 2. If there exist matrices F, L and positive matrices $P_1 > 0$, $P_2 > 0$, $Q_1 > 0$, $Q_2 > 0$ satisfying the following inequality

$$\begin{aligned} M_3 \triangleq\ & (A + BF)^T P_2 + P_2(A + BF) \\ & + P_2 A_h Q_2^{-1} A_h^T P_2 + Q_2 \\ & < 0 \end{aligned} \tag{9}$$

and inequality (2), then when input $v(t) = 0$, the closed-loop system (8) is zero-solution asymptotically stable.

If premultiply (9) by P_2^{-1}, postmultiply the result by P_2^{-1} and let $\bar{P}_2 = P_2^{-1}$, $\bar{Q}_2 = Q_2^{-1}$, $\bar{F} = F\bar{P}_2$, get another description for condition (9).

Remark 2. There exist matrix F and positive-definite matrices $P_2 > 0$, $Q_2 > 0$ satisfying inequality (9) iff there exist matrix $\bar{F}$, and positive-definite $\bar{P}_2 > 0$, $\bar{Q}_2 > 0$ satisfying the following inequality:

$$\begin{aligned} M_4 \triangleq\ & A\bar{P}_2 + \bar{P}_2 A^T + A_h \bar{Q}_2 A_h^T + \bar{P}_2 \bar{Q}_2^{-1} \bar{P}_2 \\ & + B\bar{F} + \bar{F}^T B^T \\ & < 0 \end{aligned} \tag{10}$$

or satisfying the following LMI

$$\begin{bmatrix} \bar{A} & \bar{P}_2 \\ \bar{P}_2 & -\bar{Q}_2 \end{bmatrix} < 0 \tag{11}$$

where, $\bar{A} = A\bar{P}_2 + \bar{P}_2 A^T + B\bar{F} + \bar{F}^T B^T + A_h \bar{Q}_2 A_h^T$.

Thus, design stability controller (6), it is only needed to solve the two LMIs (5) and (11) to get the output gain matrix L and state gain matrix F.

Remark 3. When $A_h = 0$ or $h = 0$, the above results are consistent with non-time-delay system.

Remark 4. One important prerequisite condition in this paper is that the delay is known. Otherwise, these results don't hold.

Example 2. Consider the state delayed linear system (1) with the following data,

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix},\ C = [0\ \ 1],$$

$$A_h = \begin{bmatrix} 0 & 0 \\ 0.2 & 0.1 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Using LMI toolbox, by solving (5) and (11), get

$$P_1 = \begin{bmatrix} 40.7117 & 22.7757 \\ 22.7757 & 37.0106 \end{bmatrix}$$

$$Q_1 = \begin{bmatrix} 28.7544 & -0.0000 \\ -0.0000 & 40.7117 \end{bmatrix}$$

$$\bar{P}_2 = \begin{bmatrix} 0.4743 & -0.2846 \\ -0.2846 & 0.4743 \end{bmatrix}$$

$$\bar{Q}_2 = \begin{bmatrix} 1.4228 & -0.0000 \\ -0.0000 & 1.4228 \end{bmatrix}$$

$$\bar{L} = \begin{bmatrix} -41.8504 \\ -10.5338 \end{bmatrix},\quad \bar{F} = \begin{bmatrix} -0.5691 & -0.0830 \end{bmatrix}$$

Thus, get the state gain matrix and the observer gain matrix.

$$F = \begin{bmatrix} -2.0391 & -1.3984 \end{bmatrix},\quad L = \begin{bmatrix} -1.3249 \\ 0.5307 \end{bmatrix}.$$

Hence,

$$\begin{cases} \dot{\xi}(t) = A\xi(t) + A_h\xi(t - h) + Bu(t) \\ \qquad + L(y(t) - C\xi(t)) \\ u(t) = F\xi(t) \\ \xi(t) = 0,\quad t \in [-h, 0] \end{cases}$$

is an observer-based stability controller for system (1) with the above data.

3.CONCLUSION

Using LMI method, this paper introduced an observer with delayed state. The dynamic output controller given in this paper not only realizes the separation of characteristic roots and makes the closed-loop transfer function equal to the transfer function of closed-loop corresponding to state feedback. Moreover, using the characteristic roots separation theorem, an output feedback controller design has been obtained that stabilizes the state delayed linear system. The controller can be given by solving two LMIs. Finally, two small examples are given to illustrate the validity of the design method.

REFERENCE

Doyle, J.C., Glover, K., Khargonekar, P.P., and Francis, B.A., (1984). State-space solution to standard H_2 and H_∞ control problem. *IEEE Trans. Auto. Contr.*, 34, 881-897.

Isidori, H., Fuksa, S., Grabowski, P., and Korytowski, A., (1989). Analysis and Synthesis of Time Delay Systems *New York, U.S.A.: Wiley.*

Joon Hwa Lee, Sang Woo Kim, and Wook Hyun Kwon, (1994). Memoryless H_∞ controllers for state delayed systems. *IEEE. Trans. Automat. Contr.*, 39, 159-162

M.S. Mahmoud and M.Zribi, (1999). H_∞ controllers for time-delay systems using linear matrix inequalities. *Journal of Optimization Theory and Applications* 100, 89-122.

Eun Tae Jeung, Jong Hae Kim, and Hong Bae Park, (1998). H_∞-output feedback controller design for linear systems with time-varying delayed state. *IEEE. Transactions on Automatic Control*, 43, 971-974.

M.Ikeda and T.Ashida, (1979). Stabilization of linear systems with time-varying delay, *IEEE Trans. Automat. Contr.*, vol. 24, pp. 369-370, Feb.

Choi, H. H. and M. J. Chung (1997). Robust observer-based H_∞ controller design for linear uncertain time-delay systems. *Automatica*, 33, 1749-1752.

M.S.Mahmoud and N.F.Al-Muthairi, (1994). Design of robust controllers for time-delay systems, *IEEE Trans. Automat. Contr.*, vol. 39, pp. 995-999, May.

J.C.Shen, B.S.Chen, and F.C.Kung, (1991). Memoryless stabilization of uncertain dynamic delay systems: Riccati equation approach, *IEEE Trans. Automat. Contr.*, vol. 36, pp. 638-640, May.

H.H.Choi and M.J.Chung, (1995). Memoryless stabilization of uncertain dynamic systems with time-varying delayed states and controls, *Automatica* vol. 31, no. 9, pp. 1349-1351.

E.T.Jeung, D.C.Oh, J.H.Kim, and H.B.Park, (1996). Robust controller design for uncertain systems with time delays: LMI approach, *Automatica* vol. 32, no. 8, pp. 1229-1231.

J.H.Kim, E.T.Jeung, and H.B.Park, (1996). Robust control for parameter uncertain delay systems in state and control input, *Automatica* vol. 32, no. 9, pp. 1337-1339.

Jian-Hua, Ge. P.M. Frank and Ching-Fang Lin (1996). H_∞ control via output feedback for state delay systems. *Int. J. Control*. 64, 1-7.

Su, H.Y, and Chu, J., (1999). Robust H_∞ control for Linear time-varying uncertain time-delay systems via dynamic output feedback. *International Journal of Systems Science*, 30, 1093-1107.

Choi, H. H. and M. J. Chung (1996). Observer-based H_∞ control design for state delayed linear systems. *Automatica*, 32, 1073-1075.