15th Triennial World Congress, Barcelona, Spain

DECENTRALIZED HIERARCHICAL STOCHASTIC CONTROL IN A LARGE SCALE STATIC SYSTEM

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Abstract: In the paper a synthesis of control law for a large scale stochastic system is presented. The large scale system composed of M linear static subsystems with an iteraction and quadratic performance index are considered. A two-level hierarchical control structure is assumed, in which a coordinator and local controllers have access to different information. A suboptimal algorithm, in which it is possible to partially decompose calculations and to realize decentralized control, is proposed. A simple example is presented. *Copyright @ 2002 IFAC*

Keywords: large scale system, stochastic system, decentralized control, optimal control

1. INTRODUCTION

This paper deals with a control design for a large scale stochastic system composed of static coupled subsystems and quadratic performance index, which should be minimized. It is obvious that quality of control depends on assumed information and control structures. In a one level structure a central decision maker determines values of control on the basis of available information collected from all subsystems. However in a large scale systems a process of transmission and transformation of information in a centralized manner can be difficult to realize. It leads to decentralization of information and control structures.

Control problems with decentralized measurement information are studied in a team decision theory, as well as in the hierarchical control (Aoki, 1973; Chong and Athans, 1971; Ho, 1980). The problems may be complicated, especially in the case of so called nonclassical information pattern, in which controllers do not have identical information. In (Witsenhausen, 1978) it is shown that a linear quadratic gaussian case is nontrivial when the information pattern is nonclassical.

Control and optimization for large scale systems are usually based on a decomposition of a global system into subsystems so as to decrease computational requirements and decrease amount of information to be transmitted to and processed by decision makers. A conflict between local controllers is softened by the coordinator on the upper level.

Decomposition and coordination methods have been developed for large scale systems. Studies on decomposition methods can be found e.g. in (Findeisen *et al.*, 1980; Lasdon, 1970; Mesarovic *et al.*, 1970).

In the present paper, a stochastic optimal control problem for a system composed of static linear subsystems interacting by means of output variables is considered. Control is realized in a two-level structure with a coordinator on the upper level and local controllers on the lower level. It is assumed that the local controllers have essential information for their subsystems, while the coordinator has aggregated information on the whole system.

[1] Supported by the Science Research Committee under Grant 8 T11A 012 19

Similar problems are formulated and solved e.g. in (Duda and Gessing, 1992; Gessing and Duda, 1995; Gessing, 1987).

The primary problem statement was discussed in (Gessing and Duda, 1990), where so called elastic constraint was introduced. The two-fold interpretation of the control variable was utilized during the derivation of the control laws. The same control variable was treated as the decision variable for the local controller and as a random variable for the coordinator. Owing to this the solution of the problem had an analytical, linear form. The present paper differs in the solution of the problem, which leads to the algorithm presented in (Gessing and Duda, 1990). The elastic constraint and the two-fold interpretation of the control varaiable are not required.

2. MODEL OF THE SYSTEM

Consider the large scale system composed of M static subsystems described by the equation

$$x_i = B_{ii}^* u_i + \sum_{\substack{j=1 \\ j \neq i}}^{M} A_{ij} x_j + w_i^* \quad (1)$$

where $x_i,\ u_i,\ w_i^*$ denote realizations of an output, control and disturbance vector variables, respectively, of the ith subsystem; B_{ii}^* and $A_{ij},\ i,j = 1,2,...,M$ are appropriate matrices. The sum appearing in (1) will be denoted by $\sum_{j \neq i}$.

The model of the measurements has the form

$$\mathbf{y_i} = \phi_i(\mathbf{w_i^*}, \mathbf{e_i}) \quad (2)$$

where $\mathbf{y_i}$ and $\mathbf{e_i}$ are the vectors of the measurements and measurement errors. It is assumed that $\mathbf{w_i^*}$ and $\mathbf{e_i}$ are random variables with given probability distribution functions and independent of $\mathbf{w_j},\ \mathbf{e_j},\ i \neq j,\ i,j = 1,2,...,M$; $\phi_i(.),\ i = 1,2,...,M$ is a given function. The realization of the random variable $\mathbf{y_i}$ will be denoted by y_i.

The performance index of the whole system has the form

$$I = E[\sum_{i=1}^{M} (\mathbf{x_i^T} Q_i \mathbf{x_i} + \mathbf{u_i^T} H_i \mathbf{u_i})_{\mathbf{u_i} = a_i(\mathbf{z_i})}] \quad (3)$$

where E denotes mean operation, $\mathbf{x_i}$ is a random variable with realizations described by (1) and $a_i(\mathbf{z_i})$ is a control law for the ith subsystem with an argument $\mathbf{z_i}$.

The argument $\mathbf{z_i}$ represents an available information, which will be defined later.

The realization of the control u_i results from the relation $u_i = a_i(z_i)$.

The problem is to design optimal control laws $\mathbf{u_i} = a_i(\mathbf{z_i}),\ i = 1,2,...,M$ for which the performance index (3) takes a minimal value under constraint (1) written in the form

$$\mathbf{x_i} = B_{ii}^* \mathbf{u_i} + \sum_{\substack{j=1 \\ j \neq i}}^{M} A_{ij} \mathbf{x_j} + \mathbf{w_i^*} \quad (4)$$

3. PROBLEM FORMULATION

The complexity and the effectiveness of a solution depends on assumed information and control structures.

Consider the control realized in a two level hierarchical structure with the coordinator on the upper level and the local controllers on the lower one. Proposed structure is justified for large scale distributed system (large M), in which transmission of information $y_i,\ i = 1,2,...,M$, to one central controller is difficult to realize.

Let us assume that the ith local controller receives from the appropriate subsystem the measurement y_i which is aggregated to the form

$$m_i = D_i y_i \quad (5)$$

where m_i is the vector of lower dimension than y_i; D_i is an appropriate matrix.

The coordinator collects the transformed measurement m_i from all local controllers and in return transmits to them the values of coordinating variables p_i. The ith local controller transfers the decision u_i to its subsystem.

Two kinds of information available for decision makers are considered.

A priori information for the problem consists of the model (2), (4), (3), as well as of the appropriate probability distribution functions.

The measurement y_i with the coordinating variable p_i and information defined by $m = [m_1^T, ..., m_M^T]^T$ represent *a posteriori* information of the ith local controller and the coordinator, respectively.

Owing to low dimension of the vector $m_i,\ i = 1,2,...,M$, the amount of information transmitted and converted by the coordinator may be decreased.

By the admissible control laws of the coordinator and the ith local controller are meant the functions $\mathbf{p_i} = b_i(\mathbf{m})$ and $\mathbf{u_i} = a_i[\mathbf{y_i}, b_i(\mathbf{m})],\ i = 1,2,...,M$, respectively. For the realization of the random variables $\mathbf{m}$ and $\mathbf{y_i}$ the realized controls determined by the coordinator and the local controllers take values $p_i = b_i(m)$ and $u_i = a_i(y_i, p_i)$.

For the system considered with the assumed control and information structures, among the admissible control laws, the optimal control laws $\mathbf{u_i} = a_i^o[\mathbf{y_i}, b_i^o(\mathbf{m})]$ and $\mathbf{p_i} = b_i^o(\mathbf{m}),\ \ i = 1, 2, ..., M$ are to be found for which the performance index (3) under constraint (4) is minimized.

4. SOLUTION TO THE PROBLEM

Denote

$$\mathbf{v_i} = \sum_{\substack{j=1 \\ j \neq i}}^{M} A_{ij}\mathbf{x_j} \tag{6}$$

In the course of the synthesis of the control laws $\mathbf{u_i} = a_i(\mathbf{y_i}, \mathbf{p_i})$ and $\mathbf{p_i} = b_i(\mathbf{m})$ let us substitute (4) with (6) into (3).

After performing some transformations the performance index (3) takes the form

$$I = E\{\sum_{i=1}^{M}[\mathbf{u_i^T}V_i\mathbf{u_i} + 2(\mathbf{v_i} + \mathbf{w_i^*})^T Q_i B_{ii}^*\mathbf{u_i} + \\ +\mathbf{v_i}^T Q_i\mathbf{v_i} + 2\mathbf{v_i^T}Q_i\mathbf{w_i^*} + \mathbf{w_i^{*T}}Q_i\mathbf{w_i^*}]\} \tag{7}$$

where $\mathbf{u_i} = a_i[\mathbf{y_i}, b_i(\mathbf{m})]$, $V_i = B_{ii}^{*T}Q_iB_{ii}^* + H_i$.

The problem is to find optimal control laws $\mathbf{u_i} = a_i^o[\mathbf{y_i}, b_i^o(\mathbf{m})]$ and $\mathbf{p_i} = b_i^o(\mathbf{m}),\ \ i = 1, 2, ..., M$, which minimize the performance index (7).

4.1 *Synthesis of local control laws*

Because of assumed available information for local controlers, the optimal control laws $b_i^o(\mathbf{m})$ and $a_i^o[\mathbf{y_i}, b_i^o(\mathbf{m})]$ can not be effectively solved by minimization of the performance index (7). Then a suboptimal solution is proposed.

In the course of the synthesis of the control laws $\mathbf{u_i} = a_i(\mathbf{y_i}, \mathbf{p_i})$ let us assume that the ith subsystem is described by the equation

$$\mathbf{x_i^*} = B_{ii}^*\mathbf{u_i} + \mathbf{v_i^*} + \mathbf{w_i^*} \tag{8}$$

where

$$\mathbf{v_i^*} = E_{|\mathbf{m}}\mathbf{v_i} \tag{9}$$

The realization of the random variable $\mathbf{v_i^*}$ denoted by v_i^* results from (9) and has the form

$$v_i^* = E_{|m}\mathbf{v_i} \tag{10}$$

where $E_{|m}$ denotes the conditional mean, given m.

If the variable $\mathbf{v_i}$ has gaussian distribution then the value of the variable v_i^* is the best estimate of the interaction, based on the information of the coordinator.

Then the performance index results from (7) and has the form

$$I^* = E\{\sum_{i=1}^{M}[\mathbf{u_i^T}V_i\mathbf{u_i} + 2(\mathbf{v_i^*} + \mathbf{w_i^*})^T Q_i B_{ii}^*\mathbf{u_i} + \\ +\mathbf{v_i^{*T}}Q_i\mathbf{v_i^*} + 2\mathbf{v_i^{*T}}Q_i\mathbf{w_i^*} + \mathbf{w_i^{*T}}Q_i\mathbf{w_i^*}]\} \tag{11}$$

where $\mathbf{u_i} = a_i[\mathbf{y_i}, b_i(\mathbf{m})]$.

The performance index (11) may be written in the form

$$I^* = E\{E_{|\mathbf{m}}\{\sum_{i=1}^{M}[\mathbf{u_i^T}V_i\mathbf{ui} + 2(\mathbf{v_i^*} + \mathbf{w_i^*})^{\mathbf{T}} Q_i B_{ii}^*\mathbf{u_i} + \\ +\mathbf{v_i^{*T}}Q_i\mathbf{v_i^*} + 2\mathbf{v_i^{*T}}Q_i\mathbf{w_i^*} + \mathbf{w_i^{*T}}Q_i\mathbf{w_i^*}]\}\} \tag{12}$$

which results from the relation $Eg(\mathbf{x}, \mathbf{y}) = EE_{|\mathbf{y}}g(\mathbf{x}, \mathbf{y}) = EE_{|\mathbf{x}}g(\mathbf{x}, \mathbf{y})$ relating to random variables $\mathbf{x}, \mathbf{y}$ and random function $g(\mathbf{x}, \mathbf{y})$.

It is easy to show that the optimal control laws $p_i = b_i^o(m)$ and $a_i^o[\mathbf{y_i}, b_i(m)], i = 1, 2, .., M$ can be found by minimization in the expression

$$\bar{I}^* = \min_{p,\mathbf{u}} E_{|m}\{\sum_{i=1}^{M}[\mathbf{u_i^T}V_i\mathbf{u_i} + 2(v_i^* + \mathbf{w_i^*})^T Q_i B_{ii}^*\mathbf{u_i} + \\ +v_i^{*T}Q_iv_i^* + 2v_i^{*T}Q_i\mathbf{w_i^*} + \mathbf{w_i^{*T}}Q_i\mathbf{w_i^*}]\} \tag{13}$$

where $\mathbf{u_i} = a_i(\mathbf{y_i}, p_i)$, $\mathbf{u} = [\mathbf{u_1^T}, ..., \mathbf{u_M^T}]^T$, $p = [p_1^T, .., p_M^T]^T$

The constraint (10) is taken into account by using the method of Lagrange multipliers.

Then the minimization problem may be transformed to the form

$$\bar{I}^{**} = min_{p,\mathbf{u}}E_{|m}\{\sum_{i=1}^{M}[\mathbf{u_i^T}V_i\mathbf{u_i} + v_i^{*T}Q_iv_i^* + \\ +2(v_i^* + \mathbf{w_i^*})^T Q_i B_{ii}^*\mathbf{u_i} + 2v_i^{*T}Q_i\mathbf{w_i^*} + \\ +\mathbf{w_i^{*T}}Q_i\mathbf{w_i^*} + 2l_i^T(v_i^* - \sum_{j\neq i} A_{ij}\mathbf{x_j^*})]\} \tag{14}$$

Consequently

$$\bar{I}^{**} = min_{p,\mathbf{u}}E_{|m}\{\sum_{i=1}^{M}[\mathbf{u_i^T}V_i\mathbf{u_i} + v_i^{*T}Q_iv_i^* + \\ +2(v_i^* + \mathbf{w_i^*})^{\mathbf{T}} Q_i B_{ii}^*\mathbf{u_i} + 2v_i^{*T}Q_i\mathbf{w_i^*} + \\ +\mathbf{w_i^{*T}}Q_i\mathbf{w_i^*} + 2l_i^T v_i^* - 2\sum_{j\neq i} l_j^T A_{ji}\mathbf{x_i^*}]\} = \\ = min_{p,\mathbf{u}}E_{|m}\{\sum_{i=1}^{M}[\mathbf{u_i^T}V_i\mathbf{u_i} + 2(v_i^{*T}Q_iB_{ii}^* + \\ +\mathbf{w_i^{*T}}Q_iB_{ii}^* - \sum_{j\neq i} l_j^T A_{ji}B_{ii}^*)\mathbf{u_i} + v_i^{*T}Q_iv_i^* +$$

$$+2v_i^{*T}Q_i\mathbf{w_i^*}+\mathbf{w_i^{*T}}Q_i\mathbf{w_i^*}+2l_i^Tv_i^*-$$
$$-2\sum_{j\neq i}l_j^TA_{ji}(v_i^*+\mathbf{w_i^*})\} \qquad (15)$$

From (15) it results that the local control law $a_i(\mathbf{y_i},p_i)$ can be found from local minimization

$$\bar{I}^{i**}=\min_{\mathbf{u_i}}E_{|m}[\mathbf{u_i^T}V_i\mathbf{u_i}+2(v_i^{*T}Q_iB_{ii}^*+$$
$$+\mathbf{w_i^{*T}}Q_iB_{ii}^*-\sum_{j\neq i}l_j^TA_{ji}B_{ii}^*)\mathbf{u_i}+v_i^{*T}Q_iv_i^*+$$
$$+2v_i^{*T}Q_i\mathbf{w_i^*}+\mathbf{w_i^{*T}}Q_i\mathbf{w_i^*}+2l_i^Tv_i^*-$$
$$-2\sum_{j\neq i}l_j^TA_{ji}(v_i^*+\mathbf{w_i^*})] \qquad (16)$$

where $l=[l_1^T,...,l_M^T]^T$ and v_i^* are treated as parameters.

It is easy to show that the optimal control law $a_i(\mathbf{y_i},p_i)$ can be found by the minimization in the expression

$$S^{i**}=\min_{u_i}E_{|m,y_i}[u_i^TV_iu_i+2(v_i^{*T}Q_iB_{ii}^*+$$
$$+\mathbf{w_i^{*T}}Q_iB_{ii}^*-\sum_{j\neq i}l_j^TA_{ji}B_{ii}^*)u_i+v_i^{*T}Q_iv_i^*+$$
$$+2v_i^{*T}Q_i\mathbf{w_i^*}+\mathbf{w_i^{*T}}Q_i\mathbf{w_i^*}+$$
$$+2l_i^Tv_i^*-2\sum_{j\neq i}l_j^TA_{ji}(v_i^*+\mathbf{w_i^*})] \qquad (17)$$

Let us notice that the minimization with respect to the function $\mathbf{u_i}=a_i(\mathbf{y_i},p_i)$ in (16) is replaced by the minimization with respect to the variable u_i.

After performing the $E_{|m,y_i}$ operation in (17) the expression takes the form

$$S^{i**}=\min_{u_i}[u_i^TV_iu_i+2(v_i^{*T}Q_iB_{ii}^*+\hat{w}_i^{*T}Q_iB_{ii}^*-$$
$$-\sum_{j\neq i}l_j^TA_{ji}B_{ii}^*)u_i+v_i^{*T}Q_iv_i^*+2v_i^{*T}Q_i\hat{w}_i^*+$$
$$+2l_i^Tv_i^*-2\sum_{j\neq i}l_j^TA_{ji}(v_i^*+\hat{w}_i^*)]+$$
$$+E_{|y_i}\mathbf{w_i^{*T}}Q_i\mathbf{w_i^*} \qquad (18)$$

where $\hat{w}_i^*=E_{|m,y_i}\mathbf{w_i^*}=E_{|y_i}\mathbf{w_i^*}$, which results from assumed properties of the random variables $\mathbf{w_i^*}$ and $\mathbf{e_i}$.

After differentiating in (18) with respect to u_i and equating to zero, the control law takes the form

$$u_i^o=V_i^{-1}[\sum_{j\neq i}B_{ii}^{*T}A_{ji}^Tl_j-B_{ii}^{*T}Q_i(\hat{w}_i^*+v_i^*)] \qquad (19)$$

Denote by

$$p_i=E_{|m}\{V_i^{-1}[\sum_{j\neq i}B_{ii}^{*T}A_{ji}^T\mathbf{l_j}-B_{ii}^{*T}Q_i(\mathbf{\hat{w}_i^*}+\mathbf{v_i^*})]\}$$

After performing mean operation given m, the above equation can be written in the form

$$p_i=V_i^{-1}[\sum_{j\neq i}B_{ii}^{*T}A_{ji}^Tl_j-B_{ii}^{*T}Q_i(\bar{w}_i^*+v_i^*)] \qquad (20)$$

where $\bar{w}_i^*=E_{|m}\mathbf{w_i^*}=E_{|m_i}\mathbf{w_i^*}$.

From (19) and (20) it results that

$$u_i^o=p_i-V_i^{-1}B_{ii}^{*T}Q_i(\hat{w}_i^*-\bar{w}_i^*) \qquad (21)$$

The value of u_i^o is the realization of control determined by the ith controller for given p_i (transmitted from the coordinator) and given y_i necessary for determination of the estimates $\hat{w}_i^*$ and $\bar{w}_i^*$.

4.2 *Synthesis of optimal control laws for the coordinator*

In the course of the synthesis of the control laws $b_i(\mathbf{m})$ for the coordinator the equation (4) will be taken into account.

Denote

$$\mathbf{x}=[\mathbf{x_1^T}\ \ \mathbf{x_2^T}..\ \ \mathbf{x_M^T}]^T,\quad \mathbf{u^o}=[\mathbf{u_1^{oT}}\ \ \mathbf{u_2^{oT}}..\ \ \mathbf{u_M^{oT}}]^T,$$
$$\mathbf{p}=[\mathbf{p_1^T}\ \ \mathbf{p_2^T}..\ \ \mathbf{p_M^T}]^T,\quad \mathbf{w}^*=[\mathbf{w_1^T}\ \ \mathbf{w_2^T}..\ \ \mathbf{w_M^T}]^T,$$
$$Q_d=diag[Q_1...Q_M],\quad H_d=diag[H_1...H_M],$$
$$V_d^{-1}=diag[V_1^{-1}...V_M^{-1}],\quad B_d=diag[B_{11}^*...B_{MM}^*],$$

and

$$B^*=\mathbf{1}-\begin{bmatrix}\mathbf{0_1} & A_{12} & ....A_{1M}\\ A_{21} & \mathbf{0_2} & ....A_{2M}\\ .... & .... & .....\\ A_{M1} & .... & ...\mathbf{0_M}\end{bmatrix} \qquad (22)$$

where $\mathbf{1}$ is a unit matrix and $\mathbf{0}_i,\ \ i=1,2,...,M$ are null matrices with appropriate dimensions.

Then (21), (3) and (4) may be written in the form

$$\mathbf{u^o}=\mathbf{p}-V_d^{-1}B_d^TQ_d(\mathbf{\hat{w}}-\mathbf{\bar{w}}) \qquad (23)$$

$$I=E[(\mathbf{x^T}Q_d\mathbf{x}+\mathbf{u^{oT}}H_d\mathbf{u^o})] \qquad (24)$$

$$\mathbf{x}=B\mathbf{u^o}+\mathbf{w} \qquad (25)$$

where

$$B=(B^*)^{-1}B_d,\quad \mathbf{w}=(B^*)^{-1}\mathbf{w}^* \qquad (26)$$

After substituting (23) and (25) into (24) the performance index takes the form

$$I = E[(\mathbf{p^T}V\mathbf{p} + 2\mathbf{p^T}B^TQ_d\bar{\mathbf{w}})_{\mathbf{p}=b(\mathbf{m})}] + s \quad (27)$$

where $V = H_d + B^TQ_dB$ and

$$s = E[(\hat{\mathbf{w}} - \bar{\mathbf{w}})^TQ_dB_dV_d^{-1}VV_d^{-1}B_d^TQ_d(\hat{\mathbf{w}} - \bar{\mathbf{w}}) + \\ + \mathbf{w^T}Q_d\mathbf{w} - 2(\hat{\mathbf{w}} - \bar{\mathbf{w}})^TQ_dB_dV_d^{-1}B^TQ_d\mathbf{w}] \quad (28)$$

The problem of the coordinator is to determine the optimal control laws $b^o(\mathbf{m})$ which minimize the performance index (27) written in the form

$$I = E\{E_{|\mathbf{m}}[(\mathbf{p^T}V\mathbf{p} + 2\mathbf{p^T}B^TQ_d\bar{\mathbf{w}})_{\mathbf{p}=b(\mathbf{m})}]\} + s \quad (29)$$

It can be transformed to the minimization of the expression (for given m)

$$S = p^TVp + 2p^TB^TQ_d\bar{w} \quad (30)$$

with respect to the variable p.

After differentiating in (30) with respect to p and equating to zero, the optimal control law takes the form

$$p^o = -V^{-1}B^TQ_d\bar{w} = -V^{-1}B^TQ_d(B^*)^{-1}\bar{w}^* \quad (31)$$

The value of p_i^o is the realization of control determined by the coordinator and transmitted to the ith local controller.

After substituting (31) into (27), the perfomance index (3) takes the value

$$I^o = s - E(\bar{\mathbf{w}}^\mathbf{T}Q_dBV^{-1}B^TQ_d\bar{\mathbf{w}}) \quad (32)$$

5. EXAMPLE

Let us consider a simple system composed of two subsystems and described by the equations

$$\mathbf{x_1} = B_{11}^*\mathbf{u_1} + A_{12}\mathbf{x_2} + \mathbf{w_1^*} \\ \mathbf{x_2} = B_{22}^*\mathbf{u_2} + A_{21}\mathbf{x_1} + \mathbf{w_2^*} \quad (33)$$

$$\mathbf{y_1} = C_1\mathbf{w_1^*} + \mathbf{e_1} \\ \mathbf{y_2} = C_2\mathbf{w_2^*} + \mathbf{e_2} \quad (34)$$

and

$$\mathbf{m_1} = D_1\mathbf{y_1} \quad (35)$$

for which

$$B_{11}^T = \begin{bmatrix} 2 & 1 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}, \\ B_{22}^T = \begin{bmatrix} 3 & 1 \end{bmatrix}, \quad A_{21} = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}, \quad (36)$$

$$C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \\ D_1 = \begin{bmatrix} 1 & 1 \end{bmatrix}, \quad (37)$$

It is assumed that no information is transmitted from the second subsystem to the coordinator.

The disturbances $\mathbf{w_1^*}, \mathbf{w_2^*}, \mathbf{e_1}$ and $\mathbf{e_2}$ have gaussian distributions defined by

$$E\mathbf{w_1^*} = \begin{bmatrix} 1 & 2 \end{bmatrix}^T, \quad E\mathbf{w_2^*} = \begin{bmatrix} 1 & 1 \end{bmatrix}^T, \\ P_{\mathbf{w_1^*}} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}, \quad P_{\mathbf{w_2^*}} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}, \quad (38)$$

$$E\mathbf{e_1} = \begin{bmatrix} 1 & 1 \end{bmatrix}^T, \quad E\mathbf{e_2} = \begin{bmatrix} 1 & 0 \end{bmatrix}^T, \\ P_{\mathbf{e_1}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad P_{\mathbf{e_2}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad (39)$$

The performance index has the form

$$I = E\sum_{i=1}^{2}(\mathbf{x_i^T}Q_i\mathbf{x_i} + \mathbf{u_i^T}H_i\mathbf{u_i}) \quad (40)$$

where

$$Q_1 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \quad H_1 = \begin{bmatrix} 1 \end{bmatrix}, \\ Q_2 = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}, \quad H_2 = \begin{bmatrix} 2 \end{bmatrix}, \quad (41)$$

The optimal control laws of the local controllers in accordance with (21) have the form $u_i = p_i + K_i(\hat{w}_i^* - \bar{w}_i^*)$ where

$$K_1 = \begin{bmatrix} -0.5 & 0.5 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -0.26 & -0.15 \end{bmatrix} \quad (42)$$

The optimal control law of the coordinator in accordance with (31) has the form $\mathbf{p} = K\bar{\mathbf{w}}^*$ where

$$K = \begin{bmatrix} -0.39 & -0.03 & 0.02 & -0.24 \\ -0.10 & 0.10 & -0.11 & 0.17 \end{bmatrix} \quad (43)$$

The estimates $\hat{w}_i^*, \quad i = 1, 2$, can be determined by using the conventional formulae

$$\hat{w}_i^* = E\mathbf{w_i^*} + P_{\mathbf{w_i^*y_i}}P_{\mathbf{y_iy_i}}^{-1}(y_i - E\mathbf{y_i}) \quad (44)$$

where $P_{\mathbf{w_i^*y_i}} = E(\mathbf{w_i^*} - E\mathbf{w_i^*})(\mathbf{y_i} - E\mathbf{y_i})^T$, $P_{\mathbf{y_iy_i}} = E(\mathbf{y_i} - E\mathbf{y_i})(\mathbf{y_i} - E\mathbf{y_i})^T$.

According to (44), the estimates take the form

$$\hat{w}_1^* = \begin{bmatrix} -0.8 \\ 0.4 \end{bmatrix} + \begin{bmatrix} 0.6 & 0.2 \\ 0.2 & 0.4 \end{bmatrix} y_1 \quad (45)$$

$$\hat{w}_2^* = \begin{bmatrix} -0.4 \\ 0.2 \end{bmatrix} + \begin{bmatrix} 0.6 & 0.2 \\ 0.2 & 0.4 \end{bmatrix} y_2 \quad (46)$$

The estimate $\bar{w}_2^* = E\mathbf{w_2^*}$ and the estimate $\bar{w}_1^*$ can be determined by using the formulae

$$\bar{w}_1^* = E\mathbf{w_1^*} + P_{\mathbf{w_1^*m_1}} P^{-1}_{\mathbf{m_1m_1}} (m_1 - E\mathbf{m_1}) \quad (47)$$

According to (47) the estimate has the form

$$\bar{w}_1^* = \begin{bmatrix} -1.14 \\ 0.57 \end{bmatrix} + \begin{bmatrix} 0.43 \\ 0.29 \end{bmatrix} m_1 \quad (48)$$

The estimate $\bar{w}$ results from (26) and has the form

$$\bar{w} = (B^*)^{-1} \begin{bmatrix} \bar{w}_1^* \\ \bar{w}_2^* \end{bmatrix} \quad (49)$$

where $\bar{w}_1^*$ results from (48) and $\bar{w}_2^* = E\mathbf{w_2^*}$.

Then

$$\bar{w} = \begin{bmatrix} -1.36 \\ 0.17 \\ -0.02 \\ -0.19 \end{bmatrix} + \begin{bmatrix} 0.07 \\ -0.17 \\ -0.26 \\ -0.0.09 \end{bmatrix} m_1 \quad (50)$$

6. CONCLUSIONS

In the paper is presented synthesis of control laws to the stochastic system composed of coupled static subsystems. Designed strategy is realized in a two-level structure. The local controllers have decentralized *a priori* information and decentralized measurements. It is shown that local control laws are linear functions of disturbance estimates and can be realized in decentralized way completely.

The coordinator collects an aggregated information from local subsystems and determines some "directions" to local controllers. The control law is linear function of disturbance estimate . Owing to aggregation, amount of information transmitted and transformed by decision makers can be decreased.

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