Preprints of the 18th IFAC World Congress
Milano (Italy) August 28 - September 2, 2011

Second-order sliding mode approaches to disturbance estimation and fault detection in fractional-order systems

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Abstract: This paper outlines some results concerning the application of second-order sliding-mode techniques to address estimation and fault detection problems involving fractional order (FO) dynamics. Perturbed and switched FO systems are dealt with throughout the paper. Simple tuning formulas for the suggested schemes are constructively developed along the paper by means of appropriate Lyapunov analysis. Simulation and experimental results confirm the expected performance.

Keywords: Fractional order systems. Sliding mode control. Observation and fault detection.

1. INTRODUCTION

Fractional–order systems (FOS), i.e. dynamical systems described using fractional (or, more precisely, non-integer) order derivative and integral operators, are studied with growing interest in recent years (Kilbas et al. (2006); Sabatier et al. (2007); Das (2008)). It has been pointed out that a large number of physical phenomena can be modeled effectively by fractional–order models. Known examples include the areas of bioengineering (Magin (2006)), transport phenomena (Atanackovic et al. (2007)), economy (Scalas (2008)) and mechanics (Atanackovic et al. (2003)), medical sciences (Ding and Ye, (2008)) and others (see (Sabatier et al. (2007))).

In the nineties, Oustaloup proposed a non-integer robust control strategy named CRONE (*Commande Robuste d'Ordre Non-Entier*) (Oustaloup et al. (1996)). Another well known fractional control algorithm is the fractional PID (FPID, or $PI^{\lambda}D^{\mu}$) controller introduced by Podlubny (Podlubny, (1999)), and its "partial" versions PI^{λ} and PD^{μ} (see e.g. the recent paper (Luo and Chen, (2009))).

To the best of the authors' knowledge, the state observer design for FOS is up to now limited to the standard application of Luenberger-like observers for perfectly known linear FOS (Dzieliski et al (2006)), and basic issues of observability are still under investigation (Bettayeb et al. (2006); Mozyrska et al. (2008)).

In the present paper, second-order sliding mode approaches are developed for estimation and fault detection purposes in the framework of linear FOS subject to unknown inputs. Both time- invariant and switched FOS will be studied in the paper. A distinctive feature of the proposed approaches is that special sliding surfaces, containing fractional order derivatives of the state variables, are used.

⋆ The authors gratefully acknowledge the financial support from the FP7 European Research Projects "PRODI - Power plants Robustification by fault Diagnosis and Isolation techniques", grant no.224233.

In the framework of FDI, faults in dynamical systems are usually modeled by uncertain exogenous signals entering the system dynamics (i.e., unknown inputs). Within this area, powerful results have been achieved in the context of the so called model based FDI by using several types of Unknown-Input Observers (UIOs) (Simani et al. (2002)). Here we follow a different direction, by representing the faults by means of abrupt changes in the system dynamics. This choice naturally leads to consider a **switched** dynamics as the mathematical model of the fractional-order system under investigation.

We address two distinct problems. A method for reconstructing an external disturbance acting on a known FOS is presented, and, furthermore, a method for estimating the discrete state of a switched FOS is illustrated along with the experimental application of the technique in the framework of FDI.

The outline of the paper is as follows. The next Section 2 recalls some preliminaries on fractional calculus. Section 3 illustrates the proposed results, namely a method for disturbance estimation in a class of FOS (Subsect. 3.1), and a method for discrete state observation in switched FOS (Subsect. 3.2). The corresponding Simulation results are presented in Section 4. Section 5 presents an experimental application of the suggested techniques to a problem of fault diagnosis in a hydraulic system. Section 6 draws some concluding remarks.

2. PRELIMINARIES ON FRACTIONAL CALCULUS

Several definitions of fractional-order differential operators have appeared in literature (Das (2008)). Here the so called **Riemann–Liouville** (RL) approach is adopted. The RL **fractional integral** of order $\alpha \geq 0$ is defined as

$$I^{\alpha}x(t) = \frac{1}{\Gamma(\alpha)} \int_0^t x(\tau)(t-\tau)^{\alpha-1} d\tau \tag{1}$$

where $x(t)$ is a scalar or a vector signal and $\Gamma(\alpha)$ is the Euler's Gamma function

$$\Gamma(\alpha) = \int_0^\infty \nu^{\alpha-1} e^{-\nu} d\nu. \tag{2}$$

For integer values of the integration order α the RL defintion is equivalent to the classical one. The RL **fractional derivative** of order $\alpha \geq 0$ is defined as

$$D^{\alpha}x(t) = \frac{d^n}{dt^n} I^{n-\alpha} \tag{3}$$

where n is the integer number such that $n-1 < \alpha \leq n$. It can be proven, although this is not trivial, that for integer values of α the RL definition of the fractional derivative coincides with the classical one. Within the present paper the set $\alpha \in (0,1)$ is of primary interest. For such values of α the definition (3) becomes

$$D^{\alpha}x(t) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t \frac{x(\tau)}{(t-\tau)^{\alpha}} d\tau \tag{4}$$

The following Lemma holds:

Lemma 1 (Pisano et al. (2010)) Consider a vector signal $z(t) \in \mathbb{R}^m$. Let $\alpha \in (0,1)$. If there exists $t_1 < \infty$ such that

$$I^{\alpha}z(t) = 0 \qquad \forall t \geq t_1 \tag{5}$$

then the next relation holds:

$$\lim_{t\to\infty} z(t) = 0. \tag{6}$$

The conditions for the asymptotic stability of fractional order dynamics are well understood for **commensurate** linear time-invariant FOS

$$D^{\alpha}z = Az, \quad z \in R^m, \quad \alpha \in (0,1) \tag{7}$$

The following Lemma holds:

Lemma 2 (Matignon (1996)) Consider system (7), and let $\lambda_i(A)$ $(i = 1,2,...,m)$ be the eigenvalues of matrix A. The system is asymptotically stable if and only if the following condition holds

$$|arg(\lambda_i(A))| > \alpha\frac{\pi}{2}, \quad i = 1,2,...,m \tag{8}$$

The above Lemma recovers the well known results of classical control theory when $\alpha = 1$, and interestingly states that asymptotically stable FOS may have one or more eigenvalues with positive real part.

3. SECOND-ORDER SLIDING MODE BASED OBSERVATION AND ESTIMATION FOR FOS

3.1 Disturbance observer for FOS

Consider the fractional-order system:

$$D^{\alpha}x(t) = Ax(t) + Bu(t) + F + d(t) \qquad 0 < \alpha < 1 \tag{9}$$

where $x(t) \in \mathbb{R}^n$ represents the system's *vector-space model* (Hartley et al. (2002)), whose components (the *internal variables*) are supposed to be available for measurement, $u(t) \in \mathbb{R}^m$ represents the known input vector, A and B are the characteristic and control matrices, F is an affine term, having appropriate dimension, and $d(t) \in \mathbb{R}^n$ is a sufficiently smooth uncertain disturbance that should satisfy the next assumption:

Assumption A_1 There exists a constant d_{Md} such that

$$\left\| \frac{d}{dt} d(t) \right\| \leq d_{Md} \tag{10}$$

It is proposed the next observer

$$D^{\alpha}\hat{x}(t) = Ax(t) + Bu(t) + F + w(t) \tag{11}$$

where the observer injection input term $w(t)$ is generated by means of the next algorithm:

$$\epsilon = I^{1-\alpha}(\hat{x} - x) \tag{12}$$
$$w = w_1 + w_2 \tag{13}$$
$$w_1 = -k_1\epsilon - k_2|\epsilon|^{1/2}\mathrm{sign}(\epsilon) \tag{14}$$
$$\dot{w}_2 = -k_3\mathrm{sign}(\epsilon) \tag{15}$$

where k_1, k_2 and k_3 are tuning constants. The next Theorem is demonstrated:

Theorem 1. Consider system (9) fulfilling the Assumption A_1 along with the observer (11)-(15) with the tuning parameters chosen according to the next inequalities:

$$k_1 > 2\sqrt{d_{Md}}, \qquad k_2 > 0, \qquad k_3 > d_{Md}\sqrt{k_1}. \tag{16}$$

Then, there is a finite $T > 0$ such that the next relation holds

$$w_2(t) = d(t), \qquad t \geq T \tag{17}$$

Proof Theorem 1 The time derivative of the error variable ϵ (12) is:

$$\dot{\epsilon} = D^{\alpha}\hat{x}(t) - D^{\alpha}x(t) \tag{18}$$

which, considering the plant and observer dynamics (9)-(15) yields the following:

$$\dot{\epsilon} = w(t) - d(t) = -k_1\epsilon - k_2|\epsilon|^{1/2}\mathrm{sign}(\epsilon) + w_2(t) - d(t) \tag{19}$$

By making the change of variable

$$z(t) = w_2(t) - d(t) \tag{20}$$

one can augment and rewrite (19) as

$$\dot{\epsilon} = -k_1\epsilon - k_2|\epsilon|^{1/2}\mathrm{sign}(\epsilon) + z(t) \tag{21}$$
$$\dot{z} = -k_3\mathrm{sign}(\epsilon) - \frac{d}{dt}d(t) \tag{22}$$

To demonstrate the stability of (21)-(22) the next family of Lyapunov functions is considered

$$V_i = \xi_i^T H \xi_i, \quad i = 1,2,...,n \tag{23}$$

$$\xi_i = \begin{bmatrix} |\epsilon_i|^{1/2}\mathrm{sign}(\epsilon_i) \\ \epsilon_i \\ z_i \end{bmatrix} \; H = \begin{bmatrix} (4k_3 + k_2^2) & k_1k_2 & -k_2 \\ k_1k_2 & k1^2 & -k_1 \\ -k_2 & -k_1 & 2 \end{bmatrix} \tag{24}$$

where ϵ_i and z_i $(i = 1,2,...,n)$ represent the i-th element of ϵ and z, respectively. The same dynamics and Lyapunov function were considered in (Moreno et al. (2008)). It was shown in (Moreno et al. (2008)) that if the tuning conditions (16) hold then the next estimation is in force for some $\gamma_1, \gamma_2 > 0$ and for all $i = 1, 2, ..., n$

$$\dot{V}_i \leq -\gamma_1 V_i - \gamma_2 \sqrt{V_i}, \tag{25}$$

that guarantees the finite time convergence of ϵ and z to zero starting from some finite moment $T > 0$. By the definition (20) of $z(t)$, it directly follows the condition (17). Theorem 1 is proven. □

3.2 Discrete-mode identification for switched FOS

Consider now a switched and unperturbed version of system (9)

$$D^\alpha x(t) = A_{j(t)} x(t) + B_{j(t)} u(t) + F_{j(t)} \quad 0 < \alpha < 1 \tag{26}$$

where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$. The so-called "discrete state" $j(t) \in \{1, 2, ..., q\}$ determines the actual system dynamics among the possible q "operation modes" represented by the triplets (A_i, B_i, F_i), $i = 1, 2, ..., q$. Consider the next expression for the discrete state

$$j(t) = j_k, \quad t_{k-1} \leq t < t_k, \quad k = 1, 2, ..., \infty \tag{27}$$

where $t_0 = 0$ and t_k are the "switching times" at which the discrete state is changing.

Let the next **dwell-time** restriction holds for the switching sequence

$$t_k - t_{k-1} \geq \Delta, \quad k = 1, 2, ..., \infty \tag{28}$$

The dwell time restrictions inhibits the occurrence of the so-called "Zeno phenomenon" for the considered switched dynamics.

Some specific operation modes in the set $\{1, 2, ..., q\}$ are supposed to correspond to **faulty conditions** for system (26), that need to be detected in real-time for monitoring and diagnosis purposes. The pair $(x(t), u(t))$ is supposed to be accessible for measurements. The task is to reconstruct the unknown discrete state $j(t)$.

A stack of q observers, one for each of the possible modes of operation, is suggested:

$$D^\alpha \hat{x}_i(t) = A_i x(t) + B_i u(t) + F_i + v_i, \quad i = 1, 2, ..., q \tag{29}$$

where v_i is the injection input for the $i-th$ observer, yet to be designed. Denote the observation error for the i-th observer as

$$e_i = \hat{x}_i - x_i \tag{30}$$

Then the switched and fractional-order error dynamics will be given by

$$\begin{aligned} D^\alpha e_i(t) = &(A_i - A_{j(t)})x(t) + (B_i - B_{j(t)})u(t) + \\ &+ (F_i - F_{j(t)}) + v_i(t) \end{aligned} \tag{31}$$

One can separately consider the error dynamics of the "**correct**" observer (i.e., that having the index i which matches the current mode of operation $j(t)$):

$$D^\alpha e_i(t) = v_i(t), \quad i = j(t) \tag{32}$$

and the error dynamics of the remaining "**wrong**" observers:

$$D^\alpha e_i(t) = \varphi_i^j(x, u, t) + v_i(t), \quad i \neq j(t) \tag{33}$$

where

$$\varphi_i^j(x, u, t) = \Delta A_i^j x(t) + \Delta B_i^j u(t) + \Delta F_i^j \tag{34}$$

$$\Delta A_i^j = A_i - A_{j(t)}, \quad \Delta B_i^j = B_i - B_{j(t)}, \quad \Delta F_i^j = F_i - F_{j(t)} \tag{35}$$

Concerning the state- and input-dependent functions $\varphi_i^j(x, u, t)$ entering the dynamics (33) of the "wrong" observers, in order to guarantee the identifiability of the correct mode they should not be identically zero. Then it is made the next assumption:

Assumption A_2

$$\left\| \varphi_i^j(x, u, t) \right\| \neq 0, \quad \forall i, j = 1, 2, ..., q, \; i \neq j \tag{36}$$

The above assumption A_2 should be understood as a constraint on the dynamics of the switched system and in particular on the resulting $(x-u)$ time evolutions. In other words, the manifolds $\varphi_i^j(x, u, t) = 0$ should not contain admissible $x(t) - u(t)$ trajectories of the switched system.

Functions $\varphi_i^j(x, u, t)$ are also supposed to be smooth enough according to the next assumption:

Assumption A_3 There is a constant Φ such that

$$\left\| \frac{d}{dt} \varphi_i^j(x, u, t) \right\| \leq \Phi, \quad \forall i, j = 1, 2, ..., q \tag{37}$$

Clearly, Assumption A_3 implies a bounded, although arbitrarily large, admissible domain for the evolution of the (x, u) trajectories in the respective space. This gives **semi-global** validity to the presented discrete mode observer.

The design of the observer injection terms is constructively developed by a technique that extends the one adopted in the previous subsection. The new "filtered" error variables σ_i are introduced

$$\sigma_i = I^{1-\alpha}(\hat{x}_i - x) = I^{1-\alpha} e_i \tag{38}$$

whose dynamics are of integer order:

$$\dot{\sigma}_i = \begin{cases} v_i(t) & i = j(t) \\ \varphi_i^j(x, u, t) + v_i(t) & i \neq j(t) \end{cases} \tag{39}$$

The observer injection terms are selected according to

$$v_i = v_{1i} + v_{2i} \tag{40}$$

$$v_{1i} = -k_1 \sigma_i - k_2 |\sigma_i|^{1/2} \mathrm{sign}(\sigma_i) \tag{41}$$

$$\dot{v}_{2i} = -k_3 \mathrm{sign}(\sigma_i) \tag{42}$$

with the next tuning inequalities imposed on the tuning coefficients:

$$k_1 > 2\Phi \quad k_2 > 0 \quad k_3 > \Phi \sqrt{k_1} \tag{43}$$

The main idea behind the proposed observer structure is that, after a finite transient starting at any switching times, the injection input $v_{2i}(t)$ will be identically zero, for the *correct* observer, and separated from zero for the *wrong* observers. This can be obtained if the finite-time convergence to zero of σ_i and $\dot{\sigma}_i$ is guaranteed for **all** the q observers ($i = 1, 2, ..., q$).

Let the maximal transient duration be denoted as T. Then, provided that $T < \Delta$, the next relationship directly derives by the conditions $\sigma_1 = \dot{\sigma}_1 = \dot{\sigma}_2 = ... = \dot{\sigma}_q = 0$:

$$v_{2i}(t) = \begin{cases} 0 & i = j(t) \\ -\varphi_i^j(x, u, t) & i \neq j(t) \end{cases}, \; t_{k-1} + T \leq t \leq t_k, \quad (44)$$

with $k = 1, 2, ...$. On the basis of (39), and by taking into account the Assumption A_2 as well, it can be estimated the actual discrete state $j(t)$ according to:

$$\hat{j}(t) = \arg\min_i \|v_{2i}(t)\| \quad (45)$$

The proposed scheme is summarized in the next Theorem:

Theorem 2. Consider the switched FOS (26), fulfilling the Assumptions A_2 and A_3, and the observer stack (29), (38), (40) -(42) with the observer gains chosen according to (43). Then, the discrete state estimation (45) will be such that

$$\hat{j}(t) = j(t), \qquad t_{k-1} + T \leq t \leq t_k, \quad k = 1, 2, ... \quad (46)$$

Proof of Theorem 2 By combining (32) and (33), the dynamics of the error variables e_i is given by:

$$D^\alpha e_i(t) = \begin{cases} v_i(t) & i = j(t) \\ \varphi_i^j(x, u, t) + v_i(t) & i \neq j(t) \end{cases} \quad (47)$$

Then, considering (38), the dynamics of the filtered error variables σ_i is of integer order, and it is given by (39). By (27), during the first switching interval $t \in (0, t_1)$ the actual mode is $j(t) = j_1$. Then (39) specializes as

$$\dot{\sigma}_{j_1} = v_{j_1}(t) \quad (48)$$
$$\dot{\sigma}_i(t) = \varphi_i^j(x, u, t) + v_i(t), \quad i = 1, 2, ..., q, \quad i \neq j_1 \quad (49)$$

Considering (40)-(42) into (48) yields

$$\dot{\sigma}_{j_1}(t) = -k_1\sigma_{j_1} - k_2|\sigma_{j_1}|^{1/2}\text{sign}(\sigma_{j_1}) + v_{2j_1}(t) \quad (50)$$
$$\dot{v}_{2j_1} = -k_3\text{sign}(\sigma_{j_1}) \quad (51)$$

$$\dot{\sigma}_i(t) = -k_1\sigma_i - k_2|\sigma_i|^{1/2}\text{sign}(\sigma_i) + v_{2i}(t) + \varphi_i^j(x, u, t) \quad (52)$$
$$\dot{v}_{2i} = -k_3\text{sign}(\sigma_i) \quad i = 1, 2, ..., q, \quad i \neq j_1 \quad (53)$$

By introducing the new coordinates

$$z_i(t) = v_{2i}(t) + \varphi_i^j(x, u, t) \quad (54)$$

one can augment and rewrite (52)-(53) as

$$\dot{\sigma}_i = -k_1\sigma_i - k_2|\sigma_i|^{1/2}\text{sign}(\sigma_i) + z_i(t) \quad (55)$$
$$\dot{z}_i = -k_3\text{sign}(\sigma_i) + \frac{d}{dt}\varphi_i^j(x, u, t), \quad i = 1, 2, ..., q, \quad i \neq j_1 \quad (56)$$

In light of the assumption A_3, systems (50)-(51) and (55)-(56) are formally equivalent to those dealt with in the proof of Theorem 1, and the same Lyapunov based stability analysis apply, which guarantees, taking into account the observer tuning conditions (43), the finite-time convergence to zero of σ_i ($i = 1, 2, ...n$), v_{2,j_1}, and z_i ($i = 1, 2, ..., q, \quad i \neq j_1$).

Let $T > 0$ be the transient time. By sufficiently increasing the Φ constant in the tuning formulas (43), the transient time can be made as small as desired (Levant (2003)), and in particular such that $T << \Delta$.

Then, considering (20), during the first switching interval the next conditions are achieved at the time $T < t < t_1$

$$v_{2j_1}(t) = 0 \quad (57)$$
$$v_{2i}(t) = -\varphi_i^j(x, u, t), \quad i = 1, 2, ..., q, \quad i \neq j_1 \quad (58)$$

By the assumption A_2, the residual-based estimation logic (45) provides the reconstruction of the discrete state after the finite transient time T, i.e.

$$\hat{j}(t) = j_1, \qquad 0 + T \leq t \leq t_1 \quad (59)$$

At the commutation time moment $t = t_1$ the discrete state will be changing. A new transient of length T is activated for the observer error dynamics, at the end of which the next conditions will be in force:

$$v_{2j_2}(t) = 0 \qquad t_1 + T < t < t_2 \quad (60)$$
$$v_{2i}(t) = -\varphi_i^j(x, u, t), \quad i = 1, 2, ..., q, \quad i \neq j_2 \quad (61)$$

Thus the estimation logic (45) still provides the reconstruction of the discrete state after the transient time, i.e.

$$\hat{j}(t) = j_2, \qquad t_1 + T < t < t_2 \quad (62)$$

By iteration on the successive switching intervals, condition (46), and so Theorem 2, is proven. □

Remark 1. The logic (45) appears not completely effective, since in some case it can happen that Assumption A_2 is violated and, as a result, also the wrong residuals $v_{2i}(t)$ ($i \neq j(t)$) can occasionally cross the zero value. Note that this event becomes less and less probable when the order n of the systems is growing and the dimension of the residuals will be growing accordingly. On the other hand, only the correct residual ($i = j(t)$) can stay at (or, more realistically, close to) zero for long time intervals. Hence the next averaged residuals can be considered

$$R_i(t) = \int_{t-\delta}^{t} \|v_{2i}(\tau)\| d\tau. \quad (63)$$

where δ is a small time delay (the width of a receding horizon window of observation for the residuals $\|v_{2i}\|$), along with the corresponding modified discrete state evaluation strategy

$$\hat{j}(t) = \arg\min_i R_i(t) \quad (64)$$

4. SIMULATION RESULTS

4.1 Disturbance estimation test

Consider the commensurate perturbed fractional-order system

$$D^{0.6}\mathbf{x} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \mathbf{u} + \mathbf{d} \quad (65)$$

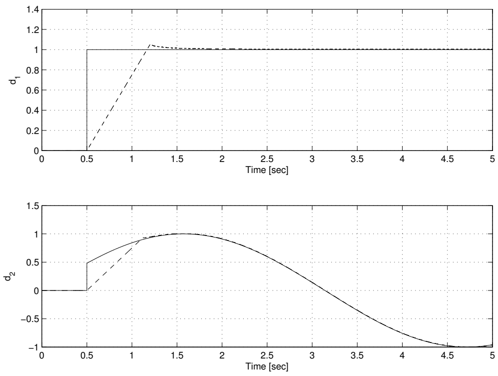

Fig. 1. Disturbance estimation test. Actual and estimated disturbance.

where $\mathbf{d}$ is a vector of unmeasurable disturbances to be estimated, selected as

$$\mathbf{d}(t) = \begin{bmatrix} 1 \\ \sin(t) \end{bmatrix} h(t - 0.5), \tag{66}$$

with $h(t)$ being the unit step signal. Clearly system (65)-(66) belongs to the class of systems (9), and the Assumption A_3 is fulfilled at any $t \geq 0.5$ with the value $d_{Md} = 1$.

The state is assumed to be fully available and the algorithm (11)-(15) can be used for disturbance estimation. The sampling time was chosen to be $T_s = 1ms$, while the parameters of the disturbance estimation algorithm were selected as $k_1 = 1.5$, $k_2 = 0.25$ and $k_3 = 2$ in accordance with the tuning formulas (16).

Figure 1 depicts the components of the actual and estimated disturbance signal. The solid line represents the actual profile of the disturbance signals. The estimated disturbances are plotted with dashed lines, which both converge to the actual disturbance profile.

4.2 Discrete state estimation test

Now let us consider the discrete state estimation problem for the affine switched FOS (26) with the order of differentiation $\alpha = 0.6$ and $q = 3$ distinct sub-models defined by the matrix triplets

$$A_1 = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \qquad F_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{67}$$

$$A_2 = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \qquad F_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{68}$$

$$A_3 = \begin{bmatrix} -6 & -4 \\ 1.5 & -1 \end{bmatrix}, \qquad B_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad F_3 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{69}$$

The discrete state is changed according to the next rule

$$j(t) = \begin{cases} 1 & 0 \leq t < 2 \\ 2 & 2 < t \leq 4 \\ 3 & 4 < t <\leq 6 \end{cases} \tag{70}$$

The parallel stage of observers (29), (38)-(42) has been implemented with the gains $k_1 = 2$, $k_2 = 2$ $k_3 = 2$. The modified residual evaluation strategy (63)-(64) has been implemented, with the width of the time window chosen as $\delta = 0.1s$. The actual and discrete state are depicted in the Figure 2 which confirms the correct performance of the discrete mode observer.

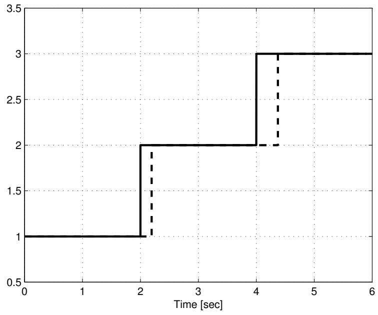

Fig. 2. Discrete state estimation test. Actual and estimated discrete state.

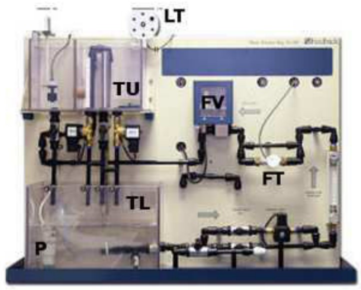

Fig. 3. "Feedback" Level/Flow Process Control System, PROCON 38-001

5. EXPERIMENTAL FAULT DETECTION OF A HYDRAULIC PLANT

The discrete state estimation algorithm previously described will now be exploited to detect certain faults in a laboratory hydraulic system. The experimental hydraulic setup is shown in Figure 3. The centrifugal pump (**P**) draws the water from the lower tank (**TL**) into the upper tank (**TU**). The flow is adjusted by the electrical servo valve (**FV**). The flow is measured using the flow meter and the pulse flow transmitter (**FT**). The level in the upper tank is measured by the float level sensor and transmitter (**LT**). The considered fault is a reduction of the pump rotating speed, which reduces the amount of flow. The fault has been reproduced in the experimental setup by reducing the pump control signals from the nominal value (healthy condition) to the 70% (faulty condition). Two fractional order models have been derived via least square identification for the healthy and faulty behaviour. Then, the suggested method for discrete state reconstruction can be applied as a fault diagnosis logic, by identifying whether the actual mode of operation is the nominal or faulty one.

Before any processing the measurements were scaled into the $[0, 1]$ interval, with 0 and 1 denoting the minimal and maximal measured value of the physical quantity. Denote the normalized flow measurements by $f(t)$ and the normalized pump control signal by $v(t)$. The measurement sampling time was selected as $T_s = 10ms$.

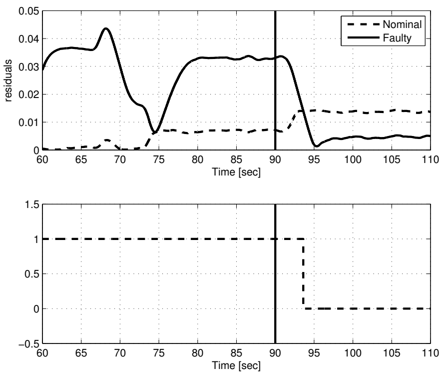

Fig. 4. Pump fault detection test. The residuals $R_1(t)$ and $R_2(t)$ (upper plot) and the estimated discrete state (lower plot)

Two scalar ($n = m = 1$) affine models have been identified for the nominal and faulty working regimes. The **nominal model** is

$$D^{0.9}f(t) = -0.0186f(t) + 0.03v(t) - 0.0043 \qquad (71)$$

and the **faulty model** was

$$D^{0.9}f(t) = -0.6532f(t) + 0.1412v(t) + 0.5082 \qquad (72)$$

The discrete state observation scheme has been implemented with the gains $k_1 = 1$, $k_2 = 2$ $k_3 = 0.5$ and $\delta = 1s$, according to the modified residual evaluation strategy (63)-(64). During the experimental test, the fault is activated at time $t = 90$. The residuals corresponding to the two observers are presented in Figure 4 along with the estimated discrete state where "1" stands for the nominal behaviour, and "0" stands for the faulty one. In both plots, the solid vertical line denotes the instant at which the fault occurs. The fault is identified after less than 4 seconds.

6. CONCLUSIONS

Second-order sliding mode techniques have been developed and experimentally tested in order to solve certain estimation and identification problems for some classes of fractional-order dynamics. The experimental verification, conducted by means of a laboratory apparatus of an hydraulic process, has shown satisfactory performance. Among the possible lines of improvement of the presented results, more general classes of linear FOS (e.g. non-commensurate ones), as well as some classes of nonlinear fractional-order dynamics could be studied.

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