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DECENTRALIZED H_∞ DESIGN OF AUTOMATIC GENERATION CONTROL

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Abstract: An H_∞ decentralized design is proposed for automatic generation control (AGC) of multi-area overlapping interconnected power systems. To derive the resulting controller, the Inclusion Principle and LMI algorithms are discussed in the H_∞ framework for the original and expanded systems. Taking a two-area overlapping interconnected power system AGC as an example, a simulation of responses to load step disturbances is used to illustrate the performance of the new H_∞ decentralized AGC design. *Copyright©2002 IFAC*

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1. INTRODUCTION

There has been an intensive research of the problem of the Automatic Generation Control (AGC) in the operation of interconnected power systems. Conventional approaches to AGC have been based on the so-called tie-line bias control concept (Cohn, 1966) and non-interaction principle (Fosha and Elgerd, 1970). Dynamic area models have been introduced in the approaches to AGC based on the modern control theory, e.g. (Calovic, 1972 and 1984; Fosha and Elgerd, 1970; Kavin *et al*., 1971). The crucial design of AGC is mainly faced with both conceptual and computational difficulties, since the necessary information for control has to be acquired from power areas and generating plants spread over large geographic territories. Attempts to overcome these difficulties have been presented by (Calovic, 1972 and 1984; Carpentier, 1985; Siljak, 1978; Wenkateswarlu and Mahalanabis, 1977), where the multi-area power system model is decomposed into several subsystems which are controlled separately by their own local AGC controllers. The inclusion principle has been found to be a convenient tool for dealing with the problem of decentralized AGC design in the deterministic context by imposing static state feedback control (Ikeda *et al*., 1981; Siljak, 1991), and in the stochastic context by imposing dynamic LQG control (Chen and Stankovic, 1996; Stankovic *et al*., 1996 and 1999). Almost all of the research in AGC so far has not addressed the problem of robustness. The H_∞ control method reinforced by the linear matrix inequality (LMI) algorithms can provide the desired results.

In the present paper, the H_∞ decentralized output feedback controller of AGC is designed for multi-area overlapping interconnected power systems to improve the system robustness; that is, the overlapping interconnected power system is decomposed as a group of two-by-two subsystems which then are decoupled by using the inclusion principle, and in the expended space, the decentralized robust H_∞ controllers are designed for the decoupled subsystems by LMI algorithm, then the resulting solution are contracted to and implemented in the original space as a decentralized sub-optimal robust controller of the original system. Simulation of the systems controlled by the new H_∞ decentralized AGC display desired performance characteristics with a robustness characteristics inherent in the H_∞ design.

2. INCLUSION PRINCIPLE IN STANDARD H_∞ STATE SPACES

Consider a pair of linear stochastic continuous-time dynamic systems in standard H_∞ state spaces represented by

$$\mathrm{S}: \begin{cases} \dot{x} = Ax + B_1 w + B_2 u \\ z = C_1 x + D_{11} w + D_{12} u \\ y = C_2 x + D_{21} w + D_{22} u \end{cases}$$
$$\tilde{\mathrm{S}}: \begin{cases} \dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{B}_1 \tilde{w} + \tilde{B}_2 u \\ \tilde{z} = \tilde{C}_1 \tilde{x} + \tilde{D}_{11} \tilde{w} + \tilde{D}_{12} u \\ \tilde{y} = \tilde{C}_2 \tilde{x} + \tilde{D}_{21} \tilde{w} + \tilde{D}_{22} u \end{cases} \qquad (1)$$

In S, the first equation describes the evolution of the state vector $x(t)\in R^n$ with $x(t_0)=x_0$, driven by control input vector $u(t) \in R^m$ and stochastic disturbance modeled by zero-mean white noise process $w(t)\in R^p$ (including input and observation noises) with covariance $R_w\delta(t_1-t_2)$; the second and third equations describe the output and observation with the controlled output vector $z(t)\in R^q$ and the measurable output $y(t)\in R^l$; it is assumed that the vector x_0 is Gaussian with mean m_0 and covariance R_0, and that x_0 and $w(t)$ are mutually independent; matrices A, B_1, B_2, C_1, C_2, D_{11}, D_{12}, D_{21}, D_{22} and R_w are constant matrices with appropriate dimensions. In $\tilde{\mathrm{S}}$, the assumptions are analogous to those in S. Our crucial assumption is that $n\le\tilde{n}$, $m\le\tilde{m}$, $l\le\tilde{l}$, $p\le\tilde{p}$, $q\le\tilde{q}$.

Generally speaking, a vector stochastic process $\alpha(t)$ is denoted by mean $m_\alpha(t)$ and covariance $R_\alpha(t_1,t_2)$. If $\alpha(t) =T\beta(t)$, $\forall\, t\ge t_0$, where $\alpha(t)$ and $\beta(t)$ are n_α- and n_β-dimensional stochastic processes, respectively, and T a full rank matrix, then it is said that $\alpha(t)$ is a strict expansion of $\beta(t)$, denoted as $\alpha(t)= E_s[\beta(t);T]$, if $n_\alpha\ge n_\beta$; $\alpha(t)$ is a strict contraction of $\beta(t)$, denoted as $\alpha(t)=C_s[\beta(t);T]$, if $n_\alpha\le n_\beta$. If, for the same processes, $m_\alpha(t)=Tm_\beta(t)$, and $R_\alpha(t_1,t_2) = TR_\beta(t_1,t_2)T^T$, $\forall\, t$, t_1, $t_2\ge t_0$, then $\alpha(t)$ is a weak expansion of $\beta(t)$, i.e. $\alpha(t)=E_w[\beta(t);T]$ if $n_\alpha \ge n_\beta$; and a weak contraction of $\beta(t)$, i.e. $\alpha(t) = C_w[\beta(t);T]$ if $n_\alpha\le n_\beta$ (Stankovic *et al.*, 1996). There also needs the quadruplet of ordered pairs of full-rank matrices $\{(U, V), (S_1, T_1), (Q_2, R_2), (S_2, T_2)\}$, satisfying $UV=I_n$, $S_1T_1=I_q$, $Q_2R_2=I_m$ and $S_2T_2=I_l$.

Definition 1. The system $\tilde{\mathrm{S}}$ includes the system S if there exists a quintuplet of full rank matrices $\{U_{n\times\tilde{n}}, V_{\tilde{n}\times n}, S_{1_{q\times\tilde{q}}}, R_{2_{\tilde{m}\times m}}, S_{2_{l\times\tilde{l}}}\}$ satisfying $UV=I_n$, such that for any x_0 and $u(t)$ in S the conditions $\tilde{x}_0= E_w[x_0;V]$ and $\tilde{u}(t)=E_s[u(t);R_2]$ imply $x(t)= C_w[\tilde{x}(t);U]$, $z(t)=C_w[\tilde{z}(t);S_1]$ and $y(t)= C_w[\tilde{y}(t);S_2]$, $\forall\, t\ge t_0$.

There are two practically important special cases of inclusion (*e.g.*, Chen and Stankovic, 1996; Stankovic *et al.*, 1996 and 1999).

Definition 2. The system S is a restriction of the system $\tilde{\mathrm{S}}$ if there exists a full rank matrix such that for any x_0 the condition $\tilde{x}_0= E_w[x_0;V]$ implies $\tilde{x}(t)= E_w[x(t);V]$, $\forall\, t\ge t_0$; and the following four relations between the inputs and the outputs are satisfied:

$$\begin{aligned}
&\tilde{u}(t)=E_s[u(t);R_2],\ \tilde{z}(t)=E_w[z(t);T_1],\ \tilde{y}(t) = E_w[y(t);T_2];\\
&\tilde{u}(t)=E_s[u(t);R_2],z(t)=C_w[\tilde{z}(t);S_1],y(t) = C_w[\tilde{y}(t);S_2];\\
&u(t)=C_s[\tilde{u}(t);Q_2],\ \tilde{z}(t)=E_w[z(t);T_1],\ \tilde{y}(t) = E_w[y(t);T_2];\\
&u(t)=C_s[\tilde{u}(t);Q_2],z(t)=C_w[\tilde{z}(t);S_1],y(t) = C_w[\tilde{y}(t);S_2],
\end{aligned} \qquad (2)$$

where Q_i, R_2, S_i, T_i, i = 1,2, are full rank matrices.

Theorem 1. The system S is a restriction of $\tilde{\mathrm{S}}$ if there exists a group of full rank matrices V, Q_i, R_2, S_i, T_i, i = 1,2, such that

$$\tilde{A}V=VA;\ VB_1R_wB_1^TV^T= \tilde{B}_1R_{\tilde{w}}\tilde{B}_1^T \qquad (3)$$

and anyone of the following restriction-type conditions holds:

$$\begin{aligned}
&\text{(a)}\ \tilde{B}_2R_2=VB_2,\ \tilde{C}_1V=T_1C_1,\ \tilde{C}_2V=T_2C_2;\\
&\text{(b)}\ \tilde{B}_2R_2=VB_2,\ S_1\tilde{C}_1V=C_1,\ S_2\tilde{C}_2V=C_2;\\
&\text{(c)}\ \tilde{B}_2=VB_2Q_2,\ \tilde{C}_1V=T_1C_1,\ \tilde{C}_2V=T_2C_2;\\
&\text{(d)}\ \tilde{B}_2=VB_2Q_2,\ S_1\tilde{C}_1V=C_1,\ S_2\tilde{C}_2V=C_2.
\end{aligned} \qquad (4)$$

Definition 3. The system S is an aggregation of the system $\tilde{\mathrm{S}}$ if there exists a full rank matrix such that for any $\tilde{x}_0$ the condition $x_0 =C_w[\tilde{x}_0;U]$ implies $x(t)=C_w[\tilde{x}(t);U]$, $\forall\, t\ge t_0$; and the following four relations between the inputs and the outputs are satisfied:

$$\begin{aligned}
&u(t)=C_s[\tilde{u}(t);Q_2],z(t)=C_w[\tilde{z}(t);S_1],y(t) =C_w[\tilde{y}(t);S_2];\\
&\tilde{u}(t)=E_s[u(t);R_2],z(t)=C_w[\tilde{z}(t);S_1],y(t) =C_w[\tilde{y}(t);S_2];\\
&u(t)=C_s[\tilde{u}(t);Q_2],\ \tilde{z}(t)=E_w[z(t);T_1],\ \tilde{y}(t)=E_w[y(t);T_2];\\
&\tilde{u}(t)=E_s[u(t);R_2],\ \tilde{z}(t)=E_w[z(t);T_1],\ \tilde{y}(t)=E_w[y(t);T_2],
\end{aligned} \qquad (5)$$

where Q_i, R_2, S_i, T_i, i = 1,2, are full rank matrices.

Theorem 2. The system S is an aggregation of $\tilde{\mathrm{S}}$ if there exists a group of full rank matrix U, Q_i, R_2, S_i, T_i, i = 1,2, such that

$$U\tilde{A}=AU;\ B_1R_wB_1^T=U\tilde{B}_1R_{\tilde{w}}\tilde{B}_1^TU^T \qquad (6)$$

and anyone of the following aggregation-type conditions holds:

$$\begin{aligned}
&\text{(a)}\ U\tilde{B}_2=B_2Q_2,\ S_1\tilde{C}_1=C_1U,\ S_2\tilde{C}_2=C_2U;\\
&\text{(b)}\ U\tilde{B}_2R_2=B_2,\ S_1\tilde{C}_1=C_1U,\ S_2\tilde{C}_2=C_2U;\\
&\text{(c)}\ U\tilde{B}_2=B_2Q_2,\ \tilde{C}_1=T_1C_1U,\ \tilde{C}_2=T_2C_2U;\\
&\text{(d)}\ U\tilde{B}_2R_2=B_2,\ \tilde{C}_1=T_1C_1U,\ \tilde{C}_2=T_2C_2U.
\end{aligned} \qquad (7)$$

For proofs of the above theorems see (Stankovic *et al.*, 1996 and 1999). It is noted that the described inclusion principle is only the transfiguration in the framework of standard H_∞ state spaces for the existing one. The inclusion principle has first been introduced into the systems by (Ikeda and Siljak, 1980; Ikeda *et al.*, 1984). The stochastic forms of inclusion have been discussed by (Hodzic and Siljak, 1983), and then by (Chen and Stankovic, 1996; Stankovic *et al.*, 1996 and 1999). The inclusion has been generalized from the state to the input and the output by (Ikeda and Siljak, 1986; Iftar, 1993a; Siljak, 1991; Chen and Stankovic, 1996; Stankovic *et al*, 1996 and 1999).

Let us associate with the systems S and $\tilde{\mathrm{S}}$ a pair of output feedback controllers described by

$$C:\begin{cases}\dot{x}_c = A_c x_c + B_c y\\ u = C_c x_c + D_c y\end{cases},$$
$$\tilde{C}:\begin{cases}\dot{\tilde{x}}_c = \tilde{A}_c \tilde{x}_c + \tilde{B}_c \tilde{y}\\ \tilde{u} = \tilde{C}_c \tilde{x}_c + \tilde{D}_c \tilde{y}\end{cases}, \qquad (8)$$

where, $x_c(t)\in R^r$ with $x_c(t_0)=x_{c0}$ and $\tilde{x}_c(t)\in R^{\tilde{r}}$ with $\tilde{x}_c(t_0)=\tilde{x}_{c0}$, denote the state vectors of controllers C and $\tilde{C}$, driven by the output vectors $y(t)\in R^l$ and $\tilde{y}(t)\in R^{\tilde{l}}$ of the systems, respectively. The matrices A_c, B_c, C_c, D_c, $\tilde{A}_c$, $\tilde{B}_c$, $\tilde{C}_c$ and $\tilde{D}_c$ are constant with proper dimensions. Although the assumption of $r\le\tilde{r}$ justified since S is a part of $\tilde{\mathrm{S}}$, and thus should not require a large dimensional controller (Ikeda and Siljak, 1986; Iftar, 1993b), we leave $r=\tilde{r}$ and only consider the contraction of inputs and outputs because there is no overlapping part of the controller corresponding to that of the system.

Theorem 3. The controller $\tilde{\mathrm{C}}$ is contractible to controller C, if the system $\tilde{\mathrm{S}}$ includes the system S and for a group of full rank matrices Q_i, R_2, S_i, T_i, i = 2, satisfying $A_c = \tilde{A}_c$ anyone of the following conditions holds:

(a) $B_cS_2 = \tilde{B}_c$, $C_c = Q_2\tilde{C}_c$, $D_cS_2 = Q_2\tilde{D}_c$;
(b) $B_cS_2 = \tilde{B}_c$, $R_2C_c = \tilde{C}_c$, $R_2D_cS_2 = \tilde{D}_c$;
(c) $B_c = \tilde{B}_c T_2$, $C_c = Q_2\tilde{C}_c$, $D_c = Q_2\tilde{D}_c T_2$;
(d) $B_c = \tilde{B}_c T_2$, $R_2C_c = \tilde{C}_c$, $R_2D_c = \tilde{D}_c T_2$. (9)

3. MODEL OF THE SYSTEM

Consider a multi-area overlapping interconnected power system (Siljak, 1978), described by the equations:

$$\mathrm{S}_i:\begin{cases}\dot{x}_i = A_i x_i + a_{ti}P_{ei} + b_i u_i + f_i\xi_i\\ \dot{P}_{ei} = \alpha_{1i}\sum\limits_{\substack{j=1\\ j\neq i}}^{n}(m_{ij}^T x_i - m_{ji}^T x_j)\\ \dot{v}_i = d_i^T x_i + P_{ei}\\ y_i = c_i x_i + \eta_i\end{cases}$$
$$i=1,2,\ldots,n. \qquad (10)$$

Where, vector x_i is the deviation of the states of i-th area (subsystem) consisting of the components: a_T, the valve opening variation of the steam turbine; P_{t1}, P_{t2} and P_{t3}, the high, intermediate and low pressure output variations of steam turbine, respectively; a_H, the gate opening variation of hydro turbine; v_H, dashpot position variation; q, water flow variation of the hydroturbine; f, frequency variation. v_i is the deviation of the variable achieving integral control; P_{ei} represents the deviation of the tie-line power exchange variations between areas; u_i is the deviation of the scalar area control input and ξ_i is immeasurable variation of the area load; $y_i = [P_T, P_H, f]_i^{\mathrm{T}}$ defined as the deviation of the local output, where P_T is the output of variation of the steam turbine and P_H is the output variation of the hydro unit; η_i represents the measurement noise vector corresponding to y_i (Calovic, 1984). The system $\mathrm{S_i}$ can be decomposed in tow-by-two overlapping interconnected power subsystems described as

$$\mathrm{S}_{ij}:\begin{cases}\dot{x} = Ax + Bu + F\xi\\ y = Cx + \eta\end{cases}$$
$$i, j=1,2,\ldots,n,\ i\neq j \qquad (11)$$

Where, $x=[x_i^{\mathrm{T}}, v_i, P_{ei}, v_j, x_j^{\mathrm{T}}]^{\mathrm{T}}$ denotes the state variation vector (note that $P_{ei} = P_{ej}$); $u=[u_i,\ u_j]^{\mathrm{T}}$ input vector, $\xi=[\xi_i,\ \xi_j]^{\mathrm{T}}$ input disturbance vector, $\eta=[\eta_i^{\mathrm{T}}, \eta_{vi}, \eta_{ei}, \eta_{vj}, \eta_j^{\mathrm{T}}]^{\mathrm{T}}$ measurement noise vector; the matrices in (11) are

$$A=\begin{bmatrix} A_{ii} & 0 & a_{ti} & 0 & 0\\ d_i^T & 0 & 1 & 0 & 0\\ \alpha_{1i}m_{ij}^T & 0 & 0 & 0 & -\alpha_{1i}m_{ji}^T\\ 0 & 0 & -\dfrac{\alpha_{1j}}{\alpha_{1i}} & 0 & d_j^T\\ 0 & 0 & -\dfrac{\alpha_{1j}}{\alpha_{1i}}a_{tj} & 0 & A_{jj}\end{bmatrix}$$

$$B=\begin{bmatrix} b_i & 0\\ 0 & 0\\ 0 & 0\\ 0 & 0\\ 0 & b_j\end{bmatrix}\quad F=\begin{bmatrix} f_i & 0\\ 0 & 0\\ 0 & 0\\ 0 & 0\\ 0 & f_j\end{bmatrix}\quad C=\begin{bmatrix} C_{ii} & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & C_{jj}\end{bmatrix}$$
$$i,\ j=1,2,\ldots,n,\ i\neq j. \qquad (12)$$

It appears that two-by-two subsystems possess overlapping parts showed by doted line in (12). Now, we consider the standard H_∞ state space description of the system S_{ij} as follows:

$$S_{ij}:\begin{cases}\dot{x}=Ax+B_1w+B_2u\\ z=C_1x+D_{11}w+D_{12}u\\ y=C_2x+D_{21}w+D_{22}u\end{cases}$$
$$i,\ j\ =1,2,\ldots,n,\ i\neq j \qquad (13)$$

where, $w=[\xi_1,\eta_{y1}^{T},\eta_{v1},\eta_{Pe},\eta_{v2},\eta_{y2}^{T},\xi_2]^{T}$ denotes input disturbance vector, the combined vector of input disturbances and output measurement noises; $z=[z_1,z_2]^{T}$ the controlled output vector, leaving the physical meaning of x, u and y as fixed above. Matrices in (13), which correspond to (11) and (12), are changed to

$$B_1=\begin{bmatrix} f_1 & 0 & 0\\ 0 & 0_{3\times9} & 0\\ 0 & 0 & f_2\end{bmatrix},$$
$$B_2=B,\ C_2=C,$$
$$C_1=\begin{bmatrix} 0 & 0.5 & 0.5 & 0.5 & 0.5 & 0 & 0.5 & 0.1 & 0.1 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1 & 0.1 & 0 & 0.5 & 0.5 & 0.5 & 0.5 & 0 & 0.5 & 0.1\end{bmatrix},$$
$$D_{11}=0_{2\times11},\quad D_{12}=I_2,$$
$$D_{21}=[\,0_{9\times1}\ \ I_9\ \ 0_{9\times1}\,],\quad D_{22}=0_{9\times2}. \qquad (14)$$

Therefore, we rewrite (13) as

$$S_{ij}:\begin{cases}\dot{x}=Ax+B_1w+B_2u\\ z=C_1x+u\\ y=C_2x+D_{21}w\end{cases}$$
$$i,\ j=1,2,\ldots,n,\ i\neq j \qquad (15)$$

For detailed amount of parameters in the expression (15) see (Calovic, 1984; Siljak, 1978). The choice of matrix C_1 for controlled output vector z allows the better performance of the designed H_∞ decentralized output feedback controllers. Other models of two-by-two area subsystems can be set as (15).

4. OVERLAPPING DECOMPOSITION AND H_∞ CONTROLLER DESIGN

For the H_∞ decentralized output feedback control of the multi-area overlapping interconnected power system, the model (10) can first be decomposed as two-by-two area subsystems (11) which can be written in the standard H_∞ form as (15). Then, the problem of the overlapping structures in the two-by-two area subsystems is solved, that is, the deviation of the tie-line power exchange variation of subsystems P_{ei} is decoupled for each subsystem in the expended space. By imposing restriction and aggregation conditions presented in section 2, one can properly choose a group of expanding matrices, except that $Q_2=R_2=I_2$, $S_1=T_1=I_2$ (since there is no overlapping part in the input $u=[u_i,\ u_j]^{T}$ and in the controlled output $z=[z_1,z_2]^{T}$), such as:

$$U=\begin{bmatrix} I_9 & 0 & 0 & 0\\ 0 & \beta & 1-\beta & 0\\ 0 & 0 & 0 & I_9\end{bmatrix},\quad V=\begin{bmatrix} I_9 & 0 & 0\\ 0 & 1 & 0\\ 0 & 1 & 0\\ 0 & 0 & I_9\end{bmatrix};$$
$$S_2=\begin{bmatrix} I_4 & 0 & 0 & 0\\ 0 & \beta & 1-\beta & 0\\ 0 & 0 & 0 & I_4\end{bmatrix},\quad T_2=\begin{bmatrix} I_4 & 0 & 0\\ 0 & 1 & 0\\ 0 & 1 & 0\\ 0 & 0 & I_4\end{bmatrix};$$
$$(16)$$

(where β is a scalar satisfying $0<\beta<1$), and appropriate complementary matrices corresponding to the matrices in (15), which satisfy the following relations:

$$\tilde{A}=VAU+M_A,\quad \tilde{B}_1=VB_1Q_1+M_{B1},\quad \tilde{B}_2=VB_2+M_{B2}, \qquad (17)$$
$$\tilde{C}_1=C_1U+M_{C1},\quad \tilde{C}_2=T_2C_2U+M_{C2},\quad \tilde{D}_{21}=T_2D_{21}Q_1+M_{D21}$$

The pair of expanded subsystems with the decomposed overlapping structure is obtained

$$\tilde{S}_{ij}:\begin{cases}\dot{\tilde{x}}=\tilde{A}\tilde{x}+\tilde{B}_1\tilde{w}+\tilde{B}_2u\\ \tilde{z}=\tilde{C}_1\tilde{x}+u\\ \tilde{y}=\tilde{C}_2\tilde{x}+\tilde{D}_{21}\tilde{w}\end{cases}$$
$$i,\ j=1,2,\ldots,n,\ i\neq j \qquad (18)$$

where, the matrices in (18) are partitioned with the block-diagonal parts denoted as the matrices in both of decoupled subsystems. The overlapping vectors are also partitioned appropriately.

In this way, the decentralized subsystems are shown in the expanded space

$$\tilde{S}_s:\begin{cases}\dot{\tilde{x}}_s=\tilde{A}_s\tilde{x}_s+\tilde{B}_{1s}w_s+\tilde{B}_{2s}u_s\\ \tilde{z}_s=\tilde{C}_{1s}\tilde{x}_s+u_s\\ \tilde{y}_s=\tilde{C}_{2s}\tilde{x}_s+\tilde{D}_{21s}w_s\end{cases}\qquad s=i,j. \qquad (19)$$

The H_∞ decentralized dynamic output feedback controller is applied to each subsystem $\tilde{S}_s$ having the structures

$$\tilde{C}_s:\begin{cases}\dot{\hat{x}}_s=\hat{A}_s\hat{x}_s+\hat{B}_sy_s\\ u_s=\hat{C}_s\hat{x}_s+\hat{D}_sy_s\end{cases}\qquad s=i,j. \qquad (20)$$

The existence condition for this kind of H_∞ controller can be established using the LMI approach of (Iwasaki and Skelton, 1994):

$$B_sG_sC_s+(B_sG_sC_s)^{T}+Q_s<0 \qquad (21)$$

where,

$$G_s = \begin{bmatrix} \hat{D}_s & \hat{C}_s \\ \hat{B}_s & \hat{A}_s \end{bmatrix}$$

$$B_s = \begin{bmatrix} \tilde{B}_{2s} & I_2 & 0 \end{bmatrix}^T, \; C_s = \begin{bmatrix} \tilde{C}_{2s}P & 0 & \tilde{D}_{21s} \end{bmatrix}$$

$$Q_s = \begin{bmatrix} \tilde{A}_s P + P\tilde{A}_s^{\mathrm{T}} & P\tilde{C}_{1s}^{\mathrm{T}} & \tilde{B}_{1s} \\ \tilde{C}_{1s}P & \gamma & 0 \\ \tilde{B}_{1s}^{\mathrm{T}} & 0 & -\gamma I_6 \end{bmatrix} \quad (22)$$

If there exist symmetric positive matrix P and a matrix G_i, such that expression (21) is satisfied, then G_i would be a candidate of parameter matrices for the H_∞ dynamic output feedback controller (20). The resulting parameter matrices for G_i are composed as a H_∞ decentralized dynamic output feedback controller

$$\tilde{C}_{ij}: \begin{cases} \dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}y \\ u = \hat{C}\hat{x} + \hat{D}y \end{cases}$$
$$i, j=1,2,\ldots,n, \; i \neq j \quad (23)$$

for the system $\tilde{\mathrm{S}}_{\mathrm{ij}}$ in the expanded space, where the parameter matrices are described as

$$\hat{A} = diag\,(\hat{A}_i, \hat{A}_j), \quad \hat{B} = diag\,(\hat{B}_i, \hat{B}_j)$$
$$\hat{C} = diag\,(\hat{C}_i, \hat{C}_j), \quad \hat{D} = diag\,(\hat{D}_i, \hat{D}_j) \quad (24)$$

Contracting the decentralized controllers described by (23) and (24) in $\tilde{\mathrm{S}}_{\mathrm{ij}}$ using conditions of Theorem 3, one can get a H_∞ decentralized dynamic output feedback controller in the original space S_{ij} described as

$$C_{ij}: \begin{cases} \dot{\hat{x}} = \hat{A}_c\hat{x} + \hat{B}_c y \\ u = \hat{C}_c\hat{x} + \hat{D}_c y \end{cases}$$
$$i, j=1,2,\ldots,n, \; i \neq j \quad (25)$$

where,

$$\hat{A}_c = \hat{A}, \quad \hat{B}_c = \hat{B}T_2, \quad \hat{C}_c = Q_2\hat{C}, \quad \hat{D}_c = \hat{D}T_2. \quad (26)$$

The controller for $\tilde{\mathrm{S}}_{\mathrm{ij}}$ can be contracted to and implemented in the original system S_{ij}. If the controller $\tilde{\mathrm{C}}_{\mathrm{ij}}$ stabilizes the system $\tilde{\mathrm{S}}_{\mathrm{ij}}$ in the expanded space, then the controller C_{ij} stabilizes the system S_{ij} in the original space. After contraction, the closed-loop system model of S_{ij} is rewritten as

$$\begin{cases} \begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = \begin{bmatrix} A + B_2\hat{D}_c C_2 & B_2\hat{C}_c \\ \hat{B}_c C_2 & \hat{A}_c \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + \begin{bmatrix} B_1 + B_2\hat{D}_c D_{21} \\ \hat{B}_c D_{21} \end{bmatrix} w, \\ y = \begin{bmatrix} C_2 & 0 \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + D_{21}w. \end{cases} \quad (27)$$

Using representation (27), a simulation of the two-by-two area system is described in the next section. When i, j = 1, 2, …, n, $i \neq j$, one can get a set of decentralized sub-optimal robust H_∞ controllers of each two-by-two area subsystems. Comparing with the results of decentralized overlapping suboptimal LQG control for the system AGC, obtained in (Chen and Stankovic, 1996; Stankovic *et al*, 1996 and 1999), the proposed H_∞ decentralized output feedback for AGC has a desired robustness features, while the response curves of the system to the load disturbance are of the same quality as the ones of the LQG approach.

5. RESULTS OF SIMULATION

For the two-by-two area system, assume i = 1, j = 2, and the parameters of the system matrices as in (12), (14), (Calovic, 1984; Chen and Stankovic, 1996; Stankovic *et al*, 1996 and 1999). Let expanding transform matrices be as in (16), H_∞ optimal performance index γ =15. We obtained the decentralized H_∞ controller in the expanded space $\tilde{\mathrm{S}}_{12}$ as in (25), which can be contracted to and implemented in the original system S_{12}. The 9-th dimensional output response curves including deviation of frequency variations $f_{1,2}$ and the tie-line power exchange variation P_{e1}, to the load disturbance are showed in Fig. 1. The controllers have important robustness characteristics, which are present in the H_∞ design, while performance curves are of the same quality as those obtained using LQG controllers (Calovic, 1984; Chen and Stankovic, 1996; Stankovic *et al*, 1996 and 1999).

6. CONCLUSIONS

A new overlapping decentralized AGC scheme is proposed using the Inclusion Principle and the H_∞ methodology. After the system is expanded into a larger space, where subsystems appear as disjoint, H_∞ controllers are designed for each individual power area using only local output feedback. Then, the system and the decentralized controllers are contracted to the initial smaller space for implementation of the controllers in the original system. Simulations of the typical interconnected power systems show that the quality of performance of the closed-loop systems in the new scheme is the same as that of the standard decentralized LQG control approach, while the closed-loop system obtains the additional robustness characteristics of controllers designed by the H_∞ methodology.

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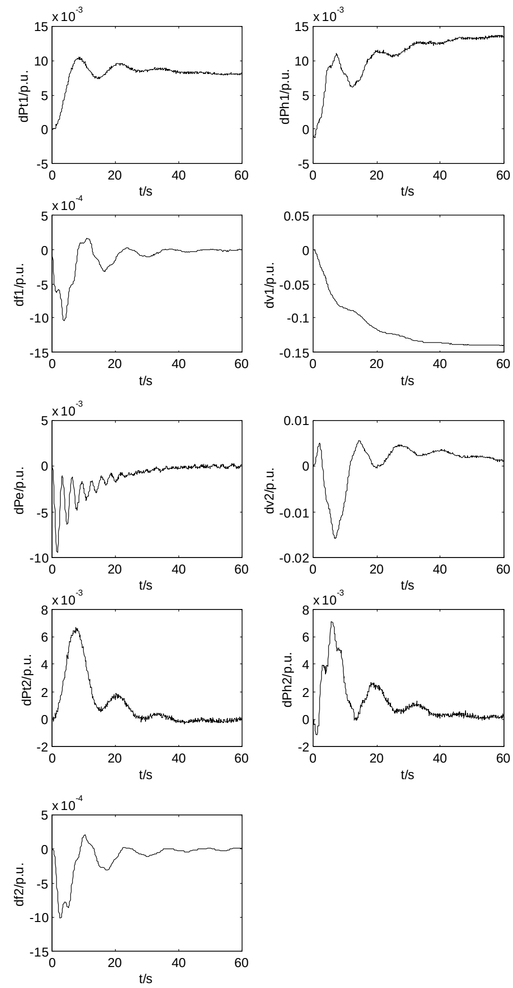

Fig. 1. 9-th dimensional output response curves to step disturbances