DESIGN OF ROBUST FAULT DETECTION AND ISOLATION OBSERVERS FOR SINGULAR SYSTEMS 1

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Abstract: This paper considers Robust Fault Detection and Isolation Observer (RDO) design for singular system with faults. Using observer-based approaches to the RDO problem, three types of faults are considered in a unified manner. Necessary and sufficient conditions for the existence of RDO and a design algorithm are given.

Keywords: fault detection, fault diagnosis, observers, singular systems, robustness

1. INTRODUCTION

In modern technical processes, such as space and air vehicles, c hemicalrectors, n uclearpower stations, as well as industrial and service robots, along with the system performance, the safety of the system equipment and human beings in the environment is necessary. The faults of equipments and systems not only effect the safe circulation, but also can lead to hurt of human being and pollution of the environment. With the increasing complexity of the control system and paying great attention to protect the environment, this leads to increasing demands on safety, reliability and of the equipment So the fault diagnosis technology has become an important field of research in control engineering. The fault diagnosis technology using analytical redundancy was first developed from American at 70s of 20c. In the early 80s, we also started at this research. In the past two decades, the fault detection and diagnosis technology has been researched deep and widespread, many realizable approaches have been developed. This technology has been applied in automatic pilot of plane, man-made earth satellite, inertia pilot,

nuclear rectors etc. territory. A large of compre-

The main ideas of observer-based fault detection is comparing the real measurement to the nominal of the mathematical model. But the existence of the unavoidable modelling errors, unknown disturbances and other uncertainties etc. non-fault factors can effect the model-based fault detec-

hensive surveys (Willsky, 1976) (Isermann, 1984) (Frank, 1990) (Zhou, et al., 1991, 1995, 1998, 2000) described the fault diagnosis problem from different viewpoint. The works in this territory have (Cui, 1996) (Wen, et al., 1998). Model-based Fault Detection and Isolation (FDI) for dynamic systems has received considerable attention in the past three decades. Among various modelbased approaches to the FDI problem, observerbased FDI methods show considerable promise for real applications in technical processes. The earliest FDI observer, called Failure Detection Filter (FDF), was first developed by (Beard, 1971). In the following three decades, this pioneering w ork w as then reformed, extended and supplied by many research using different approaches from the more general and more practical viewpoint. A more complete result and a new design method of the FDI problem was given by (Massoumnia, 1986) using a geometric approach. (White and Speyer, 1987) applied the spectral theory to the FDI problem.

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tion system strongly, even lead to give false fault alarms or some faults can not be detected. So it is important to increase the robustness of the fault detection system during the design procedure. (Ge and Fang, 1988, 1989) given a design approach of FDI observers for systems, using the robust observable concepts. (Yuan, et al., 1997) improved the above results, and given a complete algorithm to design a minimal Robust FDI Observer (RDO). Improve the real-time property of the FDI system (Hou and Müller, 1994) given a sufficient and necessary condition of the existence of RDO and a systematic design method using some advanced disturbance-decoupled observer design technique. (Patton and Hou, 1998) designed a full order observer for a observable subsystem, based on an explicit decomposition of an extended matrix pencil. Three types of faults, i.e. component, actuator, and sensor faults are considered in a unified manner. (Nikouhah, 1994) changed the failure detection problem to an innovations filter design problem, given some new results in frequency domain, and applied statistical tests to decide whether a failure has occurred. (Shen and Hsu, 1999) decoupled the faults by using the eigenstructure assignment. Moreover, the optimal gain matrix of the present diagnostic observer is obtained to improve its robustness without degrading diagnostic performance.

The above works are all completed for the linear systems, under the normal state space. For other systems (such as, bilinear systems, retarded systems, nonlinear systems), one can see (Kinnaert, 1999), (Yang and Saif 1998), (Hammouri *et al.*, 1999) and (Zhang *et al.*, 1998).

In this paper, the fault detection problem for linear singular system is discussed. In many articles, singular systems are called descriptor variable systems, generalized state space systems, semistate systems, differential-algebraic systems etc. Singular systems are more general dynamic systems than normal state systems, appear in many systems, such as engineering system (for example, power system, electrical networks, aerospace engineering, chemical processes), social economic systems, network analysis, biological systems, and so on. The works about singular system, we refer to the books of (Dai, 1989), (Aplevich, 1991).

Compare singular systems to the normal state space systems, the obvious characteristic is the multi-level of the system structure, that is their state and input are not only bound by a differential equation, but also bound by a algebraic equation. The main contributes are developing the RDO design approaches in (Yuan et al., 1997) and using it in singular system. Three types of faults, i.e. component, actuator, and sensor faults are considered in a unified manner. Given a design

algorithm of a minimal order RDO. In this paper, two test criterion are given about the existence of RDO associated with every splitting of the fault index sets. This makes the whole detection procedure simple and avoids some ineffective operation.

This paper is organized as follows. Section 2 gives the RDO design problem. Section 3 gives a design algorithm and the sufficient and necessary conditions of the existence of RDO for a linear singular system. Section 4 is a brief conclusion.

2. PROBLEM FORMULATION

Consider the following linear time-invariant singular system with faults

$$\begin{cases}
E\dot{x} = Ax + B_1u + D_1d + \sum_{i=1}^{k} L_{1i}m_i, \\
y = Cx + B_2u + D_2d + \sum_{i=1}^{k} L_{2i}m_i,
\end{cases}$$
(1)

where x(t) is the n-dimensional state vector, u(t)is the r-dimensional input vector, y(t) is the mdimensional output vector, with u(t) and y(t)measured by sensors. $E \in \mathbb{R}^{n \times n}$ is singular, $rankE = p < n. A, C, B_1, B_2, D_1, D_2, L_{1i}, L_{2i},$ i = 1, 2, cdots, k, are known constant matrices of appropriate dimensions. $\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} \cdot d(t)$ denotes the effects of the non-fault factors on the system, such as modelling errors, external disturbances etc. uncertainties, $d(t) \in R^{r_d}$ is the unknown input vector. $\begin{bmatrix} L_{1i} \\ L_{2i} \end{bmatrix} \cdot m_i(t), i = 1, 2, \dots, k$, denotes the ith failure modes, where L_{1i}, L_{2i} is the signature matrix of the ith fault, $m_i(t) \in R^{r_{k_i}}, i =$ $1, 2, \dots, k$, is the mode of the ith fault, stand for any possible faults, such as component faults, actuator faults, sensor faults. When failure i does not occur, $m_i(t) = 0$, and when failure i occurs, $m_i(t) \neq 0$. In this paper, suppose the elements of $m_i(t), i = 1, 2, \cdots, k$ are linearly independent, i.e. there does not exist non-zero constant vector of appropriate dimension α , such that

$$lpha^{ au} \left[egin{array}{c} m_1(t) \ m_2(t) \ dots \ m_k(t) \end{array}
ight] \equiv 0.$$

For simplicity, use $(f)_i$ denotes the ith element of vector f, use $(X)_i$ denotes the ith column of matrix X, use $\mathcal{R}(X)$ denotes the linear space spanned by the columns of matrix X, use $\mathcal{N}(X)$ denotes the left null space of matrix X, use $[\cdot|*]$ denotes the observability matrix of the matrix pair $(\cdot, *)$, use $(\cdot, *)$ denotes the controllability matrix of the matrix pair $(\cdot, *)$, use $(\cdot, *)$ denotes

the Moore - Penrose inverse of matrix X.

Definition 1.(Dai, 1989) Regular singular system (E, A, B) is called completely controllable (i.e. C-controllable), if for any $t_1 > 0, x(0) \in \mathbb{R}^n$, and $w \in \mathbb{R}^n$, there exists a control input u, u is smooth enough, such that $x(t_1) = w$.

Lemma 1.(Dai, 1989) Regular singular system (E, A, B) is C-controllable, if and only if:

$$rank[sE - A \quad B] = n, \quad \forall s \in C$$
 (2.1)

$$rank[E \quad B] = n \tag{2.2}$$

For simplicity, given the following assumptions:

Assumption 1. The system (1) is regular (i.e. $det(sE - A) \not\equiv 0$), and C-controllable.

Assumption 2. The output matrix C of system (1) is row-wise rank full.

Definition 2. A k-dimensional logic vector f is called an eigenvector which is associated with splitting \mathbf{w} , if its element $(f)_i$ satisfies:

$$(f)_i = \begin{cases} 1, \ i \in \mathbf{w} \\ 0, \ i \in \mathbf{k} - \mathbf{w} \end{cases} \tag{3}$$

where $\mathbf{k} = \{1, 2, \dots, k\}, \mathbf{w}, \mathbf{k} - \mathbf{w}$ are the index sets, $\mathbf{w} \subset \mathbf{k}$.

Definition 3. Suppose the k-dimensional logic vector f is an eigenvector which is associated with splitting \mathbf{w} . A linear system in the form:

$$\begin{cases} \dot{\omega} = H_1 \omega + H_2 y + (TB_1 - H_2 B_2) u \\ r = H_4 \omega + H_5 y - H_5 B_2 u \end{cases}$$
(4)

where (H_4, H_1) is observable, T satisfies:

$$TD_1 = 0 (5.1)$$

$$TL_{1i} = 0, \quad i \in \mathbf{k} - \mathbf{w}$$
 (5.2)

 $B_1, B_2, D_1, L_{1i}, i \in \mathbf{k} - \mathbf{w}$ which are described by system (1). is called a Robust Fault Detection and Isolation Observer (RDO) of system (1) with f, if $m_i(t) = 0, \forall i \in \mathbf{w}$ if and only if $r(t) \to 0, \omega(t) - TEx(t) \to 0, t \to \infty$. r(t) is called residual vector.

For simplicity, in this paper denote system (4) as (H_1, H_2, T, H_4, H_5) .

Definition 4. Suppose the k-dimensional logic vector f is an eigenvector which is associated with splitting \mathbf{w} . f is called a realizable eigenvector, if the RDO of system (1) with f exists.

Split the faults $m_i(t), i = 1, 2, \dots, k$ which act upon system (1) into two parts, one part contains the faults in the index set \mathbf{w} , which belong to be detected faults, one part contains the faults in the index set $\mathbf{k} - \mathbf{w}$, which belong to be isolated faults. By this, rewrite the system (1) as:

$$\begin{cases}
E\dot{x} = Ax + B_1 u + D_1 d + L_1 m + \bar{L}_1 \bar{m}, \\
y = Cx + B_2 u + D_2 d + L_2 m + \bar{L}_2 \bar{m},
\end{cases} (6)$$

where m(t) contains all the faults in the index set **w**, is specified to be detected, m(t) form as:

$$m(t) = \begin{bmatrix} m_{i_1}(t) \\ m_{i_2}(t) \\ \vdots \\ m_{i_s}(t) \end{bmatrix}, \quad \mathbf{w} = \{i_1, i_2, \cdots, i_s\}$$

 $L_1 \in \mathbb{R}^{n \times l}, L_2 \in \mathbb{R}^{m \times l}$ are the associated coefficient matrices; $\bar{m}(t)$ contains all the faults in the index set $\mathbf{k} - \mathbf{w}$, is specified to be isolated, $\bar{m}(t)$ form as:

$$ar{m}(t) = egin{bmatrix} m_{j_1}(t) \ m_{j_2}(t) \ dots \ m_{j_g}(t) \end{bmatrix}, \qquad \mathbf{k} - \mathbf{w} = \{j_1, j_2, \cdots, j_g\}$$

 $\bar{L}_1 \in R^{n \times \bar{l}}, \bar{L}_2 \in R^{m \times \bar{l}}$ are the associated coefficient matrices.

3. DESIGN OF ROBUST FAULT DETECTION AND ISOLATION OBSERVERS

Definition 5. Suppose the system contains faults and the external disturbances, then

- (a) Direct redundancy implies the existence of the relationship represented by algebraic equations among the elements of inputs and outputs, the relationship may be depend on the detected faults, but not depend on states, disturbances and the isolated faults.
- (b) Temporal redundancy implies the existence of the relationship represented by differential equations among the elements of inputs and outputs, the relationship may be depend on the detected faults, but not depend on states, disturbances and the isolated faults.

It is well known, the existence of analytical redundancy (contains direct redundancy and temporal redundancy) is necessary for the solution of all RDO problems based upon model-based approaches. So, we first consider the sufficient and necessary conditions about the existence of analytical redundancy of the system (1).

Definition 6. A matrix pencil -sE + A is said to be row (column) unimodular, if the pencil has

full row (column) rank for all finite $s \in C$.

Lemma 3.1 (Gantmacher, 1959) Using nonsingular constant matrices P and Q a singular matrix pencil -sE + A can be brought into the following Kronecker canonical form:

$$P(-sE+A)Q = blockdiag(-sI+J_f, -sJ_{\infty}+I, -sE_r + A_r, -sE_c + A_c, 0_{h \times k})$$
(7)

where $J_f \in R^{n_f \times n_f}$ is in Jordan block form, J_f contains all finite eigenvalues of -sE+A, form as

$$J_f = \begin{bmatrix} J_1 & & & & \\ & J_2 & & & \\ & & \ddots & & \\ & & & J_{\sigma} \end{bmatrix}, J_i = \begin{bmatrix} J_{i1} & & & & \\ & J_{i2} & & & \\ & & \ddots & & \\ & & & J_{id_i} \end{bmatrix},$$

$$J_{ik} = \begin{bmatrix} \lambda_i & 1 & & & \\ & \lambda_i & \ddots & & \\ & & \ddots & 1 & \\ & & & \lambda_i \end{bmatrix}, \qquad k = 1, 2, \cdots, d_i.$$

where J_i is the Jordan block associated with the eigenvalue λ_i , $i = 1, 2, \dots, \sigma$. $J_{\infty} \in \mathbb{R}^{n_{\infty} \times n_{\infty}}$ is a nilpotent matrix, form as:

$$J_{\infty} = \begin{bmatrix} J_{1\infty} & & & \\ & J_{2\infty} & & \\ & & \ddots & \\ & & & J_{\varepsilon\infty} \end{bmatrix}, J_{i\infty} = \begin{bmatrix} 0 & 1 & & \\ & 0 & \ddots & \\ & & \ddots & 1 \\ & & & 0 \end{bmatrix},$$

 $i=1,2,\cdots,\varepsilon$. $-sE_r+A_r$ is row unimodular, form

$$\begin{bmatrix} -sE_{r1} + A_{r1} \\ -sE_{r2} + A_{r2} \\ \vdots \\ -sE_{r\alpha} + A_{r\alpha} \end{bmatrix}, \quad \text{only if} \quad rank \begin{bmatrix} L_{1i} & D_1 & \bar{L}_1 \\ L_{2i} & D_2 & \bar{L}_2 \end{bmatrix} > rank \begin{bmatrix} D_1 & \bar{L}_1 \\ D_2 & \bar{L}_2 \end{bmatrix}, \quad i \in \mathbf{w}$$

$$-sE_{ri} + A_{ri} = \begin{bmatrix} -s & 1 & & & \\ & -s & \ddots & & \\ & & \ddots & \ddots & \\ & & & -s & 1 \end{bmatrix}_{n_{ri} \times (n_{ri}+1)},$$

 $i = 1, 2, \dots, \alpha$. $-sE_c + A_c$ is column unimodular,

$$\begin{bmatrix} -sE_{c1} + A_{c1} & & & & \\ -sE_{c2} + A_{c2} & & & & \\ & & \ddots & & \\ & & -sE_{c\beta} + A_{c\beta} \end{bmatrix}$$

form as
$$\begin{bmatrix} -sE_{c1} + A_{c1} \\ -sE_{c2} + A_{c2} \end{bmatrix}$$
 and only if the following equations satisfy.
$$TA - H_2C - H_1TE = 0$$

$$H_4TE + H_5C = 0$$

$$H_1 \text{ is stable}$$

$$H_2D_2 = 0$$

$$H_2L_{2i} = 0, \quad i \in \mathbf{k} - \mathbf{w}$$

$$H_5D_2 = 0$$

$$H_5L_{2i} = 0, \quad i \in \mathbf{k} - \mathbf{w}$$

$$\begin{bmatrix} H_4 < H_1|TL_{1i} - H_2L_{2i} > \vdots H_5L_{2i} \end{bmatrix} \neq 0,$$

$$i \in \mathbf{w}$$
where $E, A, C, D_2, L_{1i} \ i \in \mathbf{w}, L_{2i} \ i \in \mathbf{k}$
defined in (1) T, H_1, H_2, H_2, H_3 are defined in

Theorem 3.1 Analytical redundancy exists in the system (1), if and only if

(a) for the direct redundancy case.

$$rank \begin{bmatrix} E & A & B_1 & 0 & D_1 & \bar{L}_1 \\ 0 & C & B_2 & -I_m & D_2 & \bar{L}_2 \end{bmatrix} > rank \begin{bmatrix} E & A & D_1 & \bar{L}_1 \\ 0 & C & D_2 & \bar{L}_2 \end{bmatrix}$$
(8.1)

(b) for the temporal redundancy case.

$$rank \begin{bmatrix} E & A & D_1 & \bar{L}_1 \\ 0 & C & D_2 & \bar{L}_2 \end{bmatrix} > rank \begin{bmatrix} -sE + A & D_1 & \bar{L}_1 \\ C & D_2 & \bar{L}_2 \end{bmatrix}, \tag{8.2}$$

Definition 7. Suppose the system contains faults and external disturbances, the detected fault $m_{i_0}(t)$ is separable from the disturbance d(t) and the isolated faults $m_i(t), j \in \mathbf{k} - \mathbf{w}$, if there exists a relationship between the elements of the detected fault $m_{i_0}(t)$, but independent of the disturbances and the isolated faults.

The existence of analytical redundancy is only one necessary condition of the existence of RDO, in the following, given another necessary condition of the existence of RDO, which is the test condition of the separability of the detected faults $m_i(t), i \in \mathbf{w}$ from the disturbances d(t) and the isolated faults $m_i(t), i \in \mathbf{k} - \mathbf{w}$.

Theorem 3.2 The faults to be detected $m_i(t), i \in \mathbf{w}$ separate from the external disturbances d(t) and the faults to be isolated, if and

$$rank \begin{bmatrix} L_{1i} & D_1 & \bar{L}_1 \\ L_{2i} & D_2 & \bar{L}_2 \end{bmatrix} > rank \begin{bmatrix} D_1 & \bar{L}_1 \\ D_2 & \bar{L}_2 \end{bmatrix}, \quad i \in \mathbf{w}$$

$$\tag{9}$$

Then, given the necessary and sufficient conditions about the existence of RDO of the system (1) and a design algorithm.

Theorem 3.3 Given an eigenvector f, which is defined in (3), under the assumptions 1, 2, the system (1) exists a RDO, which defined in (4), if and only if the following equations satisfy.

$$TA - H_2C - H_1TE = 0 (10.1)$$

$$H_4 T E + H_5 C = 0 (10.2)$$

$$H_1 is stable$$
 (10.3)

$$H_2 D_2 = 0 (10.4)$$

$$H_2 L_{2i} = 0, \quad i \in \mathbf{k} - \mathbf{w} \tag{10.5}$$

$$H_5D_2 = 0$$
 (10.6)

$$H_5 L_{2i} = 0, \quad i \in \mathbf{k} - \mathbf{w} \tag{10.7}$$

$$\left[H_4 < H_1 | TL_{1i} - H_2 L_{2i} > : H_5 L_{2i} \right] \neq 0, \quad (10.8)$$
 $i \in \mathbf{w}$

where E, A, C, D_2, L_{1i} $i \in \mathbf{w}, L_{2i}$ $i \in \mathbf{k}$, are defined in (1), T, H_1 , H_2 , H_4 , H_5 are defined in (4), and T satisfies the equations (5.1), (5.2).

Remark 3.1 Conditions (10.1), (10.3) are commonly required by any observers for the singular system, the rest are specially required by the robust fault detection and isolation: condition (10.2) implies that r(t) is a residual vector; conditions (10.4), (10.6) guarantees that r(t) is robust to nonfault factor d(t); conditions (10.5), (10.7) make r(t) insensitive to the faults in the isolated index set $\mathbf{k} - \mathbf{w}$, guarantees the realization the isolation of faults; condition (10.8) make r(t) sensitive to the faults in the detected index set \mathbf{w} , guarantees the realization of the detection of faults. From (5.2), (10.5) and (10.7), one obtains

$$\begin{bmatrix} H_4 < H_1 | TL_{1i} - H_2L_{2i} > \vdots H_5L_{2i} \end{bmatrix} \equiv 0,$$

$$i \in \mathbf{k} - \mathbf{w}$$

Theorem 3.4 Under the assumptions 1, 2, the system (H_1, H_2, T, H_4, H_5) , satisfies equations (5.1)-(5.2) and (10.1)-(10.8), then its observable subsystem is a RDO for system (1) associated with the eigenvector f.

Under the assumptions 1, 2, designing a $RDO(H_1, H_2, T, H_4,$

 H_5) defined in (4) for the system (1) with the eigenvector f is to simultaneously solve equations (5.1) - (5.2), (10.1) - (10.8). This can be done with the following algorithm.

Algorithm 1.

Step 1. Test the separability of $m_i(t), i \in \mathbf{w}$, from $d(t), m_j(t), j \in \mathbf{k} - \mathbf{w}$, using equation (9). If it does not satisfy, the algorithm stops;

Step 2. Test the existence of analytical redundancy using equations (8.1) and (8.2). If they do not satisfy, the algorithm stops;

Step 3. Define T_1, T_2 , to be columnwise rank full matrices satisfying

$$\mathcal{R}(T_1) = \sum_{i \in \mathbf{k} - \mathbf{w}} \mathcal{R}(L_{1i}) + \mathcal{R}(D_1), \qquad (11.1)$$

$$\mathcal{R}(T_2) = \sum_{i \in \mathbf{k} - \mathbf{w}} \mathcal{R}(L_{2i}) + \mathcal{R}(D_2), \qquad (11.2)$$

Then, equations (5.1), (5.2) are equivalent to

$$TT_1 = 0,$$
 (11.3)

equations (10.4), (10.5) are equivalent to

$$H_2 T_2 = 0, (11.4)$$

equations (10.6), (10.7) are equivalent to

$$H_5 T_2 = 0. (11.5)$$

Step 4. Solve equations (10.1), (10.3) and (11.3), (11.4) with the algorithm in appendix E to get H_1 , H_2 and T. Note that, the equivalence of equations (11.3), (11.4) and (5.1) - (5.2), (10.4) - (10.5), so H_1 , H_2 , T is the solutions of equations (10.1), (10.3), (5.1) - (5.2), (10.4) - (10.5). If no H_1 , H_2 , T is designed, the algorithm stops;

Step 5. Compute H_4 , H_5 , such that H_4 , H_5 satisfy equations (10.2), (11.5), i.e. satisfy equations (10.2), (10.6) and (10.7). From (11.5), (E2), one obtains

$$H_5 = \bar{S}R_2 \tag{11.6}$$

From (10.2) and (11.6), one gets

$$H_4 T E Q = 0 \tag{11.7}$$

where Q is defined in (E5). Take H_4 as the left annihilator of TEQ, then

$$\bar{S} = -H_4 T E (R_2 C)^+ \tag{11.8}$$

From (11.6), (11.8), one gets

$$H_5 = -H_4 T E(R_2 C)^+ R_2 (11.9)$$

Step 6. Check whether H_1, H_2, T, H_4, H_5 satisfy equation (10.8), if not, the algorithm stops;

Step 7. Decompose system (H_1, H_2, T, H_4, H_5) into its observable normal form, then the observable subsystem is a RDO of system (1) with the eigenvector f.

Remark 3.2 The order of the *RDO* obtained from the algorithm 1 is not greater than the order of system (1).

Remark 3.3 Step 1, Step 2 are only the necessary test conditions, so the algorithm can be started directly from Step 3.

Remark 3.4 It is easy to proof that the algorithm is complete. That is, a RDO with the eigenvector f will be designed using the algorithm 1 as long as any exists.

On the above, one discussed the design method of RDO described by (4) with eigenvector f. The designed RDO by Algorithm 1 is usually not of its minimal order. How to degrade the compute complex and minimize the order of RDO are an important problem to increase the real-time property of the FDI system. In the next section, the minimal order RDO design problem is discussed, under Assumptions 1, 2.

4. CONCLUSION

The design problem of RDO for singular system has been discussed in this paper. Theorem 3.1, 3.2 given the necessary conditions of the existence of RDO of the singular system. It is easy to obtain that these conditions are also feasible for the non-square singular system, i.e. the system of $E \in R^{q \times n}, rankE < n$. Theorem 3.3 given a sufficient and necessary condition of the existence of RDO described by (4) for system (1). From the sufficiency of Theorem 3.3, a design algorithm for RDO is given.

The observer-based robust FDI technique require little effort to the model faults. This may be lead to give false fault alarms or some faults can not be detected. For the unrealizable eigenvectors, some optimal approximation methods may provide useful solutions. To increase the precision of the detection, one can also use the other information of faults and the knowledge-based method.

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