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Improved Kick and Loss Detection and Attenuation in Managed Pressure Drilling by Utilizing Wired Drill Pipe

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Abstract: A model based method for kick and loss detection and attenuation in Managed Pressure Drilling is presented. The drilling system is modeled as a distributed parameter system combined with a reservoir flow equation containing reservoir pressure and the so-called productivity index as uncertain parameters. A swapping-based design for state and parameter estimation utilizing bottom-hole pressure measurements available via wired drill pipe is combined with a closed loop controller for kick and loss attenuation. The performance of the proposed method is compared to earlier results on kick attenuation in a simulation, showing significant improvement.

Keywords: Distributed-parameter systems, adaptive control, parameter estimation, managed pressure drilling, kick detection

1. INTRODUCTION

1.1 Problem Statement

In order to carry cuttings to the surface and maintain an appropriate pressure barrier down-hole during drilling, a drilling mud is circulated down the drill string, through the drill bit and up in a casing surrounding the drill string (see Figure 1). In cases where pressure margins are tight, a control choke and a back pressure pump are installed at the top side end of the annulus so that pressure can be controlled quickly and precisely. The method utilizing this equipment is referred to as managed pressure drilling (MPD) with applied back pressure (ABP). To model the flow dynamics in the annulus, the following model from Landet et al. (2013) is used

$$p_t(z,t) = -\frac{\beta}{A_1} q_z(z,t) \tag{1a}$$

$$q_t(z,t) = -\frac{A_1}{\rho} p_z(z,t) - \frac{F_1}{\rho} q(z,t) - A_1 g \tag{1b}$$

where $z \in [0,l]$ and $t \geq 0$ are the independent variables of space and time respectively, l is the well depth, $p(z,t)$ is pressure, $q(z,t)$ is volumetric flow, ρ is the density of the mud, β is the bulk modulus of the mud, F_1 is the friction factor, A_1 is the cross sectional area of the annulus and g is the acceleration of gravity.

When drilling into an oil reservoir, the bottom-hole end of the drill-string is exposed to the reservoir pressure. If the reservoir pressure is higher than the bottom-hole pressure in the annulus, the result is a net inflow of formation fluids into the annulus. This situation is called a *kick*. Similarly, a *loss* is a net outflow of drilling-mud into the reservoir caused by a higher bottom-hole mud pressure in the annulus than formation pressure. To model this relationship, a simple *productivity-based-inflow* model is used. Together with the top-side actuation signal $p_l(t)$, this gives the boundary conditions

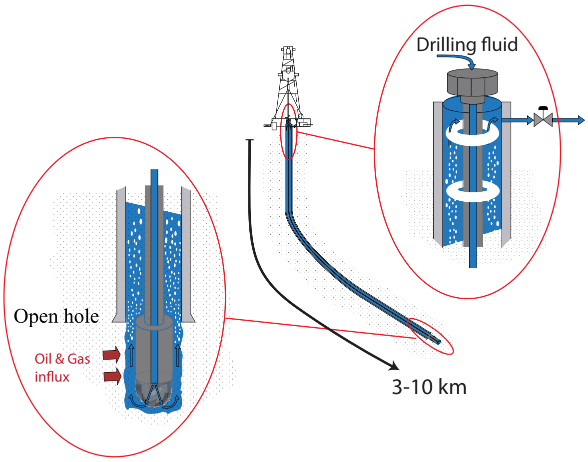

Fig. 1. Schematic of the drilling system. Courtesy of Ulf Jakob Aarsnes (Aarsnes et al. (2016b)).

$$q(0,t) = J\left(p_r - p(0,t)\right) + q_{bit} \tag{2a}$$

$$p(l,t) = p_l(t) \tag{2b}$$

where $J > 0$ is called the productivity index and is assumed unknown, p_r is the unknown reservoir pressure, and q_{bit} the known flow through the drill bit. It is assumed that p_r satisfies $0 < p_r \leq \bar{p}_r$ where $\bar{p}_r$ is some known upper bound for the reservoir pressure. Moreover, it is assumed that the choke controller has significantly faster dynamics than the rest of the system so that the actuation dynamics can be ignored and the top-side pressure p_l regarded as a control input. The design utilizes both bottom-hole pressure measurements $p(0,t)$, which are assumed available in real time when using a wired drill pipe, and the top-side flow $q(l,t)$.

The design goal is to keep the bottom-hole pressure equal to the unknown reservoir pressure, that is $p(0,t) = p_r$ such that flow from the reservoir into the drill string is zero. This implies that the flow through the annulus is equal to the drill bit flow.

1.2 Previous Work

Control of coupled distributed systems like (1) and (3) can be achieved by using the method of *infinite-dimensional backstepping* for PDEs. This method was first introduced for parabolic PDEs in Liu (2003); Smyshlyaev and Krstic (2004, 2005), where the gain kernel was expressed as a solution to a well-posed PDE. The first result for hyperbolic PDEs was in Krstic and Smyshlyaev (2008) for first order systems, for second order hyperbolic systems in Smyshlyaev et al. (2010), and to two coupled first order hyperbolic systems in Vazquez et al. (2011). The results in the latter were used in Aamo (2013) for disturbance attenuation in managed pressure drilling which has similarities to the problem considered in this paper.

Results on adaptive control for parabolic PDEs can be found in Smyshlyaev and Krstic (2010). Adaptive observers for $n+1$ hyperbolic systems using sensing collocated with the uncertain boundary parameters can be found in Anfinsen et al. (2016) using swapping filters, and in Bin and Di Meglio (2017) using a Lyapunov approach. The extension to general $m+n$ systems is given in Anfinsen et al. (2017). The extension to stabilization, without additive boundary parameter and sensing at the left boundary restricted to the form $y_0(t) = v(0,t)$, is given in Anfinsen and Aamo (2017c) ($n+1$ case) and Anfinsen and Aamo (2017b) ($m+n$ case). An adaptive observer for $n+1$ systems with a multiplicative boundary condition is developed in Di Meglio et al. (2014) and for 2×2 systems with an affine boundary condition in Anfinsen and Aamo (2016). Adaptive stabilization of the same type of systems, but without the additive parameter is considered in Anfinsen and Aamo (2017a) and with only the additive parameter unknown in Aamo (2013).

Kick attenuation in MPD has mainly been studied in the context of lumped drilling models. A lumped ODE model is applied to a gas kick detection and mitigation problem in Zhou et al. (2011) by using a method for switched control of the bottom-hole pressure. Another lumped model for estimation and control of in-/outflux is presented in Hauge et al. (2012). Kick handling methods for a first-order approximation to the PDE system is presented in Aarsnes et al. (2016a) using LMI (Linear Matrix Inequality) based controller design. An infinite-dimensional observer for kick & loss detection is presented in Hauge et al. (2013). Another observer for state and reservoir pressure estimation in under-balanced drilling is given in Di Meglio et al. (2014). In Holta et al. (2018, 2017), a distributed PDE model is combined with a model of the reservoir inflow relation (given in Equation 2a). Kick & loss detection with sensing non-collocated with control is studied in Holta et al. (2017), while a method for kick & loss attenuation for the same system is presented in Holta et al. (2018).

1.3 Contributions and Paper Structure

This paper considers both kick/loss detection and attenuation. The main contribution of this paper is an improved version of the parameter estimation scheme presented in Holta et al. (2018), better utilizing the bottom-hole pressure measurement. The estimation scheme presented in this paper is combined with the closed loop controller derived in Holta et al. (2018) and applied to the kick & loss application. In Section 2, the main results from Holta et al. (2018) are included for completeness. Section 3 presents the improved estimation scheme. In Section 4 the estimator from Section 3 is combined with the control law from Holta et al. (2018) and applied to the kick & loss problem in MPD where the performance of the new estimation scheme is compared to results from Holta et al. (2018) in a simulation.

1.4 Mapping to Riemann Invariants

To ease the design process, as well as generalize the control problem, it can be shown that system (1) with boundary conditions (2) can be transformed, through a suitable coordinate transformation (see Holta et al. (2018)), to an equivalent system written in terms of Riemann invariants as

$$u_t(x,t) + \lambda u_x(x,t) = c_1(x)v(x,t) \tag{3a}$$
$$v_t(x,t) - \mu v_x(x,t) = c_2(x)u(x,t) \tag{3b}$$
$$u(0,t) = rv(0,t) + k(\theta - y_0(t)) \tag{3c}$$
$$v(1,t) = U(t) \tag{3d}$$

defined for $x \in [0,1]$, $t \geq 0$, where u, v are the system states, $U(t)$ is the control input, $\lambda, \mu > 0$ and $c_1(x), c_2(x) \in C([0,1])$ are known, while $k \in \mathbb{R}$ and $\theta \in \mathbb{R}$ are unknown boundary parameters, but where $\operatorname{sign}(k)$ is known. The measurement collocated with actuation is given by

$$y_1(t) = u(1,t) \tag{4}$$

while the measurement anti-collocated with actuation is generated as a linear combination of the system states. That is,

$$y_0(t) = a_0 u(0,t) + b_0 v(0,t) \tag{5}$$

with $a_0 \neq 0$. The objective is generalized to stabilization in the L_2-sense and boundedness uniformly and point-wise in x. In addition, based on the design goal $p(0,t) = p_r(t)$, we select the weaker control objective

$$\lim_{t\to\infty} \int_t^{t+T} |rv(0,\tau) - u(0,\tau)|\, d\tau = 0 \tag{6}$$

for

$$r \neq -\frac{b_0}{a_0} \tag{7}$$

and for some arbitrary $T > 0$. Furthermore, it is assumed that the initial conditions $u(x,0) = u_0(x)$, $v(x,0) = v_0(x)$ satisfy $u_0,\ v_0 \in L_2([0,1])$.

2. ESTIMATION AND CONTROL WITH ADAPTATION BASED ON TOP-SIDE SENSING (OLD METHOD)

The main results on state and parameter estimation from Holta et al. (2017) are given in Section 2.1 and Theorem 1 in particular. Section 2.2 presents the main results from Holta et al. (2018) with the control law given formally in Theorem 2.

2.1 State and Parameter Estimation

In Holta et al. (2017), a swapping based design is used to generate on-line state and parameter estimates. The same swapping filters will be used in this paper. The filters are given by

$$\begin{aligned} a_t(x,t) + \lambda a_x(x,t) =& c_1(x)b(x,t) \\ &+ P_1(x)(y_1(t) - a(1,t)) \end{aligned} \tag{8a}$$
$$\begin{aligned} b_t(x,t) - \mu b_x(x,t) =& c_2(x)a(x,t) \\ &+ P_2(x)(y_1(t) - a(1,t)) \end{aligned} \tag{8b}$$
$$a(0,t) = rb(0,t) \tag{8c}$$
$$b(1,t) = U(t) \tag{8d}$$

and parameter filters

$$m_t(x,t) + \lambda m_x(x,t) = c_1(x)n(x,t) - P_1(x)m(1,t) \quad (9a)$$
$$n_t(x,t) - \mu n_x(x,t) = c_2(x)m(x,t) - P_2(x)m(1,t) \quad (9b)$$
$$m(0) = rn(0,t) + 1 \quad (9c)$$
$$n(1) = 0 \quad (9d)$$

and

$$w_t(x,t) + \lambda w_x(x,t) = c_1(x)z(x,t) - P_1(x)w(1,t) \quad (10a)$$
$$z_t(x,t) - \mu z_x(x,t) = c_2(x)w(x,t) - P_2(x)w(1,t) \quad (10b)$$
$$w(0,t) = rz(0,t) - y_0(t) \quad (10c)$$
$$z(1,t) = 0 \quad (10d)$$

where P_1, P_2 are gains to be designed. The input filters model how the control signal $U(t)$ affect the system states u, v, while the parameter filters model the effect of the boundary parameters k and θ on the system states.

Using the swapping filters, the following static relationship between the system states (u, v) and unknown parameters k and θ can be found

$$u(x,t) = a(x,t) + k\left(\theta m(x,t) + w(x,t)\right) + e(x,t) \quad (11a)$$
$$v(x,t) = b(x,t) + k\left(\theta n(x,t) + z(x,t)\right) + \epsilon(x,t) \quad (11b)$$

where e, ϵ represent the *non-adaptive* estimation error. It is shown in Holta et al. (2017) that if the gains P_1, P_2 are selected as

$$P_1(x) = \lambda P^{uu}(x,1) \quad (12a)$$
$$P_2(x) = \lambda P^{vu}(x,1), \quad (12b)$$

where (P^{uu}, P^{vu}) is the solution to

$$\lambda P^{uu}_x(x,\xi) + \lambda P^{uu}_\xi(x,\xi) = c_1(x)P^{vu}(x,\xi) \quad (13a)$$
$$\mu P^{vu}_x(x,\xi) - \lambda P^{vu}_\xi(x,\xi) = -c_2(x)P^{uu}(x,\xi) \quad (13b)$$
$$P^{vu}(x,x)\lambda + P^{vu}(x,x)\mu = c_2(x) \quad (13c)$$
$$P^{uu}(0,\xi) = rP^{vu}(0,\xi) \quad (13d)$$

defined over $\mathcal{T}_1 = \{(x,\xi) \,|\, 0 \le x \le \xi \le 1\}$, then the error terms e, ϵ will tend to zero in a finite time given by

$$t_F = \frac{1}{\lambda} + \frac{1}{\mu}. \quad (14)$$

It is shown in Coron et al. (2013) that the system (13) has a continuous, bounded and unique solution (P^{uu}, P^{vu}).

Motivated by the bilinear form of the static relationship (11), the following *adaptive* state estimates are generated:

$$\hat{u}(x,t) = a(x,t) + \hat{k}(t)\left(\hat{\theta}(t)m(x,t) + w(x,t)\right)$$
$$= u(x,t) - \hat{e}(x,t) \quad (15a)$$
$$\hat{v}(x,t) = b(x,t) + \hat{k}(t)\left(\hat{\theta}(t)n(x,t) + z(x,t)\right)$$
$$= v(x,t) - \hat{\epsilon}(x,t) \quad (15b)$$

where $\hat{e}, \hat{\epsilon}$ represent the *adaptive* estimation error, and $\hat{\theta}$ and $\hat{k}$ are estimates of θ and k, respectively.

Evaluating (15a) at $x = 1$, inserting (4) and rearranging then give

$$\hat{e}(1,t) = y_1(t) - a(1,t) - \hat{k}(t)\left(\hat{\theta}(t)m(1,t) + w(1,t)\right). \quad (16)$$

Assuming the sign of k is known, the *gradient method for bilinear parametric models* in Ioannou and Sun (2012, Theorem 4.52) can be used to minimize a cost function based on the square error $\hat{e}^2(1,t)$ and thereby forming an adaptive law for the parameter estimates $\hat{\theta}, \hat{k}$. The following theorem presents the main results from Ioannou and Sun (2012, Theorem 4.52) together with some additional properties needed to prove stability of the closed loop system.

Theorem 1. Consider the adaptive laws

$$\dot{\hat{\theta}}(t) = \gamma_1 \mathrm{sign}(k)\frac{\hat{e}(1,t)}{1 + w^2(1,t)}m(1,t) \quad (17a)$$
$$\dot{\hat{k}}(t) = \gamma_2\left[\hat{\theta}(t)m(1,t) + w(1,t)\right]\frac{\hat{e}(1,t)}{1 + w^2(1,t)} \quad (17b)$$

for $t \ge t_F$ and $\dot{\hat{\theta}}(t) = \dot{\hat{k}}(t) = 0$ for $t < t_F$, where $\gamma_1, \gamma_2 > 0$ are the adaptation gains, $m(1,t)$ and $w(1,t)$ are the filters given in (9) and (10), $\hat{e}(1,t)$ is the adaptive estimation error in (16) and t_F is defined in (14). Suppose system (3) has a unique solution u, v for all $t \ge 0$. Then, the adaptive laws (17) have the following properties:

(1) $\hat{\theta}, \hat{k}, \in \mathcal{L}_\infty$.
(2) $\dot{\hat{\theta}}, \dot{\hat{k}}, \in \mathcal{L}_\infty \cap \mathcal{L}_2$.
(3) $\hat{\theta}(t) \to \hat{\theta}(t - d_\beta)$ and $\hat{k}(t) \to \hat{k}(t - d_\beta)$.
(4) $\frac{\tilde{k}(\theta - y_0) + \hat{k}\tilde{\theta}}{\sqrt{1 + w^2(1,\cdot)}} \in \mathcal{L}_2$ where $\tilde{\theta} = \theta - \hat{\theta}$ and $\tilde{k} = k - \hat{k}$.
(5) If $w(1,\cdot) \in \mathcal{L}_\infty$ and $\hat{\theta}m(1,\cdot) + w(1,\cdot) \in \mathcal{L}_2$, then $\hat{\theta}$ converges to θ and $\hat{k}$ converges to some constant.

Proof. See Holta et al. (2017).

2.2 Closed Loop Adaptive Control

The control law from Holta et al. (2018) is given in terms of the state estimates $(\hat{u}, \hat{v})$ and parameter estimates $\hat{\theta}, \hat{k}$. The parameter estimates are generated from the adaptive laws in Theorem 1. Once these estimates are found, the adaptive relationship (15) can be used to generate state estimates.

Theorem 2. Consider system (3), the state estimates (15) and the adaptive law (17), and suppose (7) holds. Then, the control law

$$U(t) = \mathcal{K}[\hat{u}, \hat{v}](1) + \frac{1}{a_0 r + b_0}\hat{\theta}(t) \quad (18)$$

where $\mathcal{K} : L_2([0,1]) \times L_2([0,1]) \to L_2([0,1])$ is the operator given by

$$\mathcal{K}[\hat{u}, \hat{v}](x) = \hat{v}(x) - \int_0^x K^{vu}(x,\xi)\hat{u}(\xi)d\xi$$
$$- \int_0^x K^{vv}(x,\xi)\hat{v}(\xi)d\xi \quad (19)$$

defined for $x \in [0,1]$ where (K^{vu}, K^{vv}) is the unique solution to (see Coron et al. (2013))

$$K^{vu}_x(x,\xi)\mu - K^{vu}_\xi(x,\xi)\lambda = K^{vv}(x,\xi)c_2(x) \quad (20a)$$
$$K^{vv}_x(x,\xi)\mu + K^{vv}_\xi(x,\xi)\mu = K^{vu}(x,\xi)c_1(x) \quad (20b)$$
$$K^{vu}(x,x)\lambda + K^{vu}(x,x)\mu = -c_2(x) \quad (20c)$$
$$K^{vu}(x,0)\lambda r = K^{vv}(x,0)\mu \quad (20d)$$

defined over $\mathcal{T}_2 = \{(x,\xi) \,|\, 0 \le \xi \le x \le 1\}$, guarantees (6). Moreover, all signals in the closed loop system are bounded and the parameter estimate $\hat{\theta}$ converges to its true value θ in the sense

$$\int_t^{t+T} |\hat{\theta}(\tau) - \theta| d\tau \to 0 \quad (21)$$

for some $T > 0$.

3. ESTIMATION WITH ADAPTATION BASED ON BOTTOM-HOLE SENSING (NEW METHOD)

The adaptive law in Theorem 1 is designed to minimize the top-side estimation error and the bottom-hole pressure measurement is only used indirectly in the filters (8)-(10). Even though new measurements $y_0(t)$ are instantly available to the control unit by wired drill-pipe technology, the old design forces new measurements to propagate through the filter systems before the state estimates are updated, and consequently, the top-side error $\hat{e}(1,t)$ is only affected by $y_0(t-\lambda^{-1})$. In contrast, the proposed method of this section, which is the main result of the paper, utilizes the bottom-hole pressure measurement immediately and directly in the adaptive law, which is designed to minimizes the bottom-hole estimation error rather than the top-side error. The artificial time delay introduced by the filters in the old method is therefore avoided and the parameter and state estimates approach their true values significantly faster, as demonstrated in simulations in Section 4.

The new design goes as follows. Using that $e(0,t)=\epsilon(0,t)=0$ for all $t \geq t_F$ and inserting (5) into the static relationship (11), and evaluating at $x=0$ give

$$\begin{aligned} y_0(t) =& a_0 u(0,t) + b_0 v(0,t) \\ =& a_0 \left(a(0,t) + k\left(\theta m(0,t) + w(0,t)\right)\right) \\ &+ b_0 \left(b(0,t) + k\left(\theta n(0,t) + z(0,t)\right)\right). \end{aligned} \tag{22}$$

Defining

$$\breve{a}(t) = a_0 a(0,t) + b_0 b(0,t) \tag{23a}$$

$$\breve{m}(t) = a_0 m(0,t) + b_0 n(0,t) \tag{23b}$$

$$\breve{w}(t) = a_0 w(0,t) + b_0 z(0,t) \tag{23c}$$

and rearranging the terms, give the bilinear parametric model

$$y_0(t) - \breve{a}(t) = k\left(\theta \breve{m}(t) + \breve{w}(t)\right). \tag{24}$$

The same adaptive state estimates (15) will be reused here. Evaluating (15) at $x=0$, inserting (5) and defining

$$\breve{e}(t) = a_0 \hat{e}(0,t) + b_0 \hat{\epsilon}(0,t) \tag{25}$$

then give

$$\breve{e}(t) = y_0(t) - \breve{a}(t) - \hat{k}(t)\left(\hat{\theta}(t)\breve{m}(t) + \breve{w}(t)\right). \tag{26}$$

Assuming the sign of k is known, the gradient method for bilinear parametric models in Ioannou and Sun (2012, Theorem 4.52) can be used to minimize a cost function based on the square error $\breve{e}^2(t)$ and thereby forming an adaptive law for the parameter estimates $\hat{\theta}, \hat{k}$.

Theorem 3. Consider the adaptive laws

$$\dot{\hat{\theta}}(t) = \begin{cases} \gamma_1 \mathrm{sign}(k) \dfrac{\breve{e}(t)}{1+\breve{w}^2(t)} \breve{m}(t) & t \geq t_F \\ 0 & \text{otherwise} \end{cases} \tag{27a}$$

$$\dot{\hat{k}}(t) = \begin{cases} \gamma_2 \left[\hat{\theta}(t)\breve{m}(t) + \breve{w}(t)\right] \dfrac{\breve{e}(t)}{1+\breve{w}^2(t)} & t \geq t_F \\ 0 & \text{otherwise} \end{cases} \tag{27b}$$

for some adaptation gains $\gamma_1, \gamma_2 > 0$ where $\breve{m}(t)$ and $\breve{w}(t)$ are given in (23), $\breve{e}(t)$ is the adaptive estimation error (25) and t_F is defined in (14). Suppose system (3) has a unique solution u, v for all $t \geq 0$. Then, the adaptive laws (27) have the following properties:

(1) $\hat{\theta}, \hat{k} \in \mathcal{L}_\infty$.
(2) $\dot{\hat{\theta}}, \dot{\hat{k}} \in \mathcal{L}_2 \cap \mathcal{L}_\infty$.
(3) $\frac{\breve{e}}{\sqrt{1+\breve{w}^2(1,\cdot)}} \in \mathcal{L}_\infty \cap \mathcal{L}_2$
(4) If $\breve{w} \in \mathcal{L}_\infty$ and $\hat{\theta}\breve{m} + \breve{w} \in \mathcal{L}_2$, then $\hat{\theta}$ converges to θ and $\hat{k}$ converges to some constant.

Proof. Consider the Lyapunov function candidate

$$V_0 = |k| \frac{1}{2\gamma_1} \tilde{\theta}^2 + \frac{1}{2\gamma_2} \tilde{k}^2 \tag{28}$$

where $\tilde{\theta} = \theta - \hat{\theta}$ and $\tilde{k} = k - \hat{k}$. Differentiating, inserting the adaptive laws (27) for $t \geq t_F$ and using relation (26) give

$$\begin{aligned} \dot{V}_0 &= -|k| \frac{1}{\gamma_1} \tilde{\theta}\dot{\hat{\theta}} - \frac{1}{\gamma_2} \tilde{k}\dot{\hat{k}} \\ &= -\frac{\breve{e}(t)}{1+\breve{w}^2(t)} \left(|k|\tilde{\theta}\mathrm{sign}(k)\breve{m}(t) - \tilde{k}\left[\hat{\theta}(t)\breve{m}(t) + \breve{w}(t)\right]\right) \\ &= -\frac{\breve{e}(t)}{1+\breve{w}^2(t)} \left(a_0 u(0,t) + b_0 v(0,t) - a_0 \hat{u}(0,t) - b_0 \hat{v}(0,t)\right) \\ &= -\frac{\breve{e}^2(t)}{1+\breve{w}^2(t)} \leq 0 \end{aligned} \tag{29}$$

which shows that $V_0, \tilde{\theta}, \tilde{k} \in \mathcal{L}_\infty$. The adaptive estimation error $\breve{e}$ can be written on the form

$$\breve{e}(t) = \Theta(t)^T \Psi(t) \tag{30}$$

where

$$\Theta(t) = \left[\tilde{k}(t),\ \sqrt{|k|}\tilde{\theta}(t)\right]^T \tag{31a}$$

$$\Psi(t) = \left[\hat{\theta}\breve{m}(t) + \breve{w}(t),\ \mathrm{sign}(k)\sqrt{|k|}\breve{m}(t)\right]^T \tag{31b}$$

It is shown in Holta et al. (2017) that the filter system (m,n) in (9) is bounded point-wise in x. We then have

$$\frac{1}{\sqrt{1+\breve{w}^2(1,\cdot)}} \Psi(t) \in \mathcal{L}_\infty \tag{32}$$

which together with Property 1 and (30) give

$$\frac{\breve{e}}{\sqrt{1+\breve{w}^2(1,\cdot)}} \in \mathcal{L}_\infty. \tag{33}$$

Integrating (29) from $t=0$ to $t=\infty$ and using that $V_0 \geq 0$ is a non-increasing function of time give

$$\begin{aligned} \int_0^\infty \left(\frac{\breve{e}^2(1,\tau)}{1+\breve{w}^2(\tau)}\right) d\tau =& -\int_0^\infty \dot{V}_0(\tau) d\tau \\ =& V_0(0) - V_0(\infty) < \infty \end{aligned} \tag{34}$$

and therefore

$$\frac{\breve{e}(1,\cdot)}{\sqrt{1+\breve{w}^2(\cdot)}} \in \mathcal{L}_2. \tag{35}$$

From (27a), one has

$$\left|\dot{\hat{\theta}}(t)\right| \leq \gamma_1 \left|\frac{\breve{e}(t)}{\sqrt{1+\breve{w}^2(t)}}\right| \left|\frac{\breve{m}(t)}{\sqrt{1+\breve{w}^2(t)}}\right| \tag{36}$$

which together with (33), (35) and boundedness of $\breve{m}$ give $\dot{\hat{\theta}} \in \mathcal{L}_\infty \cap \mathcal{L}_2$ and the first part of Property 2. For the second part, one has similarly

$$\left|\dot{\hat{k}}(t)\right| \leq \gamma_2 \left|\frac{\breve{e}(t)}{\sqrt{1+\breve{w}^2(t)}}\right| \left|\frac{\hat{\theta}\breve{m}(t) + \breve{w}(t)}{\sqrt{1+\breve{w}^2(t)}}\right| \tag{37}$$

which together with (33), (35) and boundedness of $\breve{m}$ give $\dot{\hat{k}} \in \mathcal{L}_\infty \cap \mathcal{L}_2$ and the second part of Property 2. Inserting (26) into (27a) yields

$$\dot{\tilde{\theta}}(t) = -\frac{\gamma_1 \mathrm{sign}(k)}{1+\breve{w}^2(t)}\left(k\tilde{\theta}\breve{m}(t) + \tilde{k}(t)\left(\hat{\theta}\breve{m}(t)+\breve{w}(t)\right)\right)\breve{m}(t) \tag{38}$$

where the last term can be treated as an external input. Using that $\breve{m} \in \mathcal{L}_\infty$ and if the last term $\tilde{k}(t)\left(\hat{\theta}\breve{m}(t)+\breve{w}(t)\right)$ is square integrable, then (38) forms an *exponentially stable* system and it follows that $\tilde{\theta} \to 0$ as $t \to \infty$ or equivalently the first part of Property 4. The second part of Property 4 can be seen by applying Cauchy-Schwarz' inequality to (27b).

Remark 4. Property 4 in Theorem 3 gives sufficient conditions for parameter convergence. For $t > t_F$, we have $\breve{m}(t) = m(0,t) = 1$ and $\breve{w}(t) = w(0,t) = -y_0(t)$. The conditions are then simplified to $y_0 \in \mathcal{L}_\infty$ and $(\hat{\theta} - y_0) \in \mathcal{L}_2$. If the adaptive laws are used in conjunction with a closed loop controller guaranteeing these properties, parameter convergence will follow.

4. SIMULATION

The swapping based estimation scheme presented in Section 3, consisting of the swapping filters (8)-(10), state estimates (15) and the adaptive law of Theorem 3, is combined with the control law from Holta et al. (2018) given in Theorem 2 and implemented in MATLAB (the new method). This design is compared to the design from Holta et al. (2017) (the old method), consisting of the swapping filters (8)-(10), state estimates (15), the adaptive law of Theorem 1 and the control law of Theorem 2, which is also implemented in MATLAB. In addition, a simple controller (the simple method) where the top-side flow is kept equal to the drill bit flow $q(l,t) = q_{bit}$ is also implemented. For all control schemes, the system parameters are chosen as

$$\beta = 7317\,\mathrm{Pa}, \qquad \rho = 1250\,\mathrm{kg\,m^{-3}} \tag{39a}$$

$$l = 2500\,\mathrm{m}, \qquad A_1 = 0.024\,\mathrm{m^2} \tag{39b}$$

$$F_1 = 200, \qquad g = 9.81\,\mathrm{m\,s^{-2}} \tag{39c}$$

$$q_{bit} = 1/60\mathrm{m^3\,s^{-1}}, \qquad J = 1.1 \times 10^{-8}\,\mathrm{m^3\,s^{-1}\,Pa^{-1}}. \tag{39d}$$

The reservoir pressure is initially set to $p_r(0) = 400\,\mathrm{bar}$ and kept constant until a step to $p_r(t \geq t_0) = 450\,\mathrm{bar}$ occurs at $t_0 = 10\,\mathrm{s}$. The system is at steady state at $t = 0$ with the initial bottom-hole pressure set equal to the reservoir pressure and the bottom-hole flow equal to the drill bit flow. The adaptation gains are selected as $\gamma_1 = \gamma_2 = 5$.

Figures 2 and 3 show the bottom-hole pressure and flow when using the three methods. The figures show that all three methods are able to attenuate the kick. The bottom-hole pressure is stabilized at the reservoir pressure and the net gain into the well converges to zero. It is seen that both the new method and the old method converge in an approximately finite time after $\sim 10\,\mathrm{s}$, whereas the simple method has a much slower asymptotic convergence time. In addition, as can be seen from Figure 6, the new method offers a $\sim 35\,\%$ reduction in total accumulated inflow compared to the old method. This is due to the better utilization of the bottom-hole measurement as can be seen from the state estimation error in Figures 4, 5 and 7. Figure 7 also shows that the reservoir pressure estimates converge to the true value for both methods.

5. CONCLUSIONS AND FURTHER WORK

A new method for kick detection and attenuation in managed pressure drilling is presented. A swapping based estimator utilizing bottom-hole pressure estimates for fast parameter adaption is presented. The estimation scheme is combined with a recently developed closed loop controller for kick & loss attenuation. The new design was compared to earlier works on kick & loss attenuation in a simulation, suggesting that significant performance improvement is possible by exploiting downhole pressure measurements made available in real-time by wired drill-pipe technology. Further work include a rigorous proof of closed loop stability in the L_2-sense and convergence of the bottom-hole pressure to the desired set-point.

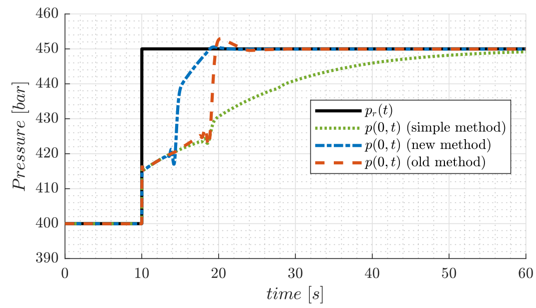

Fig. 2. Bottom-hole pressure.

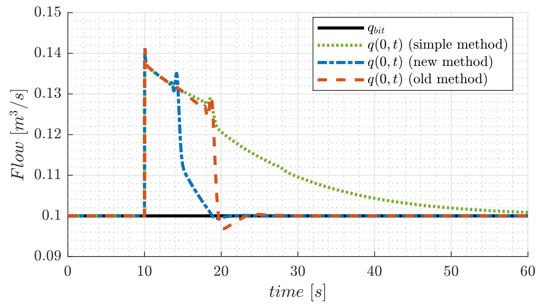

Fig. 3. Bottom-hole flow.

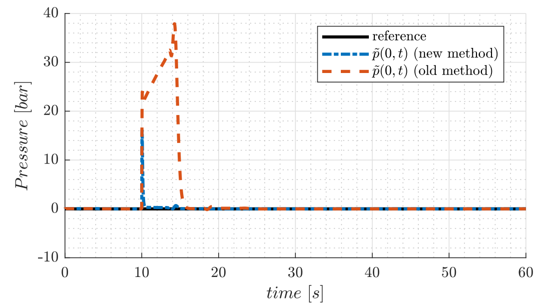

Fig. 4. Bottom-hole pressure estimation error.

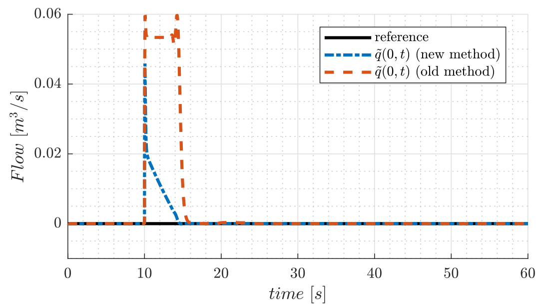

Fig. 5. Bottom-hole flow estimation error.

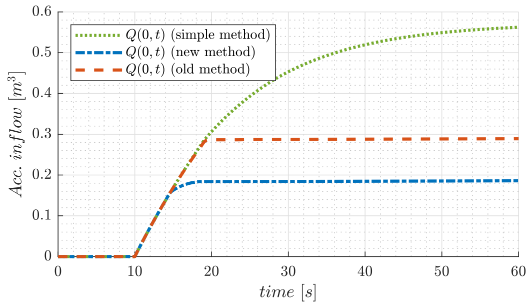

Fig. 6. Accumulated net inflow.

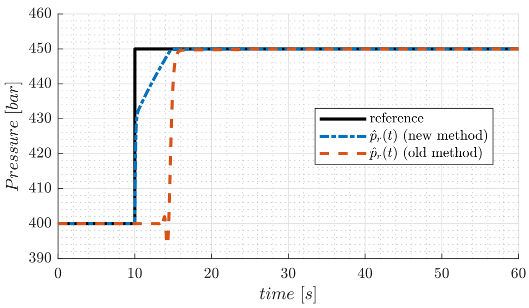

Fig. 7. Reservoir pressure estimate.

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