15th Triennial World Congress, Barcelona, Spain

INERTIA SHAPING TECHNIQUES FOR MARINE VESSELS USING ACCELERATION FEEDBACK

Thor I. Fossen [*,1] Karl-Petter Lindegaard [*,1]
Roger Skjetne [*,1]

[*] Department of Engineering Cybernetics, Norwegian University of Science and Technology, N-7491 Trondheim, Norway, e-mail: tif@itk.ntnu.no, kpl@itk.ntnu.no, skjetne@ieee.org

Abstract: The concept of energy based Lyapunov control is extended to marine vessels with a nonsymmetric system inertia matrix. Acceleration feedback is used as the main tool to symmetrize the nonsymmetric part of the system inertia matrix. The main reason for a nonsymmetric mass distribution is hydrodynamic added mass which depend on the forward speed of the vessel and the frequency of the incoming waves. This is a well known phenomenon for marine vessels moving at positive speed in waves while low-speed applications like dynamic positioning systems are fairly well described with a symmetric system inertia matrix. The main contribution of the paper is a new Lyapunov-based design technique incorporating acceleration feedback to shape the kinetic energy of the system. Acceleration feedback is implemented in conjuncture with a nonlinear PID-controller derived from vectorial backstepping. The result is a uniformly globally asymptotically stable (UGAS) closed-loop control system applicable to marine vessels with nonsymmetric system inertia matrices. Typical applications are ships in maneuvering situations, vessels in transit and high speed craft where nonsymmetric added mass effects must be compensated for. Copyright © 2002 IFAC

Keywords: Marine systems, accelerometers, ships, nonlinear control, Lyapunov analysis.

1. INTRODUCTION

The main idea of this paper is to modify the system inertia matrix of a marine vessel through acceleration feedback. By doing this it is possible to construct energy-based Lyapunov functions for marine vessels operating in different speed regimes, see Figure 1.

This is a non-trivial problem since the system inertia matrix M will be nonsymmetrical for marine vessels moving at high speed while it is symmetric at zero speed (station-keeping). The problem has not been addressed previously in the literature.

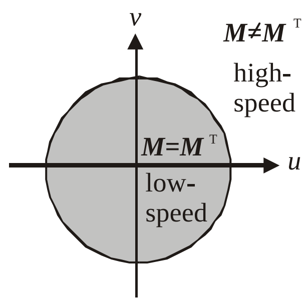

Fig. 1. Low and high speed regimes for a ship. The speed $U = \sqrt{u^2 + v^2}$ where u and v are the velocity in surge and sway.

The problem of applying the kinetic energy of a system with nonsymmetric inertia matrix as a

[1] Supported by the Norwegian Research Council through the Strategic University Program on Marine Cybernetics

Lyapunov function candidate is easiest explained by considering the following case study:

Case Study: Nonsymmetric Inertia Matrix

Consider the problem of energy-based control when the system inertia M is nonsymmetrical due to hydrodynamic added mass. Moreover, for marine vessels in transit (non-zero speed) it can be shown that:

$$M = M_{RB} + M_A \tag{1}$$

where the rigid-body system inertia matrix M_{RB} and hydrodynamic added inertia matrix M_A satisfy (Fossen, 1994):

$$M_{RB} = M_{RB}^T > 0 \tag{2}$$

$$M_A \neq M_A^T > 0 \tag{3}$$

Notice that M_A is non-symmetrical due to forward speed effects and wave-induced disturbances. This implies that the kinetic energy can be written

$$\begin{aligned} V &= \frac{1}{2}\nu^T M \nu \\ &= \frac{1}{2}\nu^T \left(\frac{1}{2}(M + M^T) + \frac{1}{2}(M - M^T) \right) \nu \\ &= \frac{1}{4}\nu^T (M + M^T)\nu \end{aligned} \tag{4}$$

since $M + M^T$ is symmetric and $M - M^T$ is skewsymmetric. Hence, time differentiation along the trajectories of ν yields:

$$\dot{V} = \frac{1}{2}\nu^T (M + M^T)\dot{\nu}$$

This approach fails for vessel models in the form:

$$M\dot{\nu} + n(\nu) = \tau \tag{5}$$

where $n(\nu)$ is a vector of nonlinear Coriolis, damping and restoring terms and τ is the control input. The main reason for this is that only $M\dot{\nu}$ in the expression for $\dot{V}$ can be be substituted with the system model (5) while the expression $M^T\dot{\nu}$ is not available from (5).

The main contribution of this paper is a solution to this problem where acceleration feedback is used to shape the system inertia matrix in such a manner that conventional Lyapunov techniques can be applied for systems with nonsymmetrical inertia matrices. Acceleration feedback is combined with nonlinear vectorial backstepping in order to obtain PID feedback control.

2. VESSEL MODELLING

2.1 Review of hydrodynamic inertia

In Lamb (1932) the concept of hydrodynamic inertia is defined in terms of fluid kinetic energy T_A which can be written as a quadratic form of the body axis velocity vector components, that is

$$T_A = \frac{1}{2}\nu^T M_A \nu \tag{6}$$

Here the body-fixed velocity vector in 6 degrees-of-freedom (DOF) is:

$$\nu = [u, v, w, p, q, r]^T \in \mathbb{R}^6 \tag{7}$$

and M_A is a 6×6 system inertia matrix defined in terms of added mass terms as:

$$\begin{aligned} M_A &= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}; \quad A_{ij} \in \mathbb{R}^{3\times 3} \\ &= -\begin{bmatrix} X_{\dot{u}} & X_{\dot{v}} & X_{\dot{w}} & X_{\dot{p}} & X_{\dot{q}} & X_{\dot{r}} \\ Y_{\dot{u}} & Y_{\dot{v}} & Y_{\dot{w}} & Y_{\dot{p}} & Y_{\dot{q}} & Y_{\dot{r}} \\ Z_{\dot{u}} & Z_{\dot{v}} & Z_{\dot{w}} & Z_{\dot{p}} & Z_{\dot{q}} & Z_{\dot{r}} \\ K_{\dot{u}} & K_{\dot{v}} & K_{\dot{w}} & K_{\dot{p}} & K_{\dot{q}} & K_{\dot{r}} \\ M_{\dot{u}} & M_{\dot{v}} & M_{\dot{w}} & M_{\dot{p}} & M_{\dot{q}} & M_{\dot{r}} \\ N_{\dot{u}} & N_{\dot{v}} & N_{\dot{w}} & N_{\dot{p}} & N_{\dot{q}} & N_{\dot{r}} \end{bmatrix} \end{aligned} \tag{8}$$

The notation of SNAME (1950) is used in this expression; for instance the hydrodynamic added mass force Y along the y-axis due to an acceleration $\dot{u}$ in the x-direction is written as:

$$Y = -Y_{\dot{u}}\dot{u} \quad \text{where} \quad Y_{\dot{u}} := \frac{\partial Y}{\partial \dot{u}}$$

Since any motion of a vessel in water will induce a motion in the otherwise stationary fluid, the fluid must move aside and then close behind the vehicle in order to allow the vessel to pass through the fluid. As a consequence, the fluid passage possesses kinetic energy T_A that it would lack if the vehicle was not in motion.

2.2 Properties

The low and high speed properties of the hydrodynamic inertia matrix can be summarized as:

Low Speed Property: For a rigid-body at rest ($U \approx 0$) under the assumption of an ideal fluid, no incident waves, no sea currents, and zero frequency, the added mass system inertia matrix is positive definite (Newman, 1977):

$$M_A = M_A^T > 0$$

This is a good assumption for low-speed maneuvers like station-keeping (dynamic positioning).

Remark: In a real fluid (not ideal) the 36 elements of M_A may all be distinct but still $M_A > 0$. Experience has shown that the numerical values of the added mass derivatives in a real fluid are usually in good agreement with those obtained from ideal theory (see Wendel, 1956). Hence, $M_A = M_A^T > 0$ is a good approximation for low speed.

High Speed Property: For surface ships moving at forward speed $U > 0$ in waves, Salvesen et al. (1970) have shown by applying strip theory that $M_A \neq M_A^T$. Consequently, the hydrodynamic system inertia matrix will depend on the forward speed U of the vessel and the wave frequency of the incoming waves.

Lyapunov based control of marine vessels have so far only addressed low-speed applications under the assumption that $M = M^T$; see Fossen and Berge (1997), Fossen and Grøvlen (1998), Fossen and Strand (2001),Fossen and Strand (1999) for instance. In the next sections this will be relaxed to $M \neq M^T$ (high speed) by introducing acceleration feedback.

3. VESSEL DYNAMICS

The dynamic equations of a ship or a floating rig can be described by the following model (Fossen 1994):

$$M\dot{\nu} + n(\nu) = \tau \tag{9}$$

with

$$M = M_{RB} + M_A \tag{10}$$

$$M_{RB} = \begin{bmatrix} mI_{3\times3} & -mS(r_g) \\ mS(r_g) & I_o \end{bmatrix}$$

where $r_g = [x_g, y_g, z_g]^T$ is the coordinates of the center of gravity, m is the mass and I_o is the inertia tensor at the body-fixed origin. The skewsymmetric matrix $S(a) = -S^T(a) \in SS(3)$ is defined such that $S(a)b = a \times b$. This yields:

$$M_{RB} = \begin{bmatrix} m & 0 & 0 & 0 & mz_g & -my_g \\ 0 & m & 0 & -mz_g & 0 & mx_g \\ 0 & 0 & m & my_g & -mx_g & 0 \\ 0 & -mz_g & my_g & I_x & -I_{xy} & -I_{xz} \\ mz_g & 0 & -mx_g & -I_{yx} & I_y & -I_{yz} \\ -my_g & mx_g & 0 & -I_{zx} & -I_{zy} & I_z \end{bmatrix}$$

The added inertia matrix M_A is defined in (8) while the nonlinear term:

$$n(\nu, \eta) = C(\nu)\nu + D(\nu)\nu + g(\eta) \tag{12}$$

is a vector of Coriolis, $C(\nu)\nu$, damping, $D(\nu)\nu$, and restoring terms, $g(\eta)$. The control input vector is denoted by τ.

4. PID AND ACCELERATION FEEDBACK

Consider the body-fixed velocity vector in 6 DOF:

$$\nu = [u, v, w, p, q, r]^T \in \mathbb{R}^6 \tag{13}$$

The main idea is to exploit the linear accelerations $\dot{u}, \dot{v}, \dot{w}$ in feedback since they are easily measured by using a conventional 3-axes accelerometer unit. Angular accelerations are, however, not available since a 3-axes gyro measures the angular rates p, q, r instead of $\dot{p}, \dot{q}, \dot{r}$. Hence, angular accelerations must be estimated in an observer in order to implement acceleration feedback in 6 DOF.

Fig. 2. The Litton LN-200 IMU.

Alternatively, $\dot{p}, \dot{q}, \dot{r}$ can be obtained by numerical differentiation of the gyro e.g. by using the filter:

$$h(s) = \frac{Ts}{Ts+1}, \quad T > 0$$

This can be done at high rate e.g. 100 Hz if an IMU (inertial measurement unit) is used to measure linear accelerations $\dot{u}, \dot{v}, \dot{w}$ and angular rates p, q, r. Several commercial IMUs are available for this purpose, e.g. the Litton LN-200 shown in Figure 2.

In the forthcoming it is assumed that all accelerations $\dot{u}, \dot{v}, \dot{w}, \dot{p}, \dot{q}, \dot{r}$ are available signals. The PID-controller with acceleration feedback is written (see Figure 3):

$$\tau = \tau_{PID} - K_a\dot{\nu} \tag{14}$$

where

$$K_a = \begin{bmatrix} K_{a11} & K_{a12} \\ K_{a21} & K_{a22} \end{bmatrix} \tag{15}$$

is a design matrix. The control law (14) applied to the system model (9), yields:

$$H\dot{\nu} + n(\nu) = \tau_{PID} \tag{16}$$

where

$$H = M_{RB} + M_A + K_a$$
$$= \begin{bmatrix} mI_{3\times3} + A_{11} + K_{a11} & -mS(r_g) + A_{12} + K_{a12} \\ mS(r_g) + A_{21} + K_{a21} & I_o + A_{22} + K_{a22} \end{bmatrix}$$

The PID controller τ_{PID} can be designed by using different methods. In Section 5 vectorial backstepping will be applied to shape the energy of the system.

4.1 Inertia Symmetrization

Two design techniques for inertia symmetrization are discussed:

Positive acceleration feedback (decreasing the system inertia)

A symmetric system inertia matrix is obtained by positive acceleration feedback:

$$K_a = -M_A < 0$$

which yields

$$H = M_{RB} = \begin{bmatrix} mI_{3\times3} & -mS(r_g) \\ mS(r_g) & I_o \end{bmatrix} \tag{17}$$

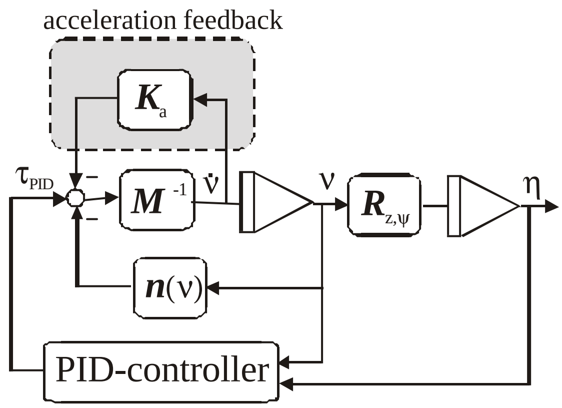

Fig. 3. Acceleration feedback and PID-controller.

Positive feedback in the inner acceleration loop will not destabilize the system since $M_{RB} > 0$. However, if M_A is uncertain, positive feedback $K_a = -M_A$ might destabilize the system if the uncertainty is in the same magnitude as the norm of M_{RB} since this can lead to $H < 0$. In this case negative acceleration feedback should be applied to avoid robustness problems.

Negative acceleration feedback (increasing the system inertia)

The system inertia can be increased by applying negative acceleration feedback:

$$K_a = M_a^T + \Delta K > 0 \quad \text{where} \quad \Delta K = \Delta K^T \geq 0$$

and with $\Delta K = 0$, this results in:

$$H = \begin{bmatrix} mI_{3\times 3} + A_{11} + A_{11}^T & -mS(r_g) + A_{12} + A_{21}^T \\ mS(r_g) + A_{12}^T + A_{21} & I_o + A_{22} + A_{22}^T \end{bmatrix}$$

The gain matrix ΔK can be used to increase the system inertia further since the feedback term $M_a^T \dot{\nu}$ ensures symmetrization. It is well known that if the inertia is increased by acceleration feedback the closed loop system will be less sensitive to external disturbances, see Lindegaard (2002) for instance.

A special solution exists for the horizontal motion of a vessel (surge, sway and yaw) since only two linear accelerometers (surge and sway) are required to symmetrize the inertia matrix. This solution is attractive both in dynamic positioning and in particular in maneuvering situations where $A_{12} \neq A_{21}^T$. The design philosophy is demonstrated by considering a floating marine vessel, a ship or a semi-submersible.

5. ENERGY SHAPING USING ACCELERATION FEEDBACK AND BACKSTEPPING DESIGNS

In this section we will demonstrate how an energy-based nonlinear controller can be designed for a floating vessel.

Fig. 4. Maneuvering of ships.

5.1 3 DOF model for ships and floating rigs

Consider the motion in surge (x-direction), sway (y-direction) and yaw (rotation about the z-axis) The 3 DOF model becomes:

$$\dot{\eta} = R_{z,\psi}\,\nu$$
$$M\dot{\nu} + C(\nu)\nu + D(\nu)\nu = \tau$$

where $\eta = [x, y, \psi]^T$ is a vector of positions and heading angle, $\nu = [u, v, r]^T$ is a vector of body-fixed velocities, and $R_{z,\psi} \in SO(3)$ is the rotation matrix in yaw, see Fossen (1994) for details. The system inertia matrix in surge, sway, and yaw (including the hydrodynamic added inertia terms $X_{\dot{u}}, Y_{\dot{v}}, Y_{\dot{r}}, N_{\dot{v}}, N_{\dot{r}}$) is:

$$M = \begin{bmatrix} m - X_{\dot{u}} & 0 & 0 \\ 0 & m - Y_{\dot{v}} & mx_g - Y_{\dot{r}} \\ 0 & mx_g - N_{\dot{v}} & I_z - N_{\dot{r}} \end{bmatrix} \quad (18)$$

where $M_{23} \neq M_{32}$ (nonsymmetric). Acceleration feedback from only $\dot{u}$ and $\dot{v}$, implies that:

$$K_a = \begin{bmatrix} K_{11} & K_{12} & 0 \\ K_{21} & K_{22} & 0 \\ K_{31} & K_{32} & 0 \end{bmatrix}$$

Hence, the system inertia matrix after acceleration feedback becomes:

$$H = M + K_a \quad (19)$$
$$= \begin{bmatrix} m - X_{\dot{u}} + K_{11} & K_{12} & 0 \\ K_{21} & m - Y_{\dot{v}} + K_{22} & mx_g - Y_{\dot{r}} \\ K_{31} & mx_g - N_{\dot{v}} + K_{32} & I_z - N_{\dot{r}} \end{bmatrix}$$

This gives us some flexibility since the acceleration feedback terms $K_{11}, K_{12}, K_{21}, K_{22}, K_{31}, K_{32}$ can be chosen such that $H = H^T > 0$. Moreover, a symmetric expression independent of hydrodynamic added mass terms is obtained by choosing the gains as:

$$K_a = \begin{bmatrix} K_{11} & K_{12} & 0 \\ K_{21} & K_{22} & 0 \\ K_{31} & K_{32} & 0 \end{bmatrix} = \begin{bmatrix} X_{\dot{u}} + \Delta K_{11} & 0 & 0 \\ 0 & Y_{\dot{v}} + \Delta K_{22} & 0 \\ 0 & N_{\dot{v}} - Y_{\dot{r}} & 0 \end{bmatrix}$$

where ΔK_{11} and ΔK_{22} can be treated as additional design parameters for the mass in the x- and y-directions. The resulting expression is:

$$H = \begin{bmatrix} m + \Delta K_{11} & 0 & 0 \\ 0 & m + \Delta K_{22} & mx_g - Y_{\dot{r}} \\ 0 & mx_g - Y_{\dot{r}} & I_z - N_{\dot{r}} \end{bmatrix} \quad (20)$$

If $\Delta K_{11} = \Delta K_{22}$ (the mass in the x- and y-directions is equal) the PID controller will be independent of the heading angle which, for instance is advantageous when tuning a ship dynamic positioning system.

The resulting model after acceleration feedback is

$$\dot{\eta} = R_{z,\psi}\nu \quad (21)$$

$$H\dot{\nu} + C(\nu)\nu + D(\nu)\nu = \tau_{PID} \quad (22)$$

Notice that H replaces M. The system (21)–(22) satisfies the following properties:

(i) $H = H^T > 0 \Rightarrow x^T H x > 0, \forall x \neq 0$

(ii) $C(\nu) = -C^T(\nu) \Rightarrow x^T C(\nu) x = 0, \forall x$

(iii) $D(\nu) > 0, \forall \|\nu\| > \varepsilon$
$\Rightarrow x^T D(\nu) x > 0, \forall \|\nu\| > \varepsilon, x \neq 0$

(iv) $R_{z,\psi}$ is the rotation matrix in yaw
$\Rightarrow R_{z,\psi}^{-1} = R_{z,\psi}^T$

5.2 Energy shaping using backstepping design

Energy-based control using backstepping design suggests that the control law is derived in two successive steps by using vectorial backstepping (see Fossen and Berge, 1997; Fossen and Grøvlen, 1998). For simplicity, a PD control law will be designed. Integral action can easily be included by using adaptive backstepping (see Fossen et al., 2001).

Consider:

$$V_1 = \frac{1}{2} z_1^T K_p z_1 \quad (23)$$

$$V_2 = V_1 + \frac{1}{2}\nu^T H \nu \quad (24)$$

where V_1 and V_2 represent the "pseudo" potential and kinetic energy, respectively. The state z_1 is defined as the tracking error:

$$z_1 = \eta_d - \eta$$

New State Variables

Assume that the reference trajectory, $\eta_d^{(3)}$, $\ddot{\eta}_d$, $\dot{\eta}_d$ and η_d, is smooth and bounded. A virtual reference trajectory is defined as:

$$\dot{\eta}_r := \dot{\eta}_d - \Lambda\tilde{\eta} \quad (25)$$

$$\nu_r := R_{z,\psi}^T \dot{\eta}_r \quad (26)$$

where $\tilde{\eta} = \eta - \eta_d$ is the tracking error and $\Lambda > 0$ is a diagonal design matrix. Furthermore let:

$$s := \dot{\eta} - \dot{\eta}_r = \dot{\tilde{\eta}} + \Lambda\tilde{\eta} \quad (27)$$

be a surface in $\mathbb{R}^3$. The vessel dynamics (21)–(22) can be transformed to (Fossen, 1993):

$$H^*(\psi)\ddot{\eta} + C^*(\nu,\psi)\dot{\eta} + D^*(\nu,\psi)\dot{\eta} = R_{z,\psi}\tau_{PD}$$

where:

$$H^*(\psi) = R_{z,\psi} H R_{z,\psi}^T$$
$$C^*(\nu,\psi) = R_{z,\psi}[C(\nu) - H R_{z,\psi}^{-1}\dot{R}_{z,\psi}]R_{z,\psi}^T$$
$$D^*(\nu,\psi) = R_{z,\psi} D(\nu) R_{z,\psi}^T$$

Hence, the vessel dynamics can be written in the following form:

$$H^*(\psi)\dot{s} = -C^*(\nu,\psi)s - D^*(\nu,\psi)s + R_{z,\psi}\tau_{PD} - H^*(\psi)\ddot{\eta}_r - C^*(\nu,\psi)\dot{\eta}_r - D^*(\nu,\psi)\dot{\eta}_r$$

or equivalently:

$$H^*(\psi)\dot{s} = -C^*(\nu,\psi)s - D^*(\nu,\psi)s + R_{z,\psi}[\tau_{PD} - H\dot{\nu}_r - C(\nu)\nu_r - D(\nu)\nu_r] \quad (28)$$

Step 1:

Consider the error dynamics:

$$\dot{\eta} - \dot{\eta}_d = R_{z,\psi}(\nu - \nu_d) \quad (29)$$

Let ν be the virtual control vector:

$$\nu := s + \alpha_1 \quad (30)$$

The position error dynamics can therefore be written:

$$\begin{aligned} \dot{\tilde{\eta}} &= R_{z,\psi}(\nu - \nu_d) \\ &= R_{z,\psi}(s + \alpha_1 - \nu_d); \quad \{\alpha_1 = \nu_r = R_{z,\psi}^T(\dot{\eta}_d - \Lambda\tilde{\eta})\} \\ &= R_{z,\psi}(s + R_{z,\psi}^T\dot{\eta}_d - R_{z,\psi}^T\Lambda\tilde{\eta} - \nu_d) \\ &= -\Lambda\tilde{\eta} + R_{z,\psi}s \end{aligned} \quad (31)$$

Hence:

$$V_1 = \frac{1}{2}\tilde{\eta}^T K_p \tilde{\eta}; \quad K_p = K_p^T > 0 \quad (32)$$

and

$$\begin{aligned} \dot{V}_1 &= \tilde{\eta}^T K_p \dot{\tilde{\eta}} \\ &= \tilde{\eta}^T K_p(-\Lambda\tilde{\eta} + R_{z,\psi}s) \\ &= -\tilde{\eta}^T K_p \Lambda \tilde{\eta} + s^T R_{z,\psi}^T K_p \tilde{\eta} \end{aligned} \quad (33)$$

Step 2:

In the second step we choose a Lyapunov function motivated by "pseudo" kinetic energy, that is:

$$V_2 = \frac{1}{2} s^T H^*(\psi) s + V_1 \tag{34}$$

$$\begin{aligned}
\dot{V}_2 &= s^T H^*(\psi)\dot{s} + \frac{1}{2} s^T \dot{H}^*(\psi) s + \dot{V}_1 \\
&= s^T \Big[-C^*(\nu,\psi)s - D^*(\nu,\psi)s \\
&\quad + R_{z,\psi}[\tau_{PD} - H\dot{\nu}_r - C(\nu)\nu_r - D(\nu)\nu_r]\Big) \\
&\quad + \frac{1}{2} s^T \dot{H}^*(\psi) s - \tilde{\eta}^T K_p \Lambda \tilde{\eta} + s^T R^T_{z,\psi} K_p \tilde{\eta}
\end{aligned}$$

Since $\dot{H}^*(\psi) = \dot{M}^*(\psi)$ (recall that $\dot{K}_a = 0$); we can use the skew-symmetric property:

$$s^T[\dot{H}^*(\psi) - 2C^*(\nu,\psi)]s = 0 \tag{35}$$

which yields:

$$\begin{aligned}
\dot{V}_2 &= s^T R_{z,\psi}[\tau_{PD} - H\dot{\nu}_r - C(\nu)\nu_r - D(\nu)\nu_r \\
&\quad + R^T_{z,\psi} K_p \tilde{\eta}] - s^T D^*(\nu,\psi) s - \tilde{\eta}^T K_p \Lambda \tilde{\eta}
\end{aligned}$$

Hence, we are ready to propose the control law:

$$\begin{aligned}
\tau_{PD} &= H\dot{\nu}_r + C(\nu)\nu_r + D(\nu)\nu_r \\
&\quad - R^T_{z,\psi}(K_p \tilde{\eta} + K_d s)
\end{aligned} \tag{36}$$

which results in:

$$\dot{V}_2 = -s^T(D^*(\nu,\psi) + K_d)s - \tilde{\eta}^T K_p \Lambda \tilde{\eta}$$

where the gain matrix K_d must be selected such that $D^*(\nu,\psi) + K_d > 0 \ \forall \nu,\psi$. Since V_2 is positive definite and $\dot{V}_2$ is negative definite it follows that the equilibrium point $(\tilde{\eta}, s) = (0, 0)$ is uniformly globally exponentially stable (UGES). Moreover, convergence of $s \to 0$ and $\tilde{\eta} \to 0 \Rightarrow \tilde{\nu} \to 0$.

The resulting control law including acceleration feedback then becomes:

$$\tau = \tau_{PD} - K_a \dot{\nu}$$

For backstepping techniques with integral action, see Fossen et al. (2001). For the integral controller only uniform global asymtotic stability (UGAS) is proven.

6. CONCLUSIONS

Acceleration feedback in conjuncture with nonlinear PID-control has been applied to control marine vessels with nonsymmetric inertia matrices. This is an important design problem since hydrodynamic added inertia is nonsymmetric for marine vessels moving at forward speed in waves. The proposed solution has been referred to as inertia shaping using acceleration feedback, and to the authors knowledge this is the first paper addressing this problem. An energy based Lyapunov design technique has been used to prove that the resulting system is UGAS under PID-control and UGES under PD-control.

REFERENCES

Fossen, T. I. (1993). Comments on "Hamiltonian Adaptive Control of Spacecraft". IEEE Transactions on Automatic Control TAC-38(4), 671–672.

Fossen, T. I. (1994). Guidance and Control of Ocean Vehicles. John Wiley and Sons Ltd.

Fossen, T. I., A. Loria and A. Teel (2001). A Theorem for UGAS and ULES of (Passive) Nonautonomous Systems: Robust Control of Mechanical Systems and Ships. International Journal of Robust and Nonlinear Control JRNC-11, 95–108.

Fossen, T. I. and Å. Grøvlen (1998). Nonlinear Output Feedback Control of Dynamically Positioned Ships Using Vectorial Observer Backstepping. IEEE Transactions on Control Systems Technology TCST-6(1), 121–128.

Fossen, T. I. and J. P. Strand (1999). A Tutorial on Nonlinear Backsteping: Applications to Ship Control. Modelling, Identification and Control MIC-20(2), 83–135. Tutorial workshop presented at IFAC CAMS'98.

Fossen, T. I. and J. P. Strand (2001). Nonlinear Passive Weather Optimal Positioning Control (WOPC) System for Ships and Rigs: Experimental Results. Automatica (Regular Paper) AUT-37(5), 701–715.

Fossen, T. I. and S. P. Berge (1997). Nonlinear Vectorial Backstepping Design for Global Exponential Tracking of Marine Vessels in the Presence of Actuator Dynamics. In: Proceedings of IEEE Conf. on Decission and Control (CDC'97). San Diego, CA. pp. 4237–4242.

Lamb, H. (1932). Hydrodynamics. Cambridge University Press. London.

Lindegaard, Karl-Petter (2002). Optimality and Nonlinear Control of Dynamically Positioned Vessels. PhD thesis. Department of Engineering Cybernetics, Norwegian University of Science and Technology. Trondheim. In progress.

Newman, J. N. (1977). Marine Hydrodynamics. MIT Press. Cambridge, MA.

Salvesen, N., E. O. Tuck and O. M. Faltinsen (1970). Ship Motions and Sea Loads. Trans. SNAME, vol. 78, pp. 250–287.

SNAME (1950). The Society of Naval Architects and Marine Engineers. Nomenclature for Treating the Motion of a Submerged Body Through a Fluid. In: Technical and Research Bulletin No. 1-5.

Wendel, K. (1956). Hydrodynamic Masses and Hydrodynamic Moments of Inertia. Technical report. TMB Translation 260.