A SIMULATION AND ANIMATION TOOL FOR STUDYING MULTIVARIABLE CONTROL

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Abstract: A simulation and animation tool for education in multivariable control is presented. The purpose of the tool is to support studies of various aspects of multivariable dynamical systems and design of multivariable feedback control systems. Different ways to use this kind of tool in control education are also presented and discussed.

Keywords: Education, Multivariable control, Simulation

1. INTRODUCTION

The overall goal in control engineering is to design control systems for real world processes (upper left plot in Figure 1). To enable a systematic approach to the control system design and to reduce the needs for experiments with the real process a mathematical model (upper right plot in Figure 1) of the process is needed. The mathematical model is an abstraction of the real world process and it can sometimes, in particular in control education situations, be difficult to get an intuitive understanding of the properties of the process by only using the model. Some properties can be investigated by computing e.g. poles or frequency functions, at least for SISO system. More insight into the system properties are gained by simulating (lower left plot in Figure 1) the system and plotting input and output signals. For MIMO systems this way of working also has limitations since the model can contain a large number of signals, whose relationships are not always obvious. In such a case an animation (lower right plot in Figure 1) of the physical process can be of great value.
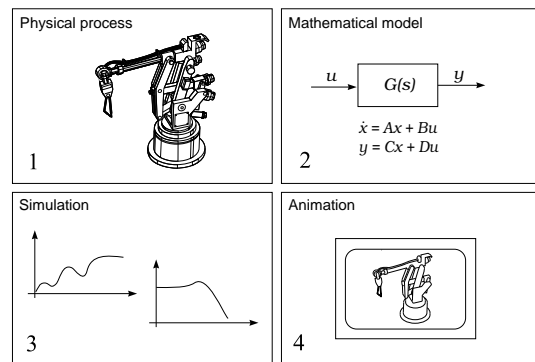

Fig. 1. Four views of the control problem.

This paper, which is based on the work in (Andersson, 2000), deals with the use of animation for studying multivariable control systems. Animation is common tool in many areas ranging from industrial use to entertainment, and several very sophisticated animations have been produced. Within the area of control education examples of the use of animation can be found in e.g. (Rios, 1999). Further examples are given in (Andersson, 2000).

The aim of this paper is to discuss some of the factors that affect the learning process in studies of multivariable control systems. These factors include the knowledge and skills that the students acquire, but also the resources that are needed

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to maintain the learning process. The solution that is presented in this paper is based on three fundamental components:

- A model of a multivariable dynamical system that makes it possible to illustrate a large number of properties of multivariable control systems.
- An animation tool that is as simple as possible in order to keep the development efforts at a reasonable level, but serves the purpose of giving a visual description of the behavior of the process.
- An educational format that is inspiring for the students and encourages independent work of the students. It is desirable that the educational format gives intuitive understanding of the different phenomena as well as hands on skills in computer aided control system design.

The paper is organized as follows. In Section 2 the model used in the simulations and animations is described. Section 3 contains a description of how the simulation model and the animation was implemented in Matlab. In Section 4 it is presented how the model has been used for studying multivariable control. Sections 5 and 6 present two examples of experiments that can be carried out using the model and the animation. Finally Section 7 contains some conclusions.

2. SIMULATION MODEL

The model used in this work is taken from (Johansson, 1999), where the original physical laboratory process is described. A schematic view of the process is given in Figure 2. The process consists of four tanks according to the figure. The system contains two pumps that pump water into the tanks. The flows into the tanks are affected by two valves, and by changing the setting of these valves the amount of flow into the tanks on the upper level can be chosen. As shown in (Johansson, 1999) different valve settings will give the overall system entirely different behavior from a multivariable control viewpoint. The tank process and its use in education is also discussed in (Johansson *et al.*, 1999).

Even though the tank process is available and can be purchased as a physical process it was here decided to use a simulation model and an animation of the process. Keeping in mind that a simulation model always is a simplification of the real world the solution with a simulation model and an animation has several advantages. Since all students have access to Matlab the simulation model and the animation can be reproduced without any extra cost. The students can hence
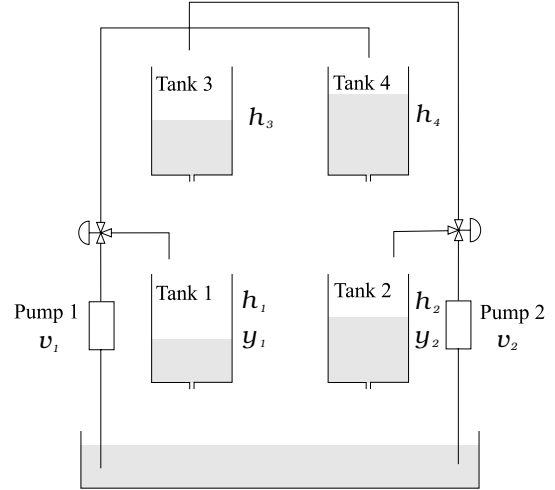

Fig. 2. Schematic view of the tank process.

work with the system independently of time and location. While the time constants of the physical process are very long, the time for simulation and animation of the process is essentially shorter. A further advantage with using the simulated and animated process is that difficulties that are present in real life, like measurement disturbances, nonlinear actuators, etc, can be introduced one at a time. The complexity of the control problem can hence be increased gradually.

The tank system can be described by the system of equations in (1).

$$
\begin{aligned}
\dot{h}_1 &= -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1}v_1 \\
\dot{h}_2 &= -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2}v_2 \\
\dot{h}_3 &= -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2}{A_3}v_2 \\
\dot{h}_4 &= -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_4}v_1
\end{aligned}
\tag{1}
$$

$$
\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} k_c & 0 & 0 & 0 \\ 0 & k_c & 0 & 0 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{bmatrix}
\tag{2}
$$

The notations used in (1) and (2) are explained in Table 2.

h_i – level in tank i	v_i – voltage to pump i
A_i – area of tank i	a_i – outlet area of tank i
k_c – sensor gain	g – gravitational constant
k_i – pump gain	γ_i – setting of valve i
y_i – measured tank level	

Table 1. Notations.

The inputs to the process are the input voltages v_i and the outputs are the measured levels in the lower tanks y_i. In the model the dynamics of the pumps and the transport delays in the pipes have been neglected.

By linearizing the nonlinear system and determining the transfer function matrix $G(s)$ of the system it is found, as shown in (Johansson, 1999), that the multivariable zeros of the system are given by

$$\det G(s) = \frac{c_1 c_2}{\gamma_1 \gamma_2 \prod_{i=1}^4 (1 + sT_i)} \tag{3}$$
$$\times \left((1 + sT_3)(1 + sT_4) - \frac{(1 - \gamma_1)(1 - \gamma_2)}{\gamma_1 \gamma_2} \right).$$

Equation (3) implies that the system has two zeros for each valve setting, where $\gamma_1, \gamma_2 \in [0, 1]$. Introduce now the parameter

$$\eta = (1 - \gamma_1)(1 - \gamma_2)/\gamma_1 \gamma_2 \tag{4}$$

which means $\eta \in [0, \infty[$. For small values of η the zeros will approach the points $-1/T_3$ and $-1/T_4$ respectively. When $\eta \to \infty$ one zero will move along the real axis towards $-\infty$ while the other will tend towards $+\infty$. For $\eta = 1$ one zero will be located in the origin, and it corresponds to $\gamma_1 + \gamma_2 = 1$. Consequently the system is non-minimum phase for $0 < \gamma_1 + \gamma_2 < 1$ while it is minimum phase for $1 < \gamma_1 + \gamma_2 < 2$. Further aspects of the model are discussed in (Johansson, 1999).

3. IMPLEMENTATION

One goal of the work has been to create an animation that is reasonably simple, in order to reduce the required efforts for development and maintenance. A second requirement has been that the simulation and animation should run equally well on different computer platforms. It was therefore decided to use Matlab both for the simulation and the animation. Since it is sufficient to use a 2D-animation for the tank process the achieved performance is reasonable. A further goal has been to find a suitable way of designing the simulation and animation environment such that the main ideas can be reused when studying models of other multivariable systems.

The nonlinear tank model given by equation (1) is implemented in Simulink. The animation itself is shown in Figure 3.

For simplicity the only parameters that are possible to alter in the animation window are the valve settings and the animation speed. Since the simulation model and the animation are used on different computers with different performance it can be useful to be able to change the animation speed.

In addition to the Simulink model and the animation two m-files are used when working with the simulation model. Using the first m-file, denoted `statpoint.m` in the current implementation, the
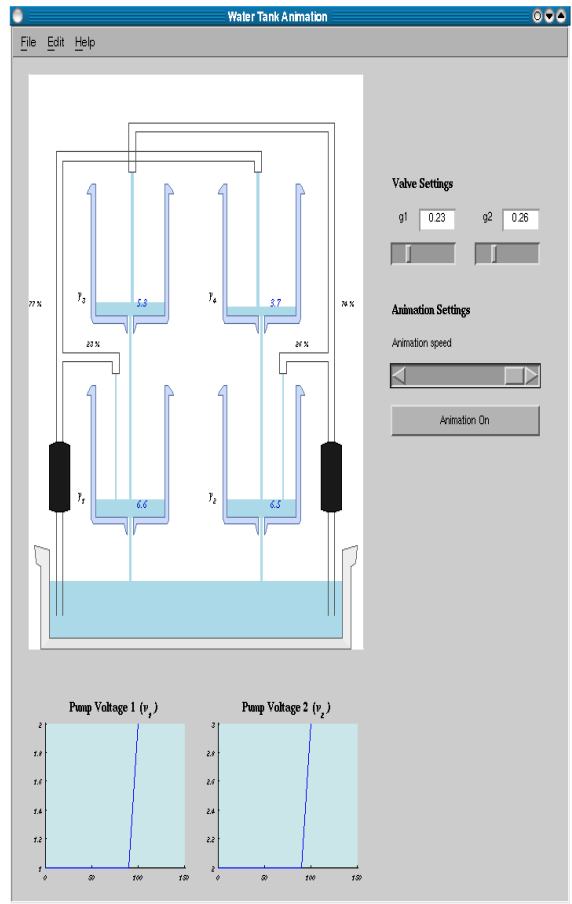

Fig. 3. Animation window. The graphs in the lower part of the window show the pump voltages.

stationary points of the tank system are computed, in two different ways. The first alternative is to give the pump voltages as inputs and get tank levels as outputs. The second alternative is to give the levels in the upper tanks as input and get the corresponding pump voltages and levels in the lower tanks as outputs. This m-file is useful for determining suitable operating points. The second m-file, denoted `tanklin.m`, is used to compute the matrices in a linearized state space model. Input to this function is the operating point in which the linearization shall be carried out. Using the obtained matrices control design can be carried out using e.g. the Control Systems Toolbox.

4. EDUCATIONAL USE

Using the multivariable tank model several aspects of multivariable dynamical systems and multivariable control systems can be illustrated. Aspects that can be illustrated are for example:

- The influence of the valve settings on the locations of the zeros.
- The influence of the locations of the zeros on the system response for different input directions.

- Design and evaluation of decentralized control.
- Design and evaluation of decoupling controllers.
- Design and evaluation of state feedback controllers for different locations of the zeros.
- Design and evaluation of observers.
- Exact linearization.

The aspects listed above are the ones that have been studied so far but there are also topics that can be studied further, like e.g. integrator windup due to actuator limitations, model predictive control, etc.

The simulation model and the animation have been used in two courses in Control Theory during the fall of 2000 and the fall of 2001 respectively. The main topics in these courses are multivariable and nonlinear control systems, and the same text book, (Glad and Ljung, 2000), is used in both courses.

For the use of the tank model in the courses a number of "tasks" have been defined. Each task consists of a main problem formulation together with some hints for finding the way to the solution. A problem formulation can for example be: "Assume that all tank levels can be measured. Design a state feedback controller and investigate if there are any limitations in the achievable performance for different valve settings." Typical examples of hints are suitable choices of operating points of the tanks, useful Matlab commands, etc. It some cases it has also been motivated to include figures showing possible realizations of the control system using Simulink. The aim has however been to pose the problems as open as possible in order to avoid controlling the solution process too much. From a pedagogical viewpoint it is also important to find a balance between the work on the control problems and work needed to implement the control systems in Simulink. For more complicated controller structures there is sometimes a risk that too much of the efforts are spent on the implementation issues.

The system has initially been used in problem solving sessions, which are supervised by a teaching assistant. By defining a number of "tasks" it can also be used as e.g. projects or home work carried out individually or in groups.

In the Control Theory courses the exam is carried out using computer support, i.e. each student has a SUN workstation with Matlab available. This is also an important component in the learning environment, since the properties of the exam have big influence on the work of the students. This aspect is further discussed in (Gunnarsson and Millnert, 1997).

5. EXAMPLE: LQ-OPTIMIZATION AND POLE PLACEMENT

In order to illustrate the use of the simulation model and the animation and point out some interesting aspects of multivariable control an example will be presented in this section. In the example the tank levels will be controlled using state feedback from measured states, where the tank levels can be measured without any measurement disturbances.

The experiments were carried out under the following conditions:

- The feedback gain vector is computed via LQ-minimization using a linearized model and the criterion
$$J = \int_0^\infty 10y_1^2(t) + 10y_2^2(t) + v_1^2(t) + v_2^2(t)dt$$
- The valves settings were $\gamma_1 = \gamma_2 = 0.3$. In the selected operating point the zeros of the system are located at 0.012 and -0.030 respectively.
- The initial state was chosen close to the equilibrium point $(x(0) = (36\ 47\ 20\ 20))$ in order to reduce the time for the tank to be filled to the desired operating point.
- The reference signals were chosen as a sum of a constant and a step occuring at time $t = 500$. The steps in each reference signal were given by $r_1(t) = -1$ and $r_2(t) = 1$, which corresponds to the direction in which the effects of the zero in the right half plane are most visible.

The simulation results are shown in Figure 4, where the non-minimum phase behavior is obvious. Using the animation it is possible to get an intuitive explanation of this phenomenon. There is also a small steady state error since no integral action is included.

Using LQ-optimization the location of the closed loop poles and the achievable performance will be determined by the location of the open loop zeros. In this particular design example the closed loop poles are $-0.1, -0.1, -0.031$, and -0.012 respectively, i.e. one pole is located at the LHP zero and one is the RHP zero mirrored in the imaginary axis. Using pole placement it is however possible to force the closed loop poles to lie in some given points. To show the effects of such a design an experiment is carried out where the gain matrix is designed using pole placement and all four poles are placed around -0.1. The simulation results are given in Figure 5. The pump velocities are very large and also overshoot in the tank levels are very large. It should be noted that the graph showing the pump velocities represents the desired pump velocity. In the simulation model there are saturations that prevent negative pump velocities
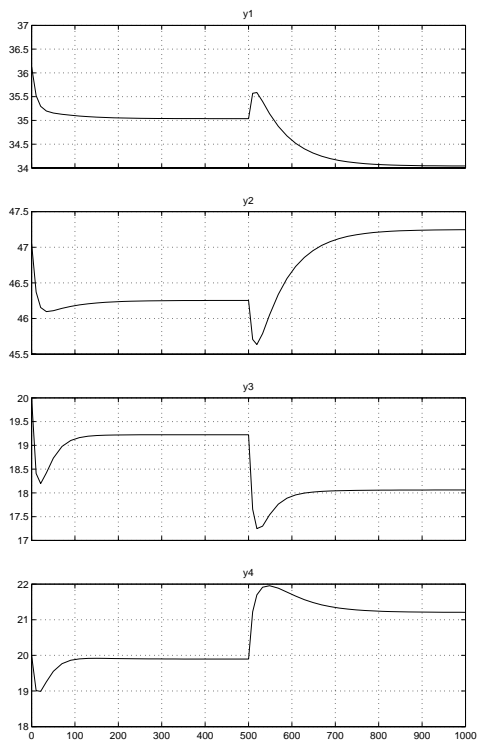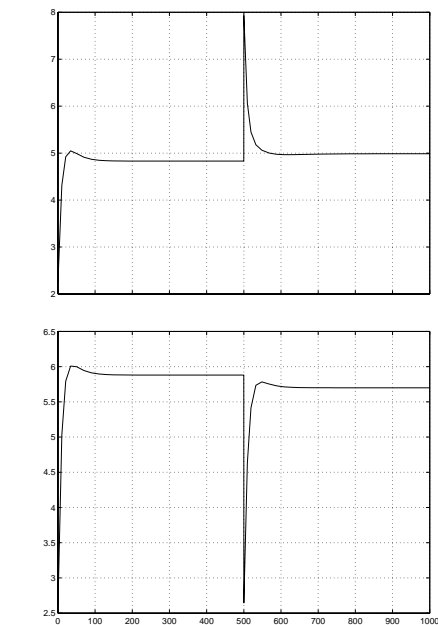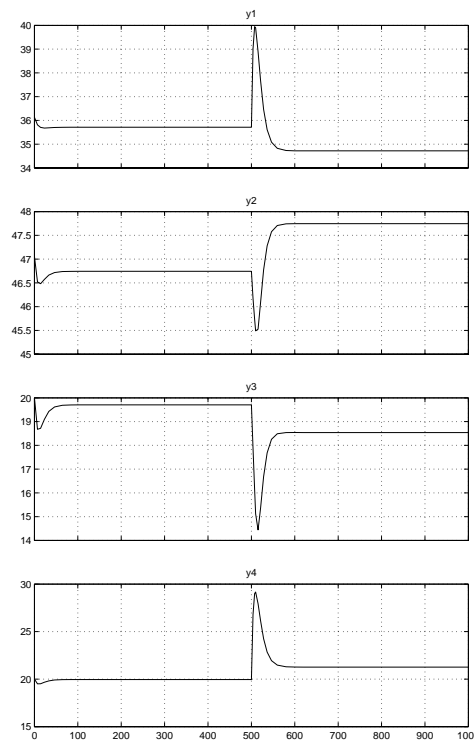
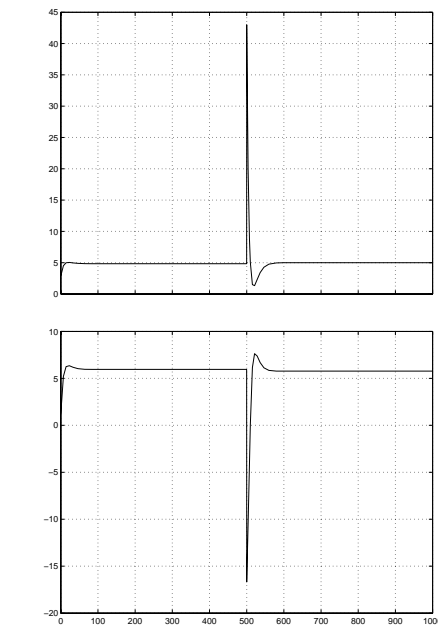Fig. 4. Tank levels and pump velocities.

since this is unrealistic. In the experiments presented here there is no upper limit on the pump velocities, but that is easily included to make the conditions even more realistic.

6. EXAMPLE: EXACT LINEARIZATION

Another interesting experiment to perform with the tanks is exact linearization. To do this for a multivariable system is outside the scope of the undergraduate courses as they are taught

Fig. 5. Tank levels and pump velocities.

presently. Instead we use the possibility of running the tank system as two uncoupled SISO-systems. This is achieved by using the valve setting $\gamma_1 = \gamma_2 = 0$. Then the two tanks to the left will be controlled from the right hand flow and the tanks to the right from the left hand flow. The model for the left hand tanks is

$$\dot{h}_1 = -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} \qquad (5)$$

$$\dot{h}_3 = -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{k_2}{A_3}v_2 \qquad (6)$$

$$y = \begin{bmatrix} k_c & 0 & 0 \end{bmatrix} h \qquad (7)$$

By introducing the variable change and feedback

$$z_1 = h_1$$
$$z_2 = -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3}$$

and

$$v_2 = \frac{A_1^2 A_3}{k_2 a_2^2 (z_2 + a_1\sqrt{2gz_1}/A_1)}$$
$$\times \left(\frac{a_2^2}{A_1 A_3} + \frac{a_1}{A_1\sqrt{2gz_1}}z_2 + u \right)$$

the system becomes a double integrator:

$$\dot{z}_1 = z_2$$
$$\dot{z}_2 = u$$

This system can now be controlled using any desired linear control law. Using the animation it is now possible to stress an important point. Looking at the exactly linearized dynamics and choosing e.g. pole placement it is tempting to draw the conclusion that one can make the tank system do anything. In Figure 6 the response of the tank levels h_1 and h_3 is drawn for a step in the reference level for h_1 when the control law is $u = --0.3z_1 - 0.02z_2$, placing the poles at -0.1, -0.2.
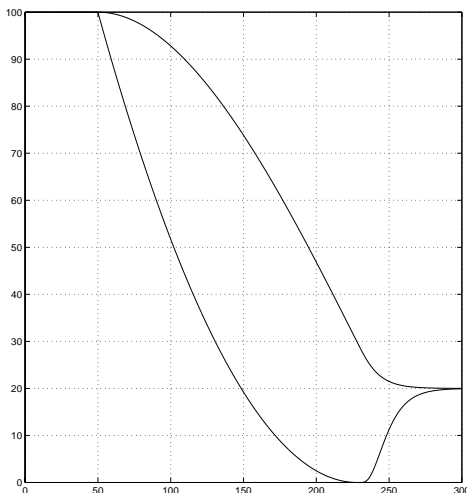

Time offset: 0

Fig. 6. Response of h_1 and h_3 with a controller based on exact linearization.

In Figure 6 it can be seen that the upper tank goes dry for a short time interval. This phenomenon becomes even more obvious by using the animation.

The real performance limitation lies in the state bounds imposed by the overflowing or emptying of the tanks (in addition to the control constraints given by maximum and minimum flows).

7. CONCLUSIONS

An educational tool for studying multivariable dynamical systems and design of multivariable control system has been presented. The tool is based on three fundamental components:

- A simulation model of a nonlinear multivariable system that enables studied of several aspects of multivariable systems.
- An animation that gives a visual description of the properties of the model.
- An educational format that encourages the students to work independent of place and time.

The tool has been used with good results in Control Theory courses during both 2000 and 2001. One of the questions for the future is how to use the tool in other learning situations and how to link it to the examination.

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