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An anti-windup framework for systems with non-standard actuator characteristics

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Abstract: We examine anti-windup architectures for linear systems containing what might be broadly called "non-standard actuators" - actuators which may not be easily approximated by simply a saturation or rate-limit, but which are a more complex interconnection between linear elements and sector-bounded nonlinearities. An "LFT" type approach is taken to modelling such actuators and a fairly generic anti-windup approach is developed, allowing for a wide class of nonlinear actuators, including, but not limited to, actuator models with rate, amplitude or deadzone limitations. We also consider the synthesis for this class of actuators using the coprime factor approach, and it is shown that, in some circumstances, the IMC approach to anti-windup compensation yields desirable closed-loop robust stability properties.

Keywords: Actuators, anti-windup, robust control

1. INTRODUCTION

For many years, anti-windup techniques have been used in industry to address problems caused by actuator saturation in control systems. However, until about 20 years ago, the research community largely ignored the process of anti-windup compensator design, and the literature contains few papers on the subject until the late 1980s. Since then, a steady increase in papers addressing anti-windup analysis and design have emerged and, although it is impossible to review all techniques here, summaries may be found in the papers Tarbouriech et al. (2007); Tarbouriech and Turner (2009); Galeani et al. (2009). It suffices to say that major improvements in the theory behind anti-windup have been made and the control engineer faced with problems involving actuator constraints now is better equipped for dealing with them.

The majority of the anti-windup literature has focussed attention on actuators which can be modelled as simple saturation elements i.e. static nonlinear maps which model the limits commonly found in actuators. There is also a significant body of work devoted to systems containing rate-limited actuators (Galeani et al. (2008); Biannic and Tarbouriech (2009)) in which simple models are used to represent the effect of the limited rate-of-change of actuator behaviour. The advantage of these approaches to modelling actuators are (i) such models are often realistic approximations of the actuators they represent, and are normally sufficient for anti-windup analysis and design purposes; (ii) the relatively simple structure of the models eases the associated mathematical analysis; and (iii) actuator behaviour is often not precisely modelled, so relatively crude limits on magnitude and rate may be all that is available to the control system designer.

Despite the clear advantages of representing actuators with relatively simple mathematical models, in certain circumstances it may be inappropriate to do so. Typically, actuators are electromechanical, hydraulic or pneumatic devices which means that representing their behaviour as simple magnitude or rate-limited devices can sometimes be a gross simplification. For example, in electromechanical actuators for high bandwidth applications, it may be necessary (or useful) to take into account the electrical/mechanical dynamics of the actuators. Similarly in aircraft control surfaces, the gearing in the control surface can cause deadzones or backlash to obstruct ideal behaviour, in addition to the position and rate-limits present. While it might be possible to include the actuator dynamics as "uncertainty" and to design a robust anti-windup compensator (Turner et al. (2007); Galeani and Teel (2006); Kerr et al. (2010); Bruckner and del Re (2010)), and while it might be possible to treat phenomena such as deadzones and backlash with other corrective strategies (Turner (2006); Tarbouriech and Prieur (2007)), there is also merit to considering more detailed actuator models and to modify the anti-windup approach accordingly.

This paper proposes a modified anti-windup scheme for systems with what might be broadly called non-standard actuators. The actuators considered are those that can be represented as interconnections between linear elements and sector bounded nonlinearities. The latter restriction could be considerably relaxed (i.e. using slope-restrictions (Turner et al. (2009))) and, while the form initially appears somewhat restrictive, it can be used to model a broad class of common nonlinear actuators. Thus, the work presented here surpasses, in a reasonably straightforward manner, the usually considered actuator limitations and models by permitting a generic actuator model in the analysis. This allows other nonlinearities such as deadzones, backlashes and such like to be incoroporated into the anti-windup framework. The work also includes that reported in Tarbouriech et al. (2006), where a specially structured class of actuators was considered, as a special case. For simplicity, the anti-windup scheme proposed is based on the coprime factorisation technique advocated in Turner et al. (2007) (see also Weston and Postlethwaite (2000); Kothare et al. (1994); Miyamoto and Vinnicombe (1996); Herrmann et al. (2008)) but in principle the technique could be extended to static and low order types of anti-windup compensation. The so-called Internal Model Control (IMC) compensator, noted for its robustness properties (Turner et al. (2007)), emerges as a special case and is examined carefully.

The paper follows a simple structure. The next section introduces the systems under consideration. Following this, a technique for synthesising anti-windup compensators for both stable and unstable systems is proposed. The special case of IMC anti-windup is then examined and some example simulation results follow. Finally, some interim conclusions are proposed. Notation in the paper is standard throughout. Bold text is used to represent linear operators, or their transfer functions; other matrices and vectors are indicated by standard text.

2. SYSTEMS UNDER CONSIDERATION

2.1 System structure

We consider the system depicted in Figure 1 where $\boldsymbol{K} = [\boldsymbol{K}_1 \quad \boldsymbol{K}_2]$ represents the linear controller, and $\boldsymbol{G} = [\boldsymbol{G}_1 \quad \boldsymbol{G}_2]$

the linear plant.[1] $\mathcal{A}(.) : \mathbb{R}^m \mapsto \mathbb{R}^m$ represents the "imperfect" (nonlinear, possibly dynamic) actuator. It is assumed, as normal, that if $\mathcal{A} \equiv I$, then the linear interconnection of $\boldsymbol{K}$ and $\boldsymbol{G}$ according to Figure 1 is stable and well-posed. The reference is $r \in \mathbb{R}^{n_r}$, the disturbance $d \in \mathbb{R}^{n_d}$, the commanded control signal $u \in \mathbb{R}^m$, the actual control signal $u_m \in \mathbb{R}^m$ and the output measurement, $y \in \mathbb{R}^p$. The anti-windup compensator, $\boldsymbol{\Theta} = [\boldsymbol{\Theta}_1{}' \quad \boldsymbol{\Theta}_2{}']'$ emits two signals, θ_1, to the controller output and θ_2, to the controller input. Although such a configuration is not the most general, it is sufficient for our purposes (e.g. Weston and Postlethwaite (2000) showed the general character of such representation). The anti-windup compensator is driven by the difference between the commanded control signal and the actual control signal, and thus it is assumed that u_m is available for measurement. Note that in standard anti-windup $\mathcal{A} \equiv \mathrm{sat}(.)$ where $\mathrm{sat}(.)$ represents the saturation function. In this case, whenever saturation does not occur, $u_m = u$ and hence the anti-windup compensator is not active unless a saturation event occurs. In the more general case, when $\mathcal{A}$ may have dynamics, it may be rare for $u_m = u$ and thus the anti-windup compensator will be active much of the time; in this sense the anti-windup compensation is of the "weakened form" (Galeani et al. (2005)) although as the anti-windup compensator is a linear system, for small differences in u_m and u, the effect of Θ on the closed loop will be minor. The actuator, $\mathcal{A}(.) : \mathbb{R}^m \mapsto \mathbb{R}^m$ is described by

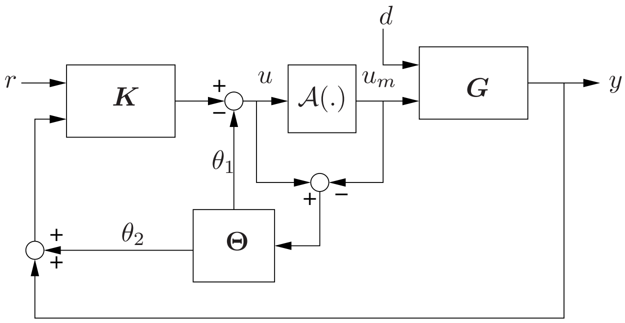

Fig. 1. System under consideration

the following state-space equations

$$\mathcal{A} \sim \begin{cases} \dot{x}_a &= A_a x_a + B_{a1} u + B_{a2} \tilde{u} \\ u_m &= C_{a1} x_a + D_{a11} u + D_{a12} \tilde{u} \\ \hat{u} &= C_{a2} x_a + D_{a21} u + D_{a22} \tilde{u} \\ \tilde{u} &= \mathcal{N}(\hat{u}) \end{cases} \tag{1}$$

where $\hat{u}, \tilde{u} \in \mathbb{R}^q$ and $q \neq m$ in general. The actuator nonlinearities are represented by the static nonlinear operator $\mathcal{N}(.) : \mathbb{R}^q \mapsto \mathbb{R}^q$. It is assumed (although this assumption can be relaxed) that the nonlinearity $\mathcal{N}(.)$ is sector bounded and, furthermore, appropriate scaling and loop-transformation (Khalil (1996)) has been performed such that $\mathcal{N} \in \mathrm{Sector}[0, I]$.

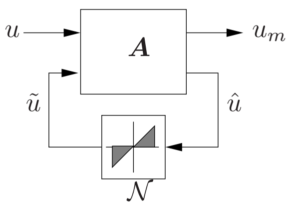

Fig. 2. "LFT" representation of actuator

It is easy to see equation (1) can be arranged in "LFT" form, as indicated by Figure 2. In this diagram $\boldsymbol{A}$ represents the linear actuator dynamics, partitioned as

$$\begin{bmatrix} u_m \\ \hat{u} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_{11} & \boldsymbol{A}_{12} \\ \boldsymbol{A}_{21} & \boldsymbol{A}_{22} \end{bmatrix} \begin{bmatrix} u \\ \tilde{u} \end{bmatrix} \tag{2}$$

[1] Note that the plant is not limited to be stable, as local stability is considered in the anti-windup analysis and synthesis.

where

$$\begin{bmatrix} \boldsymbol{A}_{11} & \boldsymbol{A}_{12} \\ \boldsymbol{A}_{21} & \boldsymbol{A}_{22} \end{bmatrix} \sim \left[\begin{array}{c|cc} A_a & B_{a1} & B_{a2} \\ \hline C_{a1} & D_{a11} & D_{a12} \\ C_{a2} & D_{a21} & D_{a22} \end{array}\right]. \tag{3}$$

2.2 A decoupled structure

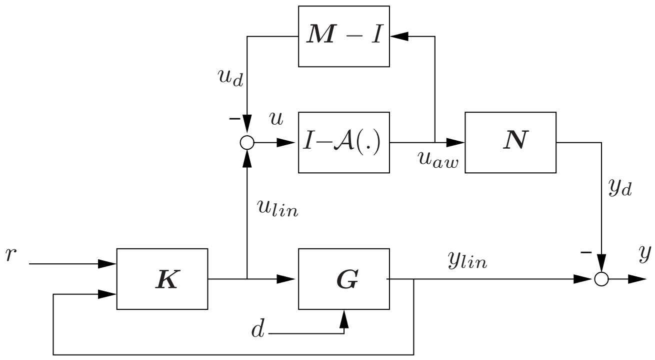

Fig. 3. System under consideration

Weston and Postlethwaite (2000) noticed that useful observations about an anti-windup compensator's properties could be obtained by assigning it a certain structure. In particular, it was shown that anti-windup could be interpreted in a manner in which the dynamics of the nominal linear system (i.e. that without anti-windup) could be decoupled from a nonlinear part. This decoupled nonlinear component represented the modification of this behaviour during and after saturation. A similar observation was made in Teel and Kapoor (1997). Several developments in anti-windup have adopted this framework; this section generalises the approach to systems with more general nonlinear actuators, $\mathcal{A}$.002a9ee4107566a3068a500311fc0aa0

Letting, $\boldsymbol{\Theta}_1 = \boldsymbol{M} - I$ and $\boldsymbol{\Theta}_2 = \boldsymbol{N}$ where $\boldsymbol{G}_2 = \boldsymbol{N}\boldsymbol{M}^{-1}$ is a coprime factorisation of the nominal plant, $\boldsymbol{G}_2$, it is possible, using simple algebra, to re-draw Figure 1 as Figure 3. It is stressed that the coprime factorisation approach is not the only way of tackling this problem, but due to space constraints, attention is focussed on this. From Figure 3, note that the deviation from linear behaviour is governed by the nonlinear mapping $\mathcal{T}_p : u_{lin} \mapsto y_d$ (for convenience re-drawn in Figure 4) and that, as $\boldsymbol{N} \in \mathcal{RH}_\infty$, and as the nominal linear system is assumed to be well-posed, the system in Figure 3 will be stable [2] if the nonlinear loop is asymptotically stable.

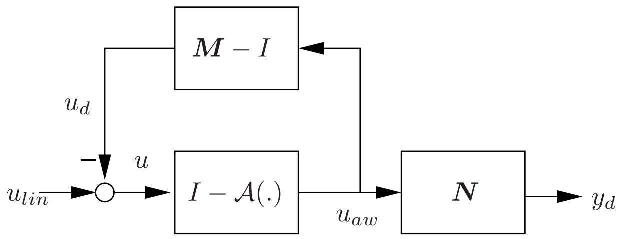

Fig. 4. Operator $\mathcal{T}_p : u_{lin} \mapsto y_d$

3. ANTI-WINDUP SYNTHESIS

3.1 Preliminary results

It was argued in the previous section that the performance and stability of the operator $\mathcal{T}_p$ is central to the performance of the (nonlinear) closed-loop system's performance. $\mathcal{T}_p$ is intimately

[2] More detail on exactly what is meant by "stable" will be given later

$$\begin{bmatrix} Q\tilde{A}_0' + \tilde{A}_0Q + \tilde{L}\tilde{B}_0' + \tilde{B}_0\tilde{L} & \tilde{B}_2U - Q\tilde{C}_{02}' - \tilde{L}\tilde{D}_{02}' + Z' & \tilde{B}_1 & Q\tilde{C}_{01}' + \tilde{L}'\tilde{D}_{01}' \\ \star & -2U - \tilde{D}_{22}U - U\tilde{D}_{22}' & -\tilde{D}_{21} & U\tilde{D}_{12}' \\ \star & \star & -\gamma I & \tilde{D}_{11}' \\ \star & \star & \star & -\gamma I \end{bmatrix} < 0 \quad (12)$$

$$\begin{bmatrix} \sigma\bar{u}_i^2 & Z_i \\ \star & Q \end{bmatrix} \geq 0 \quad \forall i \in \{1, \ldots, m\} \quad (13)$$

$$\begin{bmatrix} Q\tilde{A}_0' + \tilde{A}_0Q + Q\tilde{F}'\tilde{B}_0' + \tilde{B}_0\tilde{F}Q & \tilde{B}_2U - Q\tilde{C}_{02}' - Q\tilde{F}'\tilde{D}_{02}' + Z' & \tilde{B}_1 & Q\tilde{C}_{01}' + Q\tilde{F}'\tilde{D}_{01}' \\ \star & -2U - \tilde{D}_{22}U - U\tilde{D}_{22}' & -\tilde{D}_{21} & U\tilde{D}_{12}' \\ \star & \star & -\gamma I & \tilde{D}_{11}' \\ \star & \star & \star & -\gamma I \end{bmatrix} < 0 \quad (14)$$

$$\begin{bmatrix} \sigma\bar{u}_i^2 & Z_i \\ \star & Q \end{bmatrix} \geq 0 \quad \forall i \in \{1, \ldots, m\} \quad (15)$$

related to the choice coprime factors, $\boldsymbol{N}, \boldsymbol{M} \in \mathcal{RH}_\infty$, given by the state-space realisation

$$\begin{bmatrix} \boldsymbol{M} \\ \boldsymbol{N} \end{bmatrix} \sim \left[\begin{array}{c|c} A_p + B_pF & B_p \\ F & I \\ C_p + D_pF & D_p \end{array}\right] \quad (4)$$

where $\boldsymbol{G}_2 \sim (A_p, B_p, C_p, D_p)$ and F is a design parameter such that $(A_p + B_pF)$ is Hurwitz. This realisation can be combined with that of the actuator, (1), to obtain a state-space model of the nonlinear operator $\mathcal{T}_p$ as

$$\mathcal{T}_p \sim \begin{cases} \begin{bmatrix} \dot{\tilde{x}} \\ \hline y_d \\ \hat{u} \end{bmatrix} = \left[\begin{array}{c|cc} \tilde{A}_0 + \tilde{B}_0\tilde{F} & \tilde{B}_1 & \tilde{B}_2 \\ \hline \tilde{C}_{01} + \tilde{D}_{01}\tilde{F} & \tilde{D}_{11} & \tilde{D}_{12} \\ \tilde{C}_{02} + \tilde{D}_{02}\tilde{F} & \tilde{D}_{21} & \tilde{D}_{22} \end{array}\right] \begin{bmatrix} \tilde{x} \\ u_{lin} \\ \tilde{u} \end{bmatrix} \\ \tilde{u} = \mathcal{N}(\hat{u}) \end{cases} \quad (5)$$

where $\tilde{F} = [F \quad 0]$ and

$$\left[\begin{array}{c|c|c|c} \tilde{A}_0 & \tilde{B}_0 & \tilde{B}_1 & \tilde{B}_2 \\ \hline \tilde{C}_{01} & \tilde{D}_{01} & \tilde{D}_{11} & \tilde{D}_{12} \\ \hline \tilde{C}_{02} & \tilde{D}_{02} & \tilde{D}_{21} & \tilde{D}_{22} \end{array}\right] =$$

$$\left[\begin{array}{cc|c|c|c} A_p & -B_pC_{a1} & B_pD_{a11} & B_p(I - D_{a11}) & -B_pD_{a12} \\ 0 & A_a & -B_{a1} & B_{a1} & B_{a2} \\ \hline C_p & -D_pC_{a1} & D_pD_{a11} & D_p(I - D_{a11}) & -D_pD_{a12} \\ 0 & C_{a2} & -D_{a21} & D_{a21} & D_{a22} \end{array}\right] \quad (6)$$

Thus our goal becomes one of ensuring that $\mathcal{T}_p$ is at least locally stable and for sufficiently small $u_{lin} \in \mathcal{L}_2$ is such that $\|y_d\|_2 < \gamma\|u_{lin}\|_2$ for some, preferably small, $\gamma > 0$. There are many similar ways of proving such properties for $\mathcal{T}_p$, but for conciseness we invoke Proposition 1 of Hu et al. (2008) which enables global and local (or regional) results to be inferred. Roughly speaking, the result combines standard Circle Criterion type results (Khalil (1996)) with $\mathcal{L}_2$ gain conditions and a modified sector bound of Gomes da Silva Jr. and Tarbouriech (2005).

Proposition 1. (Hu et al. (2008)). Consider the state-space system

$$\begin{aligned} \dot{x} &= Ax + B\phi + B_ww \\ y &= Cx + D\phi + D_ww \\ z &= C_zx + D_z\phi + D_{zw}w \\ \phi &= \mathrm{Dz}(y) \end{aligned} \quad (7)$$

If there exist matrices $Q > 0$, diagonal $U > 0$ and Z, and scalars $\gamma > 0$ and $\sigma > 0$, such that the following matrix inequalities hold

$$\begin{bmatrix} QA' + AQ & BU + QC' + Z' & B_w & QC_z' \\ \star & -2U + DU + UD' & D_w & UD_z' \\ \star & \star & -\gamma I & D_{zw}' \\ \star & \star & \star & -\gamma I \end{bmatrix} < 0 \quad (8)$$

$$\begin{bmatrix} \sigma\bar{u}_i^2 & Z_i \\ \star & Q \end{bmatrix} \geq 0 \quad \forall i \in \{1, \ldots, m\} \quad (9)$$

then

- If $x(0) = 0$ and $\|w\|_2 \leq 1/\sqrt{\sigma}$
$$\|z\|_2 < \gamma\|w\|_2$$
- If $w \equiv 0$, the ellipsoid
$$\mathcal{E} = \{x \in \mathbb{R}^n : x'Px < 1/\sigma\}$$
is contained within the region of attraction.

The above result assumes that the nonlinearity $\mathcal{N}(.) = \mathrm{Dz}(.)$ is a deadzone where

$$\mathrm{Dz}(u) = [\mathrm{Dz}_1(u_1), \ldots, \mathrm{Dz}_m(u_m)]' \quad (10)$$

$$\mathrm{Dz}_i(u_i) = \mathrm{sign}(u_i)\max\{|u_i| - \bar{u}_i, 0\} \quad (11)$$

As most actuators will have limits modelled as saturation nonlinearities, which can be decomposed as $\mathrm{sat}(u) = u - \mathrm{Dz}(u)$, we shall thus limit discussion to actuators in which the nonlinearity $\mathcal{N}(.)$ is of the deadzone type. This is actually not necessary, but makes for a much easier presentation.

3.2 Main results

The main results in the paper are effectively an application of Proposition 1 to the state-space system given in the realisation (5). Equating terms in the generic state-space system (7) with those in the realisation (5), the following Corollary is obtained.

Corollary 2. Assume there exist structured matrices:

$$0 < Q = \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix} \in \mathbb{R}^{(n_p+n_a)\times(n_p+n_a)}$$

and

$$\tilde{L} = [\, L_1 \;\; 0 \,] \in \mathbb{R}^{m\times(n_p+n_a)}$$

where $Q_1 \in \mathbb{R}^{n_p\times n_p}$, $Q_2 \in \mathbb{R}^{n_a\times n_a}$, $L_1 \in \mathbb{R}^{m\times n_p}$, $Z \in \mathbb{R}^{(n_p+n_a)\times m}$ and scalars $\gamma > 0, \sigma > 0$ are such that the linear matrix inequalities (12) and (13) are satisfied. Then with $\tilde{F} = \tilde{L}\tilde{Q}^{-1} = [L_1Q_1^{-1} \quad 0]$ and $\mathcal{N}(.) = \mathrm{Dz}(.)$ it follows that

- If the initial state of operator $\mathcal{T}_p$, $\tilde{x}(0) = 0$ and $\|u_{lin}\|_2 < 1/\sqrt{\sigma}$, then $\|y_d\|_2 < \gamma\|u_{lin}\|_2$
- If $u_{lin} = 0$, the ellipsoid
$$\tilde{\mathcal{E}} := \{\tilde{x} \in \mathbb{R}^{n_p+n_a} : \tilde{x}'Q^{-1}\tilde{x} \leq 1/\sigma\}$$
is contained within the basin of attraction of the origin of $\mathcal{T}_p$.

Proof: It is easy to match terms in the generic state-space system (7) with those in the realisation (5), viz:

$$\left[\begin{array}{c|c|c} A & B & B_w \\ \hline C & D & D_w \\ \hline C_z & D_z & D_{zw} \end{array}\right] = \left[\begin{array}{c|c|c} \tilde{A}_0 + \tilde{B}_0\tilde{F} & \tilde{B}_2 & \tilde{B}_1 \\ \hline -\tilde{C}_{02} - \tilde{D}_{02}\tilde{F} & -\tilde{D}_{22} & -\tilde{D}_{21} \\ \hline \tilde{C}_{01} + \tilde{D}_{01}\tilde{F} & \tilde{D}_{12} & \tilde{D}_{11} \end{array}\right]$$

Substituting these expressions directly into inequalities (8) and (9) then gives the nonlinear inequalities (14) and (15). The product, $\tilde{F}Q$, is eliminated by defining $\tilde{L} = \tilde{F}Q$ which then yields the LMIs in the theorem. In order to recover $\tilde{F} = [F \;\; 0] = \tilde{L}Q^{-1}$ we structure $\tilde{L}$ and Q as indicated in the theorem. The remainder of the corollary follows Proposition 1. □□

The above result represents a convenient way for obtaining coprime-factor based compensators which ensure stability and performance, in some form, for the nonlinear operator $\mathcal{T}_p$, and hence for the overall nonlinear closed-loop. If the matrices A_p or A_a are unstable [3] the corollary allows one either: to optimise the size of the region of attraction, by maximising σ; to minimise the small-signal $\mathcal{L}_2$ gain of the system, by minimising γ; or to do some weighted combination of the two. If both A_p and A_a are Hurwitz, then it is possible to let $Z \equiv 0$ and to optimise the standard $\mathcal{L}_2$ gain of the system, while ensuring global exponential stability (Grimm et al. (2003)).

3.3 Special case: IMC

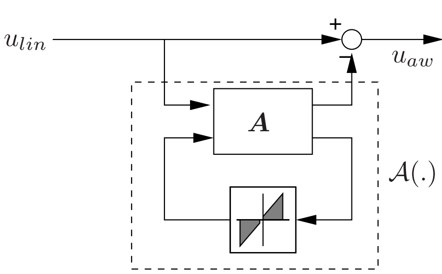

Fig. 5. Operator $\mathcal{T}_p : u_{lin} \mapsto y_d$: the IMC case

The so-called IMC anti-windup compensator is obtained by setting $\boldsymbol{M} = I$ and $\boldsymbol{N} = \boldsymbol{G}_2$ in the architecture of Figure 1. Although the compensator was introduced some time ago (Zheng and Morari (1994)), it is still intriguing to anti-windup researchers thanks to some of its rather odd properties. For stable systems, $\boldsymbol{G} \in \mathcal{RH}_\infty$ it provides a simple globally stabilising anti-windup compensator which minimises (without calculation!) certain $\mathcal{L}_2$ gain maps associated with anti-windup performance (Turner et al. (2007)). Conversely, it is also associated with very poor transient performance if the plant has lightly damped modes or other troublesome characteristics. However, it is also known to be "optimally" robust to norm-bounded additive uncertainties and, although this feature does not always carry over to more general types of uncertainty (Morales et al. (2008); Kerr et al. (2010)) this remarkable property makes it worthy of further examination.

When $\boldsymbol{M} = I$ and $\boldsymbol{N} = \boldsymbol{G}_2$, the nonlinear loop depicted in Figure 4 becomes that depicted in Figure 5. In this case, the only feedback loop which occurs is that internal to the actuators. Thus, providing that $\mathcal{A}(.)$ is itself stable, the map $u_{lin} \mapsto y_d$ is stable once $\boldsymbol{G}_2 \in \mathcal{RH}_\infty$ is assumed. Thus, when using IMC anti-windup compensation, to establish stability of the nonlinear system it is only necessary to establish stability of the actuator sub-system given in equation (1); in most practical applications, this is likely to be asymptotically stable.

In addition, the actuator model (1) may well include uncertainty. Thus, providing the actuator sub-system is stable (in some sense) in the face of these uncertainties, the mapping $u_{lin} \mapsto y_d$ would be stable. So, while the IMC scheme might lead to very poor transient behaviour, *providing the actuator sub-system is stable*, it guarantees robustness to a very large class of actuators, including those which may have uncertainty in their parameters. Therefore IMC anti-windup can be used in systems for which *nothing* (apart from stability) is known about the actuators: stability will be preserved provided the plant model is known reasonably accurately, but no further analysis is required.

[3] It is unlikely that the actuator dynamics will be exponentially unstable, but they could be polynomially unstable to a rough approximation

4. EXAMPLE

In Kahveci et al. (2008) an adaptive anti-windup scheme was applied to a glider soaring problem. We use an LTI version of this problem as an example. The nominal plant has the following matrices

$$A_p = \begin{bmatrix} -0.0803 & 0.4472 & 0 & -0.5600 & 0.00005 \\ -2.3014 & -6.8759 & 0.8436 & 0.0402 & 0.00141 \\ 0.00002 & -32.5884 & -7.8062 & -0.00007 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -0.04997 & -0.6972 & 0 & 0.6972 & 0 \end{bmatrix}$$

$$B_p = \begin{bmatrix} 0.01158 \\ -0.69954 \\ -37.47948 \\ 0 \\ 0 \end{bmatrix} \qquad C_p = I_5 \quad D_p = 0_{5\times1}$$

The nominal linear controller is taken as a linear state-feedback controller with integral action as described in Kahveci et al. (2008). Assuming perfect actuator dynamics (i.e. the actuators are purely linear), the controller gives satisfactory time-responses, as depicted in Figure 6.

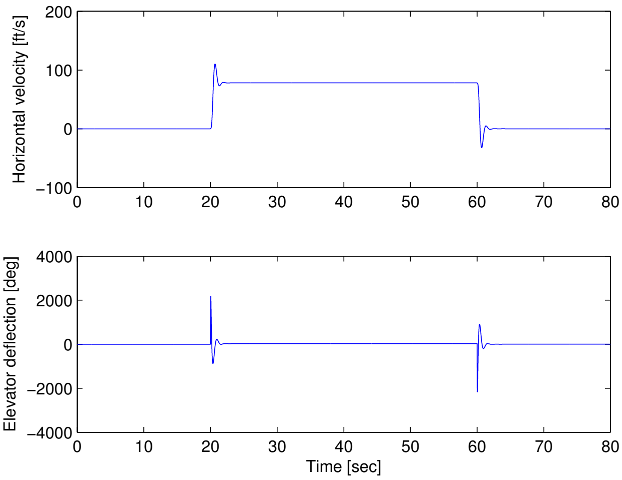

Fig. 6. Glider response: no constraints

4.1 Magnitude and rate-limited actuators

Several different models have been proposed in the literature for magnitude and rate-limited actuators, perhaps linked to the actuators from different applications (Galeani et al. (2008); Biannic and Tarbouriech (2009); Tarbouriech et al. (2007)) - a good summary can be found in Biannic and Roos (2008). In many aerospace applications, one of the actuator model is given by a series interconnection of a saturation, gain and limited integrator with unity gain feedback. The limited integrator is not convenient for analysis however, and thus can be approximated as described in Biannic and Tarbouriech (2009). A mathematical model of such a magnitude and rate-limited actuator, in the form of equation (1), can be derived as

$$\mathcal{A}_{mr} \sim \begin{cases} \begin{bmatrix} \dot{x}_a \\ \hline u_m \\ \hat{u}_1 \\ \hat{u}_2 \end{bmatrix} = \left[\begin{array}{c|c|cc} -H & H & H-\lambda I & -I \\ \hline I & 0 & -I & 0 \\ I & 0 & 0 & 0 \\ -H & H & H & 0 \end{array}\right] \begin{bmatrix} x_a \\ \hline u \\ \tilde{u}_1 \\ \tilde{u}_2 \end{bmatrix} \\ \begin{bmatrix} \tilde{u}_1 \\ \tilde{u}_2 \end{bmatrix} = \begin{bmatrix} \mathrm{Dz}(\hat{u}_1) \\ \mathrm{Dz}(\hat{u}_2) \end{bmatrix} \end{cases} \tag{16}$$

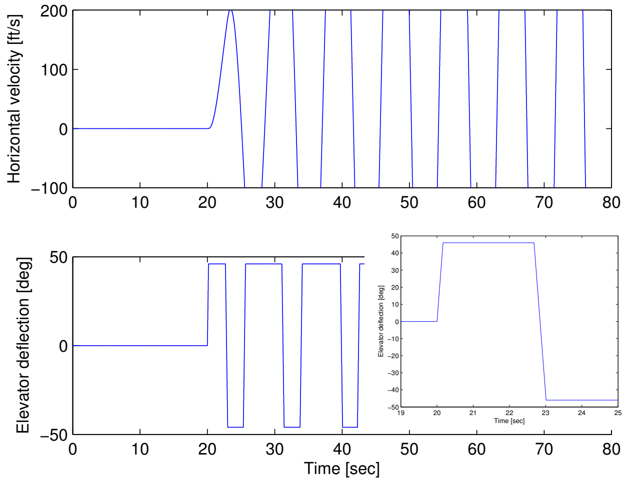

Fig. 7. Glider response: simple actuator (magnitude and rate saturation)

where H is a diagonal positive definite matrix and λ is a large scalar. In this case, the saturation limit $\bar{u}_1 = 46$, rate-limit $\bar{u}_2 = 6\bar{u}_1$, $H = 50$ and $\lambda = 100$. With this actuator model and with these limits imposed, the response of the system to a 79 degree pulse demand is that depicted in Figure 7: stability is lost and the systems enter a large magnitude limit cycle. Note that the control signal is both magnitude and rate-limited. To obtain improved behaviour during periods of both magnitude and rate-saturation, Corollary 1 is used to design a suitable coprime factor based anti-windup compensator. Similar to Turner et al. (2007); Herrmann et al. (2008), the anti-windup compensator is designed with the aid of a performance weight i.e. the $\mathcal{L}_2$ gain between u_{lin} and $\sqrt{W_p}y_d$ is minimised. This performance weight is given by:

$$W_p = \mathrm{diag}(10^{-3}, 1, 10^{-6}, 0.0006, 10^{-6})C_p \qquad (17)$$

In addition, we set $\sigma = 0.1$ and minimised γ in the LMI optimisation problem defined by the matrix inequalities in Corollary 2. The solution of these matrix inequalities yielded

$$\gamma = 5.00 \quad F = [0.030\ -0.045\ 0.033\ 0.291\ 0] \qquad (18)$$

This value of F was then inserted into equation (4) in order to obtain the anti-windup compensator. The results with this compensator are shown in Figure 8. Anti-windup compensation is initially activated by rate-saturation of the control signal, but this is then followed by short periods of magnitude saturation. The output obtained during this constrained behaviour using anti-windup is clearly much better than that without anti-windup: tracking performance is retained, although the speed of response is somewhat slower due to the presence of the rate and magnitude constraints.

It is also interesting to investigate the behaviour of the system when using IMC anti-windup compensation. Although it is not evident at first, it transpires that the actuator model $\mathcal{A}_{mr}$ is actually globally asymptotically stable (proof omitted due to space constraints). Therefore, as $\boldsymbol{G} \in \mathcal{RH}_\infty$, this implies that the system with IMC anti-windup will be globally asymptotically stable. The response with IMC anti-windup is shown in Figure 9: again rate-saturation occurs first, activating the anti-windup compensator, followed by short periods of magnitude saturation. The IMC anti-windup compensator does not lead to a particularly attractive response, but it is clearly an improvement on the response with no anti-windup. The IMC anti-windup compensator is also globally stabilising, whereas the coprime factor compensator (18) only guarantees stability for sufficiently small initial states.

4.2 A more complex actuator

In this section, we consider the same plant but assume that the actuator is somewhat more complex. In particular, it is assumed

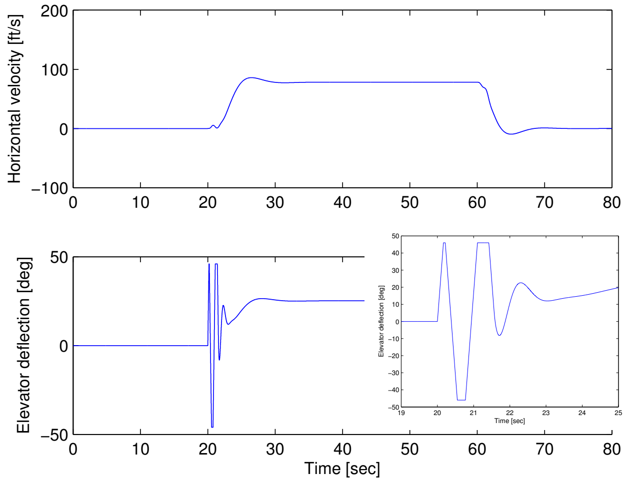

Fig. 8. Glider response: simple actuator & coprime factor anti-windup

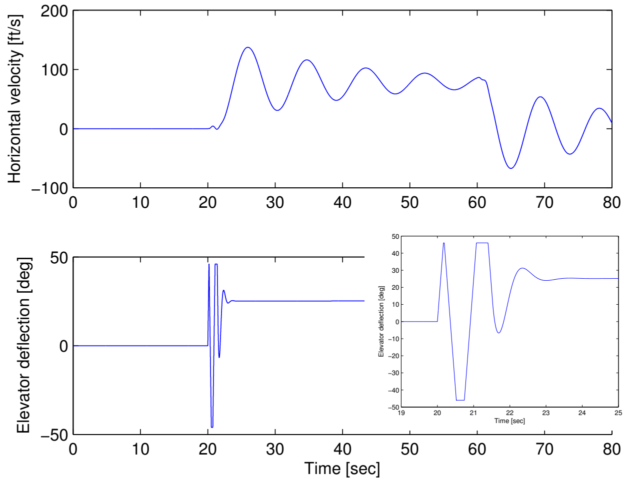

Fig. 9. Glider response: simple actuator & IMC anti-windup

that the actuator has second order dynamics and also has a deadzone, representing gearing nonlinearity, at its output. The state-space realisation of such an actuator is

$$\mathcal{A}_c \sim \left\{ \begin{array}{l} \left[\begin{array}{c} \dot{x}_{a1} \\ \dot{x}_{a2} \\ \hline u_m \\ \hat{u}_1 \\ \hat{u}_2 \\ \hat{u}_3 \end{array}\right] = \left[\begin{array}{cc|c|ccc} -\epsilon I & H_1 & 0 & -\lambda I & I & 0 \\ 0 & -H_2 & H_2 & 0 & 0 & -H_2 \\ \hline 0 & 0 & 0 & 0 & 0 & I \\ \hline I & 0 & 0 & 0 & 0 & 0 \\ 0 & H_1 & 0 & 0 & 0 & 0 \\ I & 0 & 0 & -I & 0 & 0 \end{array}\right] \left[\begin{array}{c} x_{a1} \\ x_{a2} \\ \hline u \\ \tilde{u}_1 \\ \tilde{u}_2 \\ \tilde{u}_3 \end{array}\right] \\ \left[\begin{array}{c} \tilde{u}_1 \\ \tilde{u}_2 \\ \tilde{u}_3 \end{array}\right] = \left[\begin{array}{c} \mathrm{Dz}(\hat{u}_1) \\ \mathrm{Dz}(\hat{u}_2) \\ \mathrm{Dz}(\hat{u}_3) \end{array}\right] \end{array} \right. \qquad (19)$$

where $H_1 = 50$, $H_2 = 70$, $\lambda = 100$, $\epsilon = 10^{-3}$, $\bar{u}_1 = 46$, $\bar{u}_2 = 6\bar{u}_1$ and $\bar{u}_3 = 10$. With such an actuator in place, and without magnitude and rate-limits ($\bar{u}_1 = \bar{u}_2 = \infty$) and without the deadzone at the actuator output ($\bar{u}_3 = 0$), the linear closed-loop response is almost identical to that shown in Figure 6. However, instability again ensues when the constraints and deadzone are re-introduced.

Carrying out an anti-windup design according to Corollary 2, using $\sigma = 10$ and the complex actuator model in (19), produces the following values of F and γ

$$\gamma = 209.68 \quad F = 10^{-4} \times [0.006\ 0.031\ \ -0.014\ \ -0.107\ 0] \qquad (20)$$

These can be used to construct an anti-windup compensator as given in (4). The response of the glider using the more complex actuator and the accompanying anti-windup compensator is given in Figure 10: stability is maintained but performance is worse than when the simpler actuator is used. In fact, as F given by (20) is so small, the coprime compensator is very close to the IMC compensator, and, indeed, the responses are almost identical (Figure 9). As suggested in Section 3.3, this points to an inherent robustness of the IMC compensator to actuators with different levels of complexity and dynamics.

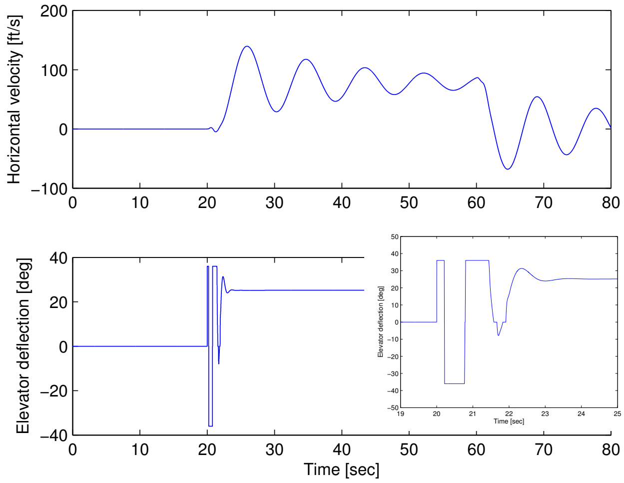

Fig. 10. Glider response: complex actuator & anti-windup

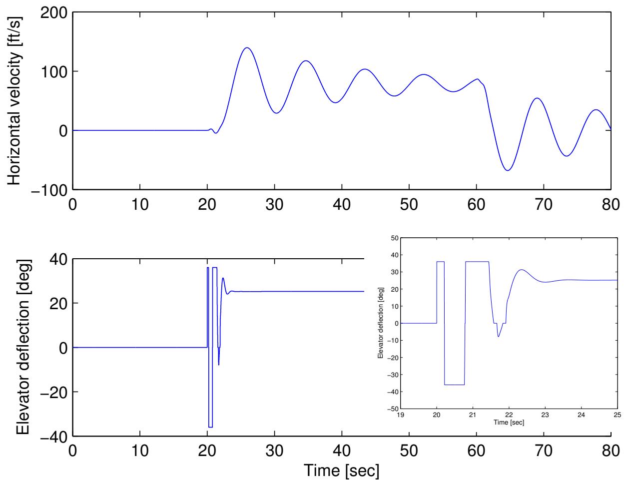

Fig. 11. Glider response: complex actuator & IMC anti-windup

5. CONCLUSION

This paper has examined a possible architecture for systems containing imperfect actuators which take a reasonably generic form. The anti-windup compensator is assumed to have access to the actuator input and output but it does not require knowledge of any signal internal to the actuator, as is sometimes assumed (Sofrony et al. (2010)). A simple method of constructing anti-windup compensators based on coprime factors was introduced and the method demonstrated on simple examples. In common with anti-windup strategies for systems where the actuator is only assumed to be magnitude or rate-limited, the IMC anti-windup compensator again emerges as a special case and, when the plant under consideration is asymptotically stable, the IMC compensator is seen to be "robust" to a large class of actuator structures.

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