

## Stability Analysis of Fuzzy Logic Control Systems for a Class of Nonlinear SISO Discrete-Time Systems

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**Abstract:** This paper suggests a new stability analysis theorem dedicated to a class of fuzzy logic control systems. The fuzzy logic control systems consist of nonlinear Single Input-Single Output (SISO) discrete-time processes controlled by Takagi-Sugeno fuzzy logic controllers. The stability analysis is conducted on the basis of Lyapunov's direct method with quadratic Lyapunov function candidates. The theorem proves that if the derivative of the Lyapunov function candidate is negative definite in the active region of each fuzzy rule then the fuzzy logic control system will be asymptotically stable in the sense of Lyapunov. Sufficient stability conditions are offered. An example related to the design of a stable fuzzy logic control system for the discrete-time Lorenz chaotic system and simulation results are included.

**Keywords:** Discrete-time systems, fuzzy logic control, Lyapunov function candidates, nonlinear systems, stability analysis.

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### 1. INTRODUCTION

The stability analysis of discrete-time fuzzy logic control systems has received considerable attention recently and many significant results have been reported recently. The stable indirect adaptive fuzzy logic control of nonlinear discrete-time systems is investigated in (Qi and Brdys, 2008). The fuzzy logic control systems with immeasurable premise variables are analyzed in (Yoneyama, 2008) and their design is treated as an  $H_\infty$  output feedback control problem. The dynamic output feedback  $H_\infty$  control of discrete-time Takagi-Sugeno (T-S) fuzzy systems is discussed in (Dong and Yang, 2009), and a switched dynamic parallel distributed compensation scheme is offered. The connection between the discrete-time fuzzy logic control systems obtained by quantization and their approximate discrete-time model is investigated in (Kim and Lee, 2010). An exponential input-to-state stability criterion for discrete-time impulsive hybrid systems is given in (Bin and David, 2010). The stabilization of Roesser type discrete-time nonlinear 2-D systems is analyzed in (Xie and Zhang, 2010) by relaxed quadratic stabilization techniques which are applied for the 2-D case. Adaptive-neuro-fuzzy-based sensorless control solutions are analyzed in (Sadighi and Kim, 2011).

The analysis of the current literature presented before outlines that the majority of theoretical approaches are based on Linear Matrix Inequalities (LMIs) derived in terms of

Lyapunov's direct method. The conservativeness of several classes of Lyapunov function candidates must be addressed with this regard as shown in (Hušek, 2008; Lam, 2009; Sala, 2009; Lendek et al., 2010).

This paper gives a new stability analysis theorem based on Lyapunov's direct method (Kalman and Bertram, 1960) dedicated to a class of nonlinear Single Input-Single Output (SISO) discrete-time processes controlled by T-S fuzzy logic controllers (FLCs). This original approach is built upon the adaptation of our continuous-time stability analysis results (Precup et al., 2009a, 2009b) to discrete-time systems. Therefore it belongs to the systematic methods needed in the development of fuzzy systems (Boukezzoula et al., 2007; Bellomo et al., 2008; Gál et al., 2009; Kurnaz et al, 2010).

The stability analysis approach offered in this paper is advantageous with respect to other similar approaches because it does not compute a common positive definite matrix, it does not require the process linearization, it is expressed in terms of convenient inequality-type stability conditions, and it offers a conservatism reduction. The stability analysis conditions are derived from the quadratic Lyapunov function candidates.

The paper is organized as follows. Section 2 recalls the Takagi-Sugeno fuzzy control systems controlling nonlinear discrete-time processes. Next Section 3 presents and proves the stability analysis theorem for nonlinear SISO discrete-

time processes controlled by T-S FLCs. An example described in Section 4 illustrates the performance achieved by the application of the theoretical results to the stable design of a fuzzy logic control system for the discrete-time Lorenz chaotic system. Section 5 concludes the paper.

## 2. DEFINITION OF FUZZY LOGIC CONTROL SYSTEMS FOR NONLINEAR SISO DISCRETE-TIME SYSTEMS

The discrete time fuzzy logic control system consists of a discrete time process which is controlled by a T-S FLC according to the structure presented in Fig. 1. The controlled process is characterized by the by the SISO discrete-time state-space mathematical model

$$\begin{aligned} \mathbf{x}(t+1) &= \mathbf{f}(\mathbf{x}(t)) + \mathbf{b}(\mathbf{x}(t))u(t), \quad t \in N, \mathbf{x}(0) = \mathbf{x}_0 \in X, \\ y(t) &= g(\mathbf{x}(t)), \end{aligned} \quad (1)$$

where  $r$  is the reference input,  $y$  is the controlled output,  $\mathbf{x}(t) = [x_1(t) \ x_2(t) \ \dots \ x_n(t)]^T \in X \subset R^n$  is the state vector,  $n \in N$ ,  $n \geq 1$ ,  $X$  is the universe of discourse,  $T$  stands for matrix transposition, the time variable  $t$  (with the initial time moment  $t_0 = 0$ ) will be omitted as follows for simplicity,  $\mathbf{x}_0$  are the initial conditions, the continuous functions

$$\begin{aligned} \mathbf{f}, \mathbf{b} : R^n &\rightarrow R^n, \\ \mathbf{f}(\mathbf{x}(t)) &= [f_1(\mathbf{x}(t)) \ f_2(\mathbf{x}(t)) \ \dots \ f_n(\mathbf{x}(t))]^T, \\ \mathbf{b}(\mathbf{x}(t)) &= [b_1(\mathbf{x}(t)) \ b_2(\mathbf{x}(t)) \ \dots \ b_n(\mathbf{x}(t))]^T, \end{aligned} \quad (2)$$

and  $g : R^n \rightarrow R$  describe the dynamics of the process, and  $u$  is the control signal produced by the FLC.

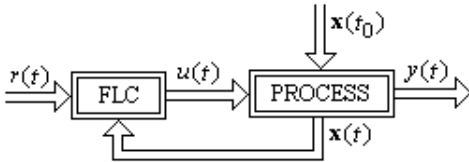


Fig. 1. Fuzzy logic control system structure.

The disturbance input is absent in Fig. 1 because it does not affect the internal stability of the fuzzy logic control system. The reference input fulfils the condition  $r(t) = \text{const}$  for stabilizing control systems.

The rule base of the FLC consists of  $p$  fuzzy rules. The  $k$ -th rule in the rule base of the T-S FLC is

$$\begin{aligned} \text{Rule } k : \\ \text{IF } x_1(t) \text{ IS } \tilde{X}_{1k} \text{ AND } x_2(t) \text{ IS } \tilde{X}_{2k} \text{ AND } \dots x_n(t) \text{ IS } \tilde{X}_{nk} \\ \text{THEN } u(t) = u_k(\mathbf{x}(t)), \quad k = \overline{1, p}, \quad p \in N, \quad p \geq 2, \end{aligned} \quad (3)$$

where  $\tilde{X}_{ik}$  are fuzzy sets (with the universes  $X_{ik}$ ,  $i = \overline{1, n}$ ) that describe the linguistics terms (LTs) of input variables,  $u = u_k(\mathbf{x})$  is the control signal calculated as the output of the rule  $k$ , and the function AND is a t-norm.  $u_k(\mathbf{x}(t))$  can be a

constant or a function of the state vector  $\mathbf{x}(t)$ . Each fuzzy rule generates a firing degree  $\alpha_k$ ,  $0 \leq \alpha_k \leq 1$ , according to

$$\alpha_k(\mathbf{x}(t)) = \text{AND}(\mu_{\tilde{X}_{1k}}(x_1(t)), \mu_{\tilde{X}_{2k}}(x_2(t)), \dots, \mu_{\tilde{X}_{nk}}(x_n(t))), \quad (4)$$

$$k = \overline{1, p},$$

where  $\mu_{\tilde{X}_{ik}} : X_{ik} \rightarrow [0, 1]$ ,  $k = \overline{1, p}$ , are the membership functions of the LTs associated to the state variables  $x_i(t)$ ,  $\bigcup_{k=1}^p X_{ik} = X_i$ ,  $i = \overline{1, n}$ , and  $X_1 \times X_2 \times \dots \times X_n = X$ .

It is assumed next that

$$\sum_{k=1}^p \alpha_k(\mathbf{x}(t)) > 0, \quad \forall \mathbf{x}(t) \in X. \quad (5)$$

The control signal  $u$  is a function of  $\alpha_k$ , and it is calculated as follows as a result of the weighted sum defuzzification method:

$$u(t) = \frac{\sum_{k=1}^p (\alpha_k(\mathbf{x}(t))u_k(t))}{\sum_{k=1}^p \alpha_k(\mathbf{x}(t))}. \quad (6)$$

The fuzzy logic control systems defined here are used in various applications and controller structures (Škrjanc et al., 2004; Zhang et al., 2004; Boucher et al., 2007; Mok and Chan, 2008; Silva et al., 2008; Ferreira et al., 2009; Pizzileo et al., 2009; Vrančić et al., 2010).

*Definition 1:* An active region of the fuzzy rule  $k$ ,  $k = \overline{1, p}$ , is defined as the set  $X_k^A = \{\mathbf{x}(t) \in X \mid \alpha_k(\mathbf{x}(t)) > 0\}$ .

## 3. STABILITY ANALYSIS THEOREM

This section is dedicated to the formulation and proof of the new stability analysis theorem based on Lyapunov's direct method. The theorem is supported by the following well acknowledged result.

*Theorem 1* (Khalil, 2002): Consider the dynamical system (1). If there exists a continuous radially unbounded Lyapunov function candidate  $V : R^n \rightarrow R$  such that  $V(\mathbf{x}) > 0$ ,  $\forall \mathbf{x} \neq \mathbf{0}$ ,  $V(\mathbf{0}) = 0$ , and

$$V(\mathbf{x}(t+1)) < V(\mathbf{x}(t)), \quad (7)$$

then the equilibrium point at the origin  $\mathbf{x}(t) = \mathbf{0} = [0 \ 0 \ \dots \ 0]^T \in R^n$  of the system (1) will be globally asymptotically stable.

Let the controlled process be characterized by the SISO discrete-time state-space mathematical model defined in (1) and let the radially unbounded function  $V$  satisfy the conditions in the hypothesis of Theorem 1 i.e.  $V : R^n \rightarrow R$  such that  $V : R^n \rightarrow R$ ,  $V(\mathbf{x}) > 0$ ,  $\forall \mathbf{x} \neq \mathbf{0}$ ,  $V(\mathbf{0}) = 0$ . The first difference of the function  $V(\mathbf{x}(t))$  along the trajectory of

(1), denoted by  $\Delta V(\mathbf{x}(t))$ , is supposed to fulfil the condition (7) which is transformed into

$$\Delta V(\mathbf{x}(t)) = V(\mathbf{x}(t+1)) - V(\mathbf{x}(t)) < 0. \quad (8)$$

Using the notation  $V_k(\mathbf{x}(t))$  for the Lyapunov function candidate  $V(\mathbf{x}(t))$  which is considered along the trajectory of the system (1) for  $u(t) = u_k(\mathbf{x}(t))$ , the relations (7) and (8) lead to the following condition for the first difference of  $V_k(\mathbf{x}(t))$ ,  $\Delta V_k(\mathbf{x}(t))$ :

$$\Delta V_k(\mathbf{x}(t)) = V_k(\mathbf{x}(t+1)) - V_k(\mathbf{x}(t)), \forall \mathbf{x} \in X_k^A, k = \overline{1, p}. \quad (9)$$

The next stability analysis theorem dedicated to the accepted class of fuzzy logic control systems is derived from Theorem 1. This theorem proves that if the derivative of the Lyapunov function candidate is negative definite for each fuzzy rule then the controlled system will be globally asymptotically stable in the sense of Lyapunov (ISL).

The stability theorem presented as follows offers sufficient conditions for the global asymptotical stability of fuzzy logic control systems. The theorem enables the design of T-S FLCs for a class of nonlinear SISO discrete-time systems with guaranteed closed-loop stability. The premise of the stability criterion in the following theorem concerns the application of each rule individually to the plant leading to a closed-loop subsystem (that consists of the process and the T-S FLC with a single rule  $k$ ) that is stable ISL in the rule's active region, and each rule shares a common Lyapunov function. The theorem proves that if each subsystem is stable ISL under a common Lyapunov function, the fuzzy logic control system will be also stable ISL. The convexity of the defuzzification method guarantees that any of the (stable) rules of the T-S FLC can stabilize the system. This is very advantageous as no fuzzy model of the process is involved, the number of subsystems is relatively small, and the common Lyapunov function can be found easily.

*Theorem 2:* Let the fuzzy logic control system consist of the T-S FLC defined in Section 2 and the nonlinear SISO discrete-time process modelled by the state-space mathematical model defined in (1). Let

$$V : X \rightarrow R, V(\mathbf{x}(t)) = \mathbf{x}^T(t) \mathbf{P} \mathbf{x}(t), \quad (10)$$

where  $\mathbf{P}$  is an  $n \times n$  positive definite matrix such that

$$\Delta V_k(\mathbf{x}(t)) < 0, \forall \mathbf{x} \in X_k^A, k = \overline{1, p}. \quad (11)$$

Then all state vectors  $\mathbf{x}(t)$  will converge globally asymptotically to the origin, viz.  $\mathbf{x}(t) = \mathbf{0}$ , as  $t \rightarrow \infty$ .

*Proof:* The hypothesis (11) of Theorem 2 leads to

$$\Delta V_k(\mathbf{x}(t)) = V_k(\mathbf{x}(t+1)) - V_k(\mathbf{x}(t)) < 0, \forall \mathbf{x} \in X_k^A, k = \overline{1, p}. \quad (12)$$

The term  $\mathbf{x}(t+1)$  is next substituted from (1) into (12):

$$\begin{aligned} \Delta V_k(\mathbf{x}(t)) &= V_k(\mathbf{f}(\mathbf{x}(t)) + \mathbf{b}(\mathbf{x}(t))u_k(t)) - V_k(\mathbf{x}(t)) \\ &= [\mathbf{f}(\mathbf{x}(t)) + \mathbf{b}(\mathbf{x}(t))u_k(t)]^T \mathbf{P} \\ &\quad [\mathbf{f}(\mathbf{x}(t)) + \mathbf{b}(\mathbf{x}(t))u_k(t)] - \mathbf{x}^T(t) \mathbf{P} \mathbf{x}(t) \\ &= \mathbf{f}^T(\mathbf{x}(t)) \mathbf{P} \mathbf{f}(\mathbf{x}(t)) - \mathbf{x}^T(t) \mathbf{P} \mathbf{x}(t) \\ &\quad + \mathbf{b}^T(\mathbf{x}(t)) \mathbf{P} \mathbf{b}(\mathbf{x}(t))u_k^2(t) \\ &\quad + [\mathbf{f}^T(\mathbf{x}(t)) \mathbf{P} \mathbf{b}(\mathbf{x}(t)) + \mathbf{b}^T(\mathbf{x}(t)) \mathbf{P} \mathbf{f}(\mathbf{x}(t))]u_k(t) < 0. \end{aligned} \quad (13)$$

The multiplication of (13) by  $\alpha_k(\mathbf{x}(t))$  and the calculation of the sum result in

$$\begin{aligned} &[\mathbf{f}^T(\mathbf{x}(t)) \mathbf{P} \mathbf{f}(\mathbf{x}(t)) - \mathbf{x}^T(t) \mathbf{P} \mathbf{x}(t)] \sum_{k=1}^p \alpha_k(\mathbf{x}(t)) \\ &+ \mathbf{b}^T(\mathbf{x}(t)) \mathbf{P} \mathbf{b}(\mathbf{x}(t)) \sum_{k=1}^p [\alpha_k(\mathbf{x}(t))u_k^2(t)] \\ &+ [\mathbf{f}^T(\mathbf{x}(t)) \mathbf{P} \mathbf{b}(\mathbf{x}(t)) + \mathbf{b}^T(\mathbf{x}(t)) \mathbf{P} \mathbf{f}(\mathbf{x}(t))] \\ &\quad \sum_{k=1}^p [\alpha_k(\mathbf{x}(t))u_k(t)] < 0. \end{aligned} \quad (14)$$

The relation (14) is divided by  $\sum_{k=1}^p \alpha_k(\mathbf{x}(t)) > 0$ , and the sums are manipulated as follows:

$$\begin{aligned} &\mathbf{f}^T(\mathbf{x}(t)) \mathbf{P} \mathbf{f}(\mathbf{x}(t)) - \mathbf{x}^T(t) \mathbf{P} \mathbf{x}(t) + \mathbf{b}^T(\mathbf{x}(t)) \mathbf{P} \mathbf{b}(\mathbf{x}(t)) \\ &\quad \left\{ \sum_{k=1}^p [\alpha_k(\mathbf{x}(t))u_k^2(t)] \right\} / \left\{ \sum_{k=1}^p \alpha_k(\mathbf{x}(t)) \right\} \\ &+ [\mathbf{f}^T(\mathbf{x}(t)) \mathbf{P} \mathbf{b}(\mathbf{x}(t)) + \mathbf{b}^T(\mathbf{x}(t)) \mathbf{P} \mathbf{f}(\mathbf{x}(t))] \\ &\quad \left\{ \sum_{k=1}^p [\alpha_k(\mathbf{x}(t))u_k(t)] \right\} / \left\{ \sum_{k=1}^p \alpha_k(\mathbf{x}(t)) \right\} < 0. \end{aligned} \quad (15)$$

Accounting for (6) the inequality (15) becomes

$$\begin{aligned} &\mathbf{f}^T(\mathbf{x}(t)) \mathbf{P} \mathbf{f}(\mathbf{x}(t)) - \mathbf{x}^T(t) \mathbf{P} \mathbf{x}(t) + \mathbf{b}^T(\mathbf{x}(t)) \mathbf{P} \mathbf{b}(\mathbf{x}(t)) \\ &\quad \left\{ \sum_{k=1}^p [\alpha_k(\mathbf{x}(t))u_k^2(t)] \right\} / \left\{ \sum_{k=1}^p \alpha_k(\mathbf{x}(t)) \right\} \\ &+ [\mathbf{f}^T(\mathbf{x}(t)) \mathbf{P} \mathbf{b}(\mathbf{x}(t)) + \mathbf{b}^T(\mathbf{x}(t)) \mathbf{P} \mathbf{f}(\mathbf{x}(t))]u(t) < 0. \end{aligned} \quad (16)$$

Cauchy-Buniakovski-Schwarz's inequality results next in

$$\begin{aligned} &\left\{ \sum_{k=1}^p [\sqrt{\alpha_k(\mathbf{x}(t))}]^2 \right\} \left\{ \sum_{k=1}^p [\sqrt{\alpha_k(\mathbf{x}(t))}u_k(t)]^2 \right\} \\ &\geq \left\{ \sum_{k=1}^p [\sqrt{\alpha_k(\mathbf{x}(t))}\sqrt{\alpha_k(\mathbf{x}(t))}u_k(t)] \right\}^2. \end{aligned} \quad (17)$$

which is equivalent to

$$\begin{aligned} &\sum_{k=1}^p [\alpha_k(\mathbf{x}(t))u_k^2(t)] \geq \left\{ \sum_{k=1}^p [\alpha_k(\mathbf{x}(t))u_k(t)] \right\}^2 \\ &\quad / \left\{ \sum_{k=1}^p \alpha_k(\mathbf{x}(t)) \right\}. \end{aligned} \quad (18)$$

The division of (18) by  $\sum_{k=1}^p \alpha_k(\mathbf{x}(t)) > 0$  using (6) leads to the result

$$\left\{ \sum_{k=1}^p [\alpha_k(\mathbf{x}(t))u_k^2(t)] \right\} / \left\{ \sum_{k=1}^p \alpha_k(\mathbf{x}(t)) \right\} \geq \left\{ \left\{ \sum_{k=1}^p [\alpha_k(\mathbf{x}(t))u_k(t)] \right\} / \left\{ \sum_{k=1}^p \alpha_k(\mathbf{x}(t)) \right\} \right\}^2 = u^2(t). \quad (19)$$

The expression of  $\Delta V(\mathbf{x}(t))$  results from the equations (1) and (8), and it is similar to (13):

$$\begin{aligned} \Delta V(\mathbf{x}(t)) &= \mathbf{f}^T(\mathbf{x}(t)) \mathbf{P} \mathbf{f}(\mathbf{x}(t)) - \mathbf{x}^T(t) \mathbf{P} \mathbf{x}(t) \\ &+ \mathbf{b}^T(\mathbf{x}(t)) \mathbf{P} \mathbf{b}(\mathbf{x}(t))u^2(t) \\ &+ [\mathbf{f}^T(\mathbf{x}(t)) \mathbf{P} \mathbf{b}(\mathbf{x}(t)) + \mathbf{b}^T(\mathbf{x}(t)) \mathbf{P} \mathbf{f}(\mathbf{x}(t))]u(t). \end{aligned} \quad (20)$$

Next the following inequality is obtained from (19) and (20):

$$\begin{aligned} \Delta V(\mathbf{x}(t)) &< \mathbf{f}^T(\mathbf{x}(t)) \mathbf{P} \mathbf{f}(\mathbf{x}(t)) - \mathbf{x}^T(t) \mathbf{P} \mathbf{x}(t) \\ &+ \mathbf{b}^T(\mathbf{x}(t)) \mathbf{P} \mathbf{b}(\mathbf{x}(t)) \left\{ \sum_{k=1}^p [\alpha_k(\mathbf{x}(t))u_k^2(t)] \right\} / \left\{ \sum_{k=1}^p \alpha_k(\mathbf{x}(t)) \right\} \\ &+ [\mathbf{f}^T(\mathbf{x}(t)) \mathbf{P} \mathbf{b}(\mathbf{x}(t)) + \mathbf{b}^T(\mathbf{x}(t)) \mathbf{P} \mathbf{f}(\mathbf{x}(t))]u(t). \end{aligned} \quad (21)$$

Finally (16) and (21) result in

$$\Delta V(\mathbf{x}(t)) < 0. \quad (22)$$

Therefore the equilibrium point at the origin  $\mathbf{x} = \mathbf{0}$  will be globally asymptotically stable. The proof is now complete. In other words, if every rule of the T-S FLC applied individually to the process (1) gives a stable subsystem ISL in the active region of the fuzzy rule subject to a common Lyapunov function and the defuzzification method is carried out in terms of (6), the fuzzy logic control system will be stable ISL.

Concluding, the above stability theorem ensures sufficient stability conditions concerning the accepted class of fuzzy logic control systems defined in Section 2. The dependency between the FLC and the process is outlined by (13) that highlight the stability analysis of each subsystem in terms of calculations and conditions related to  $u_k$ .

#### 4. DESIGN OF STABLE FUZZY LOGIC CONTROL SYSTEM

This section presents an example that deals with the stable design of a T-S FLC for the discrete-time Lorenz chaotic system. The discrete-time Lorenz system is defined usually in terms of the following system of three coupled ordinary differential equations ( $n = 3$ ):

$$\mathbf{x}(t+1) = \begin{pmatrix} \sigma[x_2(t) - x_1(t)] \\ x_1(t)[\rho - x_3(t)] - x_2(t) \\ x_1(t)x_2(t) - \beta x_3(t) \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} u(t), \mathbf{x}(0) = \mathbf{x}_0, \quad (23)$$

that model the convective motion of fluid cell which is warmed from below and cooled from above. This system is obtained by discretizing the continuous-time Lorenz system (Stuart and Humphries, 1998) in terms of considering the sampling period  $T_s = 0.01s$ .

The parameters  $\sigma, \rho, \beta > 0$  in (23) are referred to as the Prandtl number, the Rayleigh number, and the physical

proportion, respectively. The state-space equations defined in (23) exhibit chaotic behaviour i.e. they are extremely sensitive to initial conditions. A small change in the initial conditions leads quickly to large differences in the corresponding solutions. The classical values used to illustrate the chaos are  $\sigma = 10$  and  $\beta = 8/3$ .

Let the universe of discourse be

$$X = [-40, 40] \times [-40, 40] \times [-40, 40]. \quad (24)$$

The fuzzification module of the T-S FLC is set according to Fig. 2 which shows the membership functions that describe the LTs of the linguistic variables  $x_1$  and  $x_2$ . P, Z and N indicate the LTs representing "Positive", "Zero" and "Negative" values, respectively. The inference engine employs the fuzzy logic operators AND and OR implemented by the *min* and *max* functions, respectively.

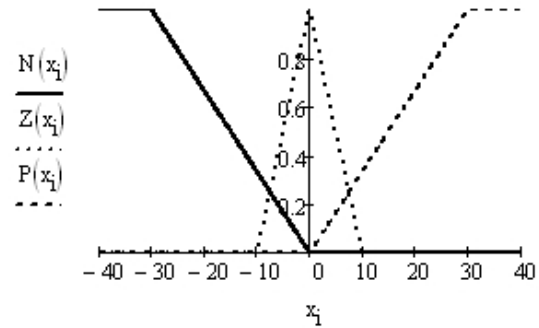


Fig. 2. Input membership functions of the Takagi-Sugeno fuzzy logic controller.

The weighted sum defuzzification method is used, and the inference engine is assisted by the complete rule base ( $p = 9$ )

- Rule 1: IF  $x_1$  IS P AND  $x_2$  IS P THEN  $u = u_1$ ,
- Rule 2: IF  $x_1$  IS N AND  $x_2$  IS N THEN  $u = u_2$ ,
- Rule 3: IF  $x_1$  IS P AND  $x_2$  IS N THEN  $u = u_3$ ,
- Rule 4: IF  $x_1$  IS N AND  $x_2$  IS P THEN  $u = u_4$ ,
- Rule 5: IF  $x_1$  IS P AND  $x_2$  IS Z THEN  $u = u_5$ ,
- Rule 6: IF  $x_1$  IS N AND  $x_2$  IS Z THEN  $u = u_6$ ,
- Rule 7: IF  $x_1$  IS Z AND  $x_2$  IS P THEN  $u = u_7$ ,
- Rule 8: IF  $x_1$  IS Z AND  $x_2$  IS N THEN  $u = u_8$ ,
- Rule 9: IF  $x_1$  IS Z AND  $x_2$  IS Z THEN  $u = u_9$ .

Theorem 2 will be applied in the sequel in order to derive the control laws in the consequents of the rules  $u_k, k = \overline{1,9}$ . Let the Lyapunov function candidate be

$$V(\mathbf{x}(t)) = 0.5[x_1^2(t) + x_2^2(t) + x_3^2(t)], \quad (26)$$

which is a continuously differentiable positive function on the domain  $X$ . The first difference of the function  $V(\mathbf{x})$  along the trajectory of the system defined in (23), denoted by  $\Delta V(\mathbf{x}(t))$ , is

$$\Delta V(\mathbf{x}(t)) = -\sigma x_1^2(t) - x_2^2(t) - \beta x_3^2(t) + x_1(t)x_2(t)(\sigma + \rho) + x_1(t)u(t), \quad (27)$$

therefore

$$\Delta V_k(\mathbf{x}(t)) = -\sigma x_1^2(t) - x_2^2(t) - \beta x_3^2(t) + x_1(t)x_2(t)(\sigma + \rho) + x_1(t)u_k(t), k = 1, 9. \quad (28)$$

The control laws in the consequents of the rules defined in (25) obtain the following expressions such that to fulfil the condition (11) in Theorem 2:

$$\begin{aligned} u_1(t) &= -40(\sigma + \rho), u_2(t) = 40(\sigma + \rho), \\ u_3(t) &= 0, u_4(t) = 0, u_5(t) = -10(\sigma + \rho), \\ u_6(t) &= -10(\sigma + \rho), u_7(t) = -x_2(t)(\sigma + \rho), \\ u_8(t) &= -x_2(t)(\sigma + \rho), u_9(t) = -x_2(t)(\sigma + \rho). \end{aligned} \quad (29)$$

Concluding, due to Theorem 2 it results that the fuzzy logic control system composed by this T-S FLC and the Lorenz process modelled by (23) is globally asymptotically stable in the sense of Lyapunov at the origin. Considering the values of the process parameters  $\sigma = 10$ ,  $\rho = 28$  and  $\beta = 8/3$ , the initial state  $x_1(0) = 1$ ,  $x_2(0) = -1$  and  $x_3(0) = 1$ , the response of  $x_3$  versus time and the 3D phase portrait are presented re presented in Fig. 3 and Fig. 4, respectively.

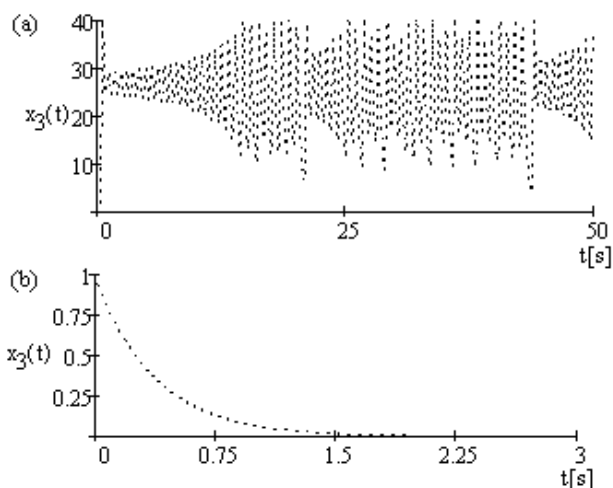


Fig. 3. State variable  $x_3$  versus time for the Lorenz system (a) and for the fuzzy logic control system (b).

## 5. CONCLUSIONS

The paper has proposed a simple and effective Takagi-Sugeno fuzzy logic controller meant for stabilizing a class of nonlinear SISO discrete-time systems. A new theorem is formulated and proved. The sufficient stability conditions offered in this theorem guarantee the global asymptotic stability of the fuzzy logic control systems.

The new approach suggested in this paper decomposes the stability analysis to the analysis of each rule. Therefore the complexity and the conservatism are reduced in comparison with the popular LMI approaches.

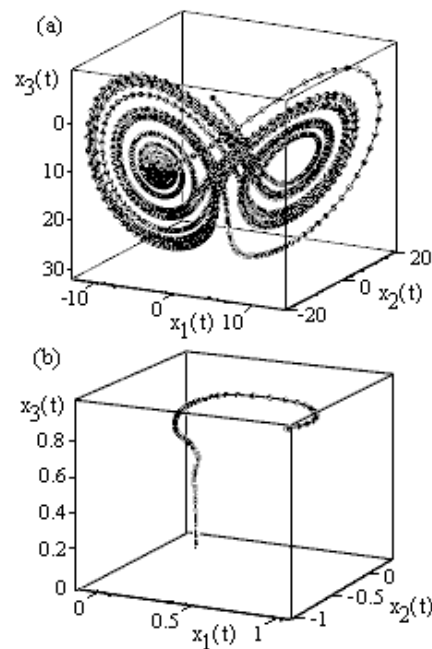


Fig. 4. 3D phase portrait of the Lorenz system (a) and of the fuzzy logic control system (b).

Compared to the use of PDC controllers for the same process (Asemani and Majd, 2009; Baek, 2010; Yang and Cai, 2010), our stability analysis approach is more simple. It does not require the fuzzy model of the plant and the eventual proof of the sliding mode existence conditions specific to sliding mode fuzzy controllers.

Our approach allows a wide area of fuzzy logic control system applications. The further research will be dedicated to the generalization of the proposed theorem to other fuzzy logic controller structures and applications.

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