

Antiphase Formation Swimming for Autonomous Robotic Fish^{*}

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Abstract: This paper proposes distributed control laws for formations of swimming robotic fish generating antiphase sinusoidal body waves. The control laws are inspired by the mathematical model for the hydrodynamics of schools of cruising fish, which reveals that fish swimming in diamond-shape formations with synchronized antiphase body waves can benefit greatly from energy saving. The phase dynamics of the body waves of the robotic fish are modeled by coupled Kuramoto oscillators and the stability analysis for the phase dynamics are carried out for coupling topologies corresponding to diamond-shape formations. It is proven that the body waves can be synchronized in specific antiphase patterns. Simulations and experiments further validate the effectiveness of the proposed control laws.

Keywords: multi-agent systems, robotic fish, formation control, antiphase synchronization

1. INTRODUCTION

Teams of mobile robots have been utilized more and more often for a growing variety of tasks, such as environmental monitoring, search and rescue, and maintenance in harsh environments (Bullo et al. [2009], Correll and Martinoli [2009]). Among these applications, the underwater sensing and sampling tasks have attracted considerable attention due to both the great need for exploring the sea and the technical challenges arising with the control, communication and coordination of robotic systems in the sea (Leonard et al. [2007]). Different designs of underwater robots have been proposed, among which teams of robotic fish are of particular interest because of the vast opportunities to learn from fish schooling behaviors in nature (Sfakiotakis et al. [1999]).

The study of design principles for individual robotic fish can be traced back to at least the early 1990's (Triantafyllou and Triantafyllou [1995]). More recently, different central pattern generator (CPG) models have been utilized to control the locomotions of robotic fish (Ijspeert et al. [2007], Crespi et al. [2008], Wang et al. [2011]) and some results have been reported to use proper sensing and planning to control multiple robotic fish (Shao et al. [2008], Hu et al. [2009]). However, less effort has been made to study how robotic fish can benefit from collective hydrodynamics they generate while they are swimming together *cooperatively*. While a detailed study on how fish adjust their own motions to exploit vortices to reduce locomotion energy costs can be too complex to be used in the design of coordination control strategies for robotic fish (Liao et al. [2003]), one can nevertheless gain insight from such

biological study into the relationship between the patterns of fish collective motion and the corresponding energy costs.

It was first pointed out by Weihs (Weihs [1973, 1975]) that diamond-shape formations are hydrodynamically advantageous for fish to improve propulsion efficiency while cruising. It was further analyzed in Stocker [1999] through mathematical modeling that fish in a diamond-shape formation need to get synchronized with antiphase body waves in order to reduce drag and benefit from the propulsion associated with the generated reverse Karman vortex street; attempts have also been made to relate the synchronized collective swimming pattern with fish's senses through eyes and lateral lines. Numerical studies in computational fluid dynamics have investigated the interactions of vortices in the wakes of biomimetic fish schools and discussed how fish might adjust the frequencies and amplitudes of their body waves to keep the fixed inphase or antiphase swimming formation (Deng and Shao [2006], Wu and Wang [2010]).

In this paper, inspired by the observation just mentioned that formations of synchronized fish may swim with less energy consumption, we design distributed control laws for teams of robotic fish to lock the phases of their sinusoidal body waves in an antiphase fashion. We model the phase dynamics of the body waves of the robotic fish by coupled Kuramoto oscillators. We prove that when such phase dynamics are coupled through real-time communications with a diamond-shape topology, they can be synchronized with the desirable relative phase differences of zero or π to mimic the fish swimming patterns predicted in the corresponding biological studies. We perform both digital simulations and physical experiments to show the effectiveness and robustness of the proposed control strategies.

The rest of the paper is organized as follows. We present the coupled oscillator model for the robotic fish that we have developed in Section 2. Then we discuss in Section 3 how distributed control laws can guide oscillators coupled in diamond-shape formations to get synchronized in an antiphase fashion. In

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Section 4, results from simulations and experiments are demonstrated.

2. COUPLED OSCILLATOR MODEL FOR ROBOTIC FISH

The robotic fish that we have developed is shown in Fig. 1, which consists of a streamlined head, a flexible body and a caudal fin (Shao et al. [2008], Wang et al. [2011]).

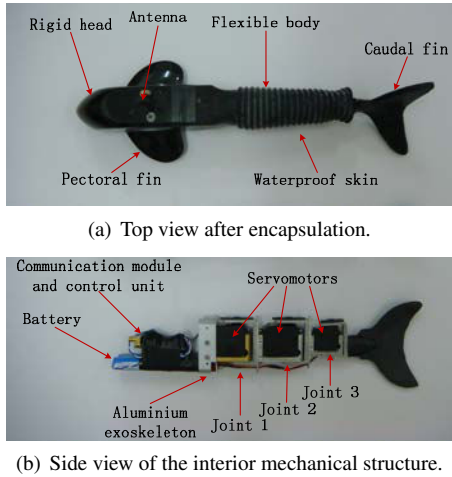


Fig. 1. Mechanical configuration of a biomimetic robotic fish.

The total length of the robotic fish is 40cm. The head is made of fiberglass, which accommodates an onboard control unit, a duplex wireless serial-port communication module and a set of rechargeable batteries. The battery pack is placed at the bottom to lower the center of mass and consequently stabilize the vertical posture (Figure 1(b)). A pair of fixed pectoral fins is used to ensure the roll stability when the robotic fish swims. Since this pair of fins is rigidly attached to the sides of the head, the robotic fish's locomotion is confined roughly to a horizontal two-dimensional space. The flexible body contains three revolute joints that are linked together by aluminium exoskeletons. Each joint is driven by a R/C servomotor, which controls its relative joint angle with respect to those of its adjacent joints. The caudal fin is attached to the third joint, whose shape is designed to enhance the swimming efficiency. The whole body from the end of the head to the tail is covered by tailor-made waterproof rubber. In order to make the robotic fish swim just below the water surface for the purpose of keeping the antenna above the water, we inject an appropriate amount of air so that the density of the robotic fish is just a little bit smaller than that of the water.

The propulsion of each robotic fish is achieved through generating a traveling wave traversing the robotic fish's body towards the tail. The traveling wave (Lighthill [1960]) can be described by

$$y(x, t) = (c_1 x + c_2 x^2) \sin(kx + \omega t + \theta_0) \quad (1)$$

where x denotes the displacement along the main axis that starts at the first joint and points towards the opposite direction of motion, y is the body displacement with respect to the axis, c_1 and c_2 are the constants for the amplitude of the envelope of the traveling wave, k is a constant called the body wave number, ω is the body wave frequency, θ_0 is the initial phase and $\omega t + \theta_0$ is the phase. In the sequel, we use θ to denote $\omega t + \theta_0$. An illustrative drawing is shown below.

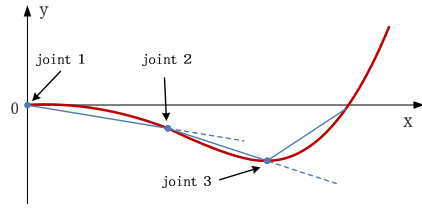


Fig. 2. Three links connected by revolute joints that generate traveling waves.

Obviously, the hydrodynamics that the robotic fish generates are mainly determined by the amplitude and phase of the traveling wave, which, in fact, can be controlled by implementing CPG-based model. We will discuss in Section 4 how to control the three joints once the desired values of the amplitude and phase of the body wave are set. In this paper, we are less concerned with the lower level control to adjust the motors at the three joints to obtain a desired body wave, but rather concentrate on how to find the desired values of the phases of the body waves of the robotic fish in a formation such that the collective hydrodynamics of the formation of robotic fish to the advantage of the fish to swim forward.

For the sake of clarity of analysis, we assume that the traveling waves associated with all the robotic fish are identical except that they may have different phases. In other words, the parameters c_1 , c_2 and k in (1) are all the same for all robotic fish. The setting can be further idealized by assuming that the frequencies ω of all the robotic fish are also the same. We will show in Section 4 that such simplifying assumptions are meaningful and can help us gain insight into how robotic fish can be coordinated together by just focusing on the key determining factor of their phase dynamics. Consequently the only state that we need to control is θ . Consider a formation of n robotic fish labeled by $1, \dots, n$. Then the dynamics of the phase of robotic fish i , $1 \leq i \leq n$, are

$$\dot{\theta}_i = \omega + u_i \quad (2)$$

where u_i is the control input. So the aim of this paper is to design the distributed control law u_i such that the collective dynamics of the formation of robotic fish may evolve into a "desirable" equilibrium and here by the *desirable* dynamics we mean the collective behaviors that have been proven to be advantageous through mathematical modeling for schools of real fish in nature. Note that the system (2) itself has the form of the dynamics of coupled oscillators with the phases as the system's state.

3. ANTIPHASE SYNCHRONIZATION IN DIAMOND-SHAPE FORMATIONS

It was reported and analyzed in Weihs [1973, 1975] and Stocker [1999] that fish swimming in elongated diamond-shape formations can save locomotion energies by up to 20%. The main ideas of the explanation are as follows. Each swimming fish sheds vortices behind its body. Now consider the vortex streets behind a column of fish swimming parallel to each other as shown in Fig. 3, the fish that is swimming laterally midway behind the two fish in the column that is immediately in front of it can utilize the favorable flow at the sides of the vortex streets. This hydrodynamical advantage is strengthened when the phases of the body waves of the neighboring fish differ by π , i.e. the body waves of these fish are antiphase synchronized. This leads to the fact that the elongated diamond-shape

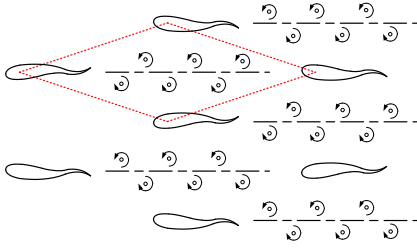


Fig. 3. Fish swimming in elongated diamond-shape formations with neighboring fish being synchronized in antiphase (Adapted from Weihs [1975]).

patterns with antiphase synchronized body waves are preferred basic structure for cruising fish schools. Hence, the goal of this section is to design u_i in (2) utilizing only the information of the phases of robotic fish i 's neighbors in the diamond-shape formation topologies such that when the relative positions of the robotic fish have been determined by elongated diamond-shape building blocks, one can guarantee that the body waves of the neighboring fish in the same column of the diamond formation are antiphase synchronized and as a result the formation of robotic fish can maintain the elongated diamond-shape patterns and fully utilize the benefit of energy saving from the collective hydrodynamics of the robotic fish formation.

3.1 Control law

For a real fish in a diamond-shape fish school, the phases of its adjacent fish can be acquired through visual information by eyes and fluid pressure information by lateral lines. Usually one side of the vision is preferred than the other (namely the two eyes have asymmetric sensing capabilities), so only that adjacent fish that is on the preferred side is taken as the fish's neighbor (Stocker [1999]); on the other hand, there is usually no preferences between the lateral lines on the two sides of the body, so both of those two fish that are adjacent in the same column are the fish's neighbors. Hence, in this biological model a fish may have at most three neighbors in the school. To mimic the neighbor relationships in real fish schools, we adopt the same local rule of picking neighbors in the model of real fish school for the robotic fish formation. For robotic fish i , let $\mathcal{N}_i$ denote the set of indices of its neighbors. Then there are at most three elements in $\mathcal{N}_i$. Here, without loss of generality, a robotic fish always takes the front left adjacent robotic fish (if there is one) as its neighbor instead of the one on the front right, although these two have similar distances to this robotic fish. This particular choice of neighbors turns out to be key in proving stability in Section 3.2.

Let graph $\mathbb{G}$ with vertex set $\mathcal{V} = \{1, \dots, n\}$, where vertex i in the graph corresponds to robotic fish i in the formation, be the graph associated with the diamond-shape robotic fish formation describing the neighbor relationships. Then for $i, j \in \mathcal{V}$, there is a directed edge (i, j) in the edge set $\mathcal{E}$ of $\mathbb{G}$ if and only if $i \in \mathcal{N}_j$. For the clarity of expression, we denote the robotic fish formation by $\mathbb{F}$. We emphasize below two facts about the edges of $\mathbb{G}$ that are inherited from the properties of neighbor relationships in $\mathbb{F}$.

- (1) If (i, j) is an edge in $\mathbb{G}$ and i, j are in the same column of $\mathbb{F}$, then (j, i) is also an edge in $\mathbb{G}$. This is due to the fact that the neighbor relationships in the same columns of $\mathbb{F}$ are symmetric.

- (2) If (i, j) is an edge in $\mathbb{G}$ and i, j are in different columns of $\mathbb{F}$, then (j, i) is *not* an edge of $\mathbb{G}$. This is due to the fact that the neighbor relationships in different columns of $\mathbb{F}$ are asymmetric.

We say that a graph is *connected* if each of its vertices can reach every other vertex and that a graph *contains a spanning tree* if there is a vertex in the graph that can reach all the other vertices.

After clarifying the neighbor relationships in $\mathbb{F}$ and defining the graph $\mathbb{G}$, we are ready to present the control law that is used in this paper:

$$u_i = \sum_{j \in \mathcal{N}_i} \sin(\theta_i - \theta_j), \quad i = 1, \dots, n. \quad (3)$$

This control law is in the form of the coupling terms in the Kuramoto model for oscillators coupled through sinusoidal signals (Acebron et al. [2005]). Kuramoto model has been studied intensively in the past two decades mainly to study the *in-phase* synchronization behavior where, in the case of all the oscillators having the same frequency, the phases of all the oscillators become the same asymptotically. Recently oscillator models have been used to stabilize different patterns of collective motion for multi-agent systems (Paley et al. [2007]). In this paper, to study the behavior of the phase dynamics of robotic fish formation $\mathbb{F}$, we are in fact exploring the stability properties of coupled Kuramoto oscillators with a specific local neighbor relationship graph $\mathbb{G}$ and with respect to a specific type of equilibrium point corresponding to the "antiphase synchronization". Here, we say that a connected network of coupled oscillators has reached *antiphase synchronization* if any pair of neighboring oscillators has a phase difference of $2k\pi + \pi$ for some integer k .

3.2 Stability analysis

Denote the number of columns in $\mathbb{F}$ by a positive number m . Let $\mathbb{F}_i$, $1 \leq i \leq m$, denote the sub-formation in $\mathbb{F}$ that corresponds to the i th column from the front. Correspondingly let graph $\mathbb{G}_i$ be the subgraph of $\mathbb{G}$ associated with $\mathbb{F}_i$. Denote the number of elements in $\mathbb{V}_i$ by l_i , and thus $l_1 + \dots + l_m = n$. We label the robotic fish in $\mathbb{F}$ following the rule that a fish in the front always has a smaller index than another in the back and if two fish are in the same column, the one on the left always has a smaller index than the one on the right. Hence, the fish in $\mathbb{F}_1$, from the left to right, are assigned with indices $1, \dots, l_1$, those in $\mathbb{F}_2$ with indices $l_1 + 1, \dots, l_1 + l_2$, and so on.

Note that as a standard procedure by a simple argument using coordinate transformation, the systems

$$\dot{\theta}_i = \omega + \sum_{j \in \mathcal{N}_i} \sin(\theta_i - \theta_j), \quad i = 1, \dots, n \quad (4)$$

and

$$\dot{\theta}_i = \sum_{j \in \mathcal{N}_i} \sin(\theta_i - \theta_j), \quad i = 1, \dots, n \quad (5)$$

have the same stability properties. In the sequel, we take (5) to be the system of interest.

It is easy to check that there is no edge in $\mathbb{G}$ that starts from a vertex in $\bigcup_{i=2}^m \mathbb{G}_i$ and ends at a vertex in $\mathbb{G}_1$. So the phase dynamics of the robotic fish in $\mathbb{F}_1$ are not influenced by the robotic fish in $\bigcup_{i=2}^m \mathbb{F}_i$. This motivates us to study the collective phase dynamics of $\mathbb{F}_1$ first.

Proposition 1. For almost all initial phase configurations and all $l_1 > 1$, the phases of the robotic fish in $\mathbb{F}_1$ reach antiphase synchronization asymptotically if $\mathbb{G}_1$ is connected.

To prove this proposition, we first look at the possible equilibrium points of the system. Let

$$\mathcal{E}_1 \triangleq \{ [\theta_1 \dots \theta_{l_1}]^T \in \mathbb{R}^{l_1} \mid (\theta_i - \theta_{i+1}) \bmod \pi = 0, \\ i = 1, \dots, l_1 - 1 \}.$$

Then it is easy to check the following result.

Lemma 1. When $\mathbb{G}_1$ is connected, the set of all the equilibrium points of the phase dynamics of $\mathbb{F}_1$ is $\mathcal{E}_1$.

It can be further checked that the antiphase synchronized state is in $\mathcal{E}_1$.

Lemma 2. When $\mathbb{F}_1$ reaches antiphase synchronization, it is at an equilibrium point in $\mathcal{E}_1$.

Proof of Proposition 1: For the phases θ_i and θ_j , we define the distance between them to be

$$d_{ij} \triangleq \sin^2 \frac{\theta_i - \theta_j}{2}.$$

Let D denote the sum of the distances between all pairs of neighboring robotic fish in $\mathbb{F}_1$

$$D \triangleq \sum_{i \in \mathcal{V}_1} \sum_{j \in \mathcal{N}_i} d_{ij}.$$

Then for $\mathbb{F}_1$ with a connected $\mathbb{G}_1$, D can be further written as

$$D = 2 \sum_{i=1}^{l_1-1} \sin^2 \frac{\theta_i - \theta_{i+1}}{2} = \sum_{i=1}^{l_1-1} [1 - \cos(\theta_i - \theta_{i+1})] \geq 0,$$

which reaches its maximum

$$D_{\max} = 2l_1 - 2$$

when $(\theta_i - \theta_{i+1}) \bmod 2\pi = \pi$ for all $1 \leq i < l_1$. Consider the following Lyapunov function

$$V \triangleq 2l_1 - 2 - D. \quad (6)$$

Then V reaches its minimum zero when $D = D_{\max}$ in which case $\mathbb{F}_1$ reaches antiphase synchronization. Furthermore,

$$\dot{V} = - \sum_{i=1}^{l_1-1} \dot{\theta}_i^2 \leq 0$$

where the equality signs holds if and only if $[\theta_1 \dots \theta_{l_1}]^T \in \mathcal{E}_1$. Since the antiphase synchronized state corresponds to the unique global minimum of V and in view of Lemma 2, we know that the antiphase synchronized state is asymptotically stable.

Let $\bar{\mathcal{E}}_1$ denote the subset of $\mathcal{E}_1$ that contains all the equilibrium points not corresponding to antiphase synchronization. To show further that for almost all initial conditions, the phases of $\mathbb{F}_1$ can be antiphase synchronized, it suffices to show that all the equilibrium points in $\bar{\mathcal{E}}_1$ are not stable. For any $\bar{\theta} \triangleq [\bar{\theta}_1 \dots \bar{\theta}_{l_1}]^T \in \bar{\mathcal{E}}_1$, one can always find $1 \leq i < l_1$ such that $(\bar{\theta}_i - \bar{\theta}_{i+1}) \bmod 2\pi = 0$. Although $V(\bar{\theta}) = 0$, the value of V always decreases if we perturb $\bar{\theta}$ in a direction that changes the difference between $\bar{\theta}_1$ and $\bar{\theta}_2$. Hence, $\bar{\theta}$ cannot be stable. $\square$

After investigating the stability of the antiphase synchronized state of the sub-formation $\mathbb{F}_1$, we continue to look at the stability of the phase dynamics of the overall formation $\mathbb{F}$.

Let

$$\theta \triangleq [\theta_1 \dots \theta_n]^T. \quad (7)$$

We define the set of antiphase states $\mathcal{E}^*$ to be

$$\mathcal{E}^* \triangleq \{ \theta \mid (\theta_i - \theta_j) \bmod 2\pi = \pi, i = 1, \dots, n \text{ and } j \in \mathcal{N}_i \}. \quad (8)$$

We first examine the local stability of $\mathcal{E}^*$ using the linearization technique.

Theorem 1. When $\mathbb{G}$ contains a spanning tree, any point in $\mathcal{E}^*$ is locally stable.

Proof: We provide the main ideas of the proof because of the page limit. We linearize system (5) at any point $\theta^* \in \mathcal{E}^*$. We define an $(n-1)$ -dimensional column vector $\tilde{\theta} \triangleq [\theta_1 - \theta_2 \dots \theta_n - \theta_{n-1}]^T$. One can check that it holds for the system after linearization that

$$\dot{\tilde{\theta}} = -A(\tilde{\theta} - \tilde{\theta}^*)$$

where $\tilde{\theta}^*$ is defined with respect to θ^* and A is the projection matrix that projects $\mathbb{R}^n$ to the eigenspace of all the nonzero eigenvalue of $\mathbb{G}$'s Laplacian matrix. Since $\mathbb{G}$ contains a spanning tree, from the existing result (Ren and Beard [2005]) we know that L has a simple zero eigenvalue and all its other eigenvalues are strictly positive. Hence, all the eigenvalues of A are positive and thus $\tilde{\theta}$ converges to $\tilde{\theta}^*$. So we have proved that θ^* is stable. Since this holds for any θ^* in $\mathcal{E}^*$, we arrive at the conclusion. $\square$

In view of the almost global convergence result in Proposition 1 for the sub-formation $\mathbb{F}_1$, we want to study the global stability of $\mathcal{E}^*$.

Theorem 2. For almost all initial phase configurations, the phases of the robotic fish in $\mathbb{F}$ reach antiphase synchronization asymptotically if $\mathbb{G}$ contains a spanning tree.

Proof: We provide the main ideas of the proof because of the page limit. We prove by induction. As shown in Proposition 1, the robotic fish in $\mathbb{F}_1$ can reach antiphase synchronization. Now we look at the phase dynamics of the fish in $\mathbb{F}_2$, it can be checked that the only possible stable equilibrium points for $\mathbb{F}_2$ are those corresponding to the case when the phases of each robotic fish in $\mathbb{F}_2$ are antiphase synchronized with respect to all its neighbors. Following an argument similar to the proof of Proposition 1, we know that all the robotic fish in $\mathbb{F}_1 \cup \mathbb{F}_2$ can reach antiphase synchronization. Now we assume that for any $1 < p < m$, the robotic fish in $\bigcup_{i=1}^p \mathbb{F}_i$ can reach antiphase synchronization. Then one can again check that the only possible stable equilibrium points for $\mathbb{F}_{p+1}$ are those corresponding to the case when the phases of all robotic fish in $\mathbb{F}_{p+1}$ are antiphase synchronized with respect to all its neighbors. So using the argument similar to that in the proof of Proposition 1 again, one can shown that all the robotic fish in $\bigcup_{i=1}^{p+1} \mathbb{F}_i$ can reach antiphase synchronization. This completes the proof. $\square$

In the next section, we carry out simulations and experiments to verify the theoretical results that we have obtained in this section.

4. SIMULATIONS AND EXPERIMENTS

We first simulate the phase dynamics of a formation of fourteen fish as shown in Fig. 4. The initial phases take random values in $[0, 2\pi)$. The phases of the robotic fish, without considering the shared ωt term, are presented in Fig. 5 and the phase differences for all pairs of neighboring robotic fish are shown in Fig. 5 as well. The results show that the robotic fish school swimming in diamond-shape formations can achieve antiphase synchronization under the proposed phase control law.

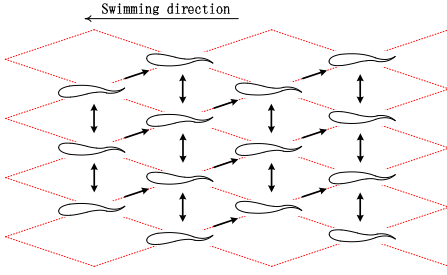


Fig. 4. A formation of fourteen robotic fish.

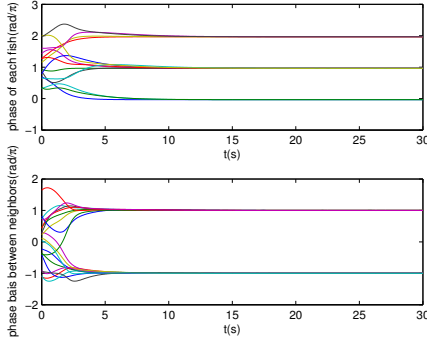


Fig. 5. The phases and phase differences of the robotic fish formation.

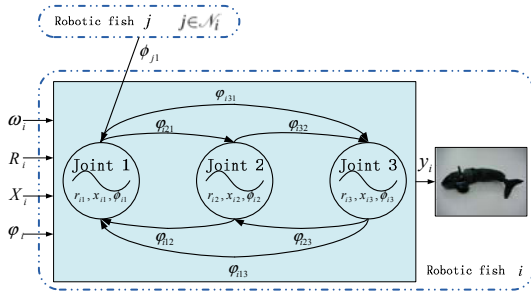


Fig. 6. Diagram of CPG-based locomotion control architecture.

We further test the proposed phase control law using two robotic fish that have been discussed in Section 2. For each robotic fish, all the onboard electronic devices are powered by four 5V rechargeable Ni-MH batteries. The onboard control unit is based on a micro-controller, Atmel ATmega 128, which runs the CPG model with control law (see Sections 3 and 4) and generates the control commands in the form of Pulse Width Modulation (PWM) to drive the three R/C servomotors at the three joints in real time. The servomotors at the first two joints are Futaba S3003 with a maximum recommended torque of $3.2\text{kg} \cdot \text{cm}$ and a working speed of $0.23\text{sec}/60\text{deg}$ at 4.8V . The smaller servomotor of the third joint is Futaba S3102 with a maximum recommended torque of $3.7\text{kg} \cdot \text{cm}$ and a working speed of $0.25\text{sec}/60\text{deg}$ at 4.8V . The realtime communication with other robotic fish is achieved through the communication module WAP200B.

To implement the lower level control for the servomotors of robotic fish, we implement a modified CPG model that can be described by the diagram in Fig. 6. The detailed equations for such a CPG model can be written as follows.

$$\ddot{r}_{im} = \alpha[\alpha(R_{im} - r_{im}) - 2\dot{r}_{im}], \text{ for } m = 1, 2, 3$$

$$\ddot{x}_{im} = \beta[\beta(X_{im} - x_{im}) - 2\dot{x}_{im}], \text{ for } m = 1, 2, 3$$

$$\ddot{\phi}_{im} = \sum_{l=1, l \neq m}^3 \mu[\mu(\phi_{il} - \phi_{im} - \varphi_{iml}) - 2(\dot{\phi}_{im} - \omega_{im})],$$

for $m = 2, 3$

$$\ddot{\phi}_{i1} = \sum_{l=2,3} \mu[\mu(\phi_{il} - \phi_{i1} - \varphi_{i1l}) - 2(\dot{\phi}_{i1} - \omega_{im}) - \gamma \sum_{j \in \mathcal{N}_i} \sin(\phi_{i1} - \phi_{j1})]$$

$$y_{im} = x_{im} + r_{im} \cos(\phi_{im}), \text{ for } m = 1, 2, 3 \quad (9)$$

Here, $r_{im}(t)$, $x_{im}(t)$ and $\phi_{im}(t)$ are the amplitude, the offset and the phase of the m th joint of the i th robotic fish at time t respectively. $y_{im}(t)$ are the desired deflection angle of the m th joint of the i th robotic fish at time t . The parameters ω_{im} , R_{im} and X_{im} are the desired frequency, amplitude and offset of the m th joint of the i th robotic fish respectively. The parameter φ_{iml} are the desired phase difference between joints m and l of the i th robotic fish. Finally, α , β , μ and γ are structural parameters that affect the transient dynamics. The CPG-based model is used to generate the desired traveling wave for each robotic fish. Note that the coupling $-\gamma \sum_{j \in \mathcal{N}_i} \sin(\phi_{i1} - \phi_{j1})$ is

used for antiphase synchronization. We choose the parameters $\omega_{im} = 4.69\text{rad/s}$, $R_{i1} = 0.26\text{rad}$, $R_{i2} = 0.44\text{rad}$, $R_{i3} = 0.52\text{rad}$, $\varphi_{i12} = -1.33\text{rad}$, $\varphi_{i13} = -1.85\text{rad}$ ($m = 1, 2, 3$ and $i = 1, 2$), $\alpha = \beta = 8.72/\text{s}$, and $\mu = 4.36/\text{s}$ for both of the two robotic fish. Additionally, we set $\gamma = 1.5/\text{s}$ to couple the phases of body waves of the robotic fish. We refer to (Wang et al. [2011]) for readers who are interested in more details about the CPG model and its settings.

We run the above detailed robotic fish model through simulations and experiments with initial conditions $\phi_{i1}(0) = 2.79\text{rad}$, $\phi_{i2}(0) = 2.88\text{rad}$, namely the two robotic fish start swimming almost in-phase. The controllers are activated at the time instance 2s. For the simulations, we show in Fig. 7 both the phases of the six joints of the two robotic fish and the phase differences between the first joints. The snapshots during the experiments are provided in Fig. 8. The data collected per 0.2s from the experiments are shown in Fig. 9 where we present the phases of the first joints and the phase difference of the first joints.

The results of simulations and experiments show that the two robotic fish can achieve antiphase synchronization using the proposed phase control law together with our CPG-based model. In Fig. 9, the final value of phase differences is not exactly equal to π because of the low-precision data format we have used to reduce the communication cost.

5. CONCLUSION

We have shown an effective antiphase synchronization strategy in the form of the coupled Kuramoto model that can coordinate the body waves of a formation of swimming robotic fish. While the achieved antiphase body waves of the neighboring robotic fish have a clear biological explanation from the existing hydrodynamical models for fish school swimming in nature, much more questions remain to be answered. For example, what are the best combination of the parameters for lower level motor

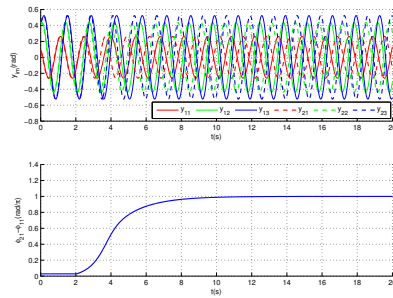


Fig. 7. Two simulated robotic fish in the antiphase synchronization process.

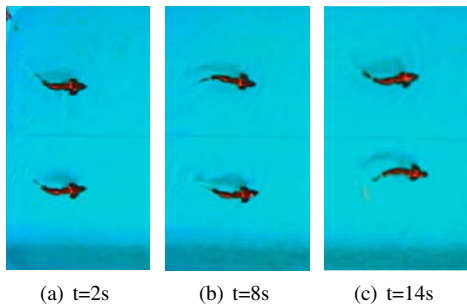


Fig. 8. Snapshots of the experiments.

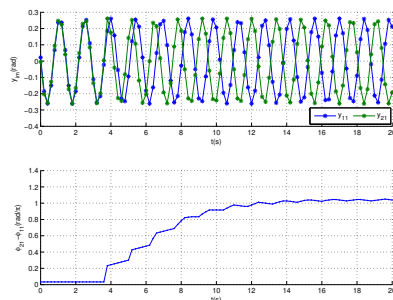


Fig. 9. Two robotic fish in the antiphase synchronization process.

control such that the best antiphase body waves can be achieved in terms of the energy efficiency.

We are also exploring to study a more comprehensive coordination control strategy that considers not only the phase dynamics but the amplitudes and frequencies of the oscillator model as well. Although by relying on a lower level CPG-type of strategy we have obtained an acceptable performance of the robotic fish decoupling the phase dynamics from the rest, a joint consideration of all aspects of the robotic fish motion dynamics that may affect their interactions in water is of a clear advantage. The main challenge in such a comprehensive study lies in the possible complicated analysis for fluid dynamics that are related to the motion of robotic fish.

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