# A GENERAL DISJUNCTIVE MODEL FOR THE RETROFIT OF MULTIPRODUCT BATCH PLANTS

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*Abstract*

The retrofit of multiproduct batch plants treats the modification of the original configuration of the plant to satisfy new production conditions (new products, new demand pattern, etc.). A disjunctive model is presented to solve this problem that includes all the usual alternatives in this kind of problems. A disjunction is generated for each batch stage considering all the feasible configuration alternatives for old and new units. Each disjunction term contains all constraints to model the batch stage options: operation time, units sizing and cost, etc. In a similar way, a disjunction is generated for the allocation of intermediate storage tanks. Several examples were solved, the computational performance of this approach is compared with the conventional formulation.

*Keywords*

Multiproduct batch plants, Disjunctive programming, Retrofit.

## Introduction

In the retrofit problem the batch plant structure is changed in order to satisfy the new production requirements due to the development of new products, new demands, etc. The options considered include the allocation of new units, the sale of useless old units and the configuration of new units.

The differences between previous works (Vaselenak et al., 1987; Fletcher et al., 1991and Yoo et al., 1999) are the available options to configure the units at each batch stage. The objective is to maximize the plant benefits subject to a new demand pattern. Previous papers solve a MINLP model, except Van den Heaver and Grossman (1999) whose proposed disjunctive multiperiod program, but they do not include all the options of the work of Yoo et al (1999). Finally, Montagna et al. (2001) present a model considering storage tanks.

A general model using the Generalized Disjunctive Programming (GDP) approach is presented for the retrofit problem. All the available alternatives to configure the old and new batch units are maintained; the allocation of intermediate storage tanks is included. The advantages of the new model are analyzed and the performance of the conventional versus the GDP approach is compared.

## Generalized Disjunctive Programming

The Generalized Disjunctive Programming (GDP) problem is shown (Lee and Grossmann, 2000):

$$\begin{aligned}
& Min\ z = \sum_{k \in K} c_k + f(x) \\
& s.t.\ \ r(x) \le 0 \\
& \bigvee_{j \in J_k} \begin{bmatrix} Y_{jk} \\ g_{jk}(x) \le 0 \\ c_k = \gamma_{jk} \end{bmatrix},\ k \in K \qquad (1) \\
& \Omega(Y) = True \\
& x \ge 0,\ c_k \ge 0,\ Y_{jk} \in \{true,\ false\}
\end{aligned}$$

In this model, $x \in R^n$ are continuous variables and $Y_{jk}$ are boolean variables. $c_k \in R^1$ are continuous variables and

$\gamma_{jk}$ are fixed charges. f: $R^n \rightarrow R^1$ is the term of the objective function that depend on the variables x. r: $R^n \rightarrow R^q$ are constraints that hold regardless of the discrete decisions. This general model assumes that f(x) and r(x) are convex functions. Finally $\Omega(Y)$ =True is a logical constraint set relating the boolean variables Y.

**Multiproduct batch plant retrofit**

In a multiproduct batch plant i=1,...,P products are processed following the same sequence over the j=1,…,N batch stages of the plant. A set of $N_j$ units is available in stage j, with $N_j^{OLD}$ existing units and $N_j^{NEW}$ new units to be added in the plant, so:

$$N_j = N_j^{OLD} + N_j^{NEW} \qquad \forall j \qquad (2)$$

The units configuration in stage j can be different for each product i. The units can be arranged in phase or out of phase. In the last case, the batch is separated among all the units that conform the group operating in phase. Figure 1 shows stage j with four units. $V_{jk}$ is the size of unit k in stage j. The superposed units operate in phase, like units 1 and 3 conforming the group 1 and units 2 and 4 in the group 2. Both groups operate out of phase.

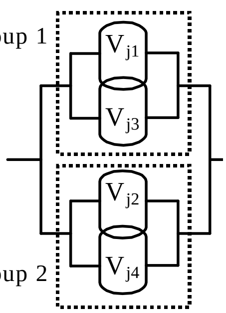


*Figure 1. Groups in stage j*

The configuration of the batch units must be determined at each stage for every product. A configuration option h is an arrangement of units. Fig. 1 is one option for a stage with 4 units arranged into two groups. For example, in a stage with 3 units, there are 14 feasible options:

$h_1$ = [1]; $h_2$ = [2]; $h_3$ = [3]; $h_4$ = [1,2]; $h_5$ = [1,3];
$h_6$ = [2,3]; $h_7$ = [1,2,3]; $h_8$ = [1]+[2]; $h_9$ = [1]+[3];
$h_{10}$ = [2]+[3]; $h_{11}$ = [1,2]+[3]; $h_{12}$ = [1]+[2,3];
$h_{13}$ = [1,3]+[2]; $h_{14}$ = [1]+[2]+[3]

Considering that the units between brackets belong to a group, there are 7 options with an only group ($h_1$ a $h_7$), 6 options with 2 groups ($h_8$ a $h_{13}$) and one option with 3 groups ($h_{14}$). The designer a priori can discard options not optimal or not feasible.

To represent the available options the following disjunction is proposed for each product and stage, where the Boolean variable $Y_{ijh}$ indicates the term h of the disjunction that is true. Only one of them can be true:

$$\bigvee_{h \in H_j} \left[ \begin{array}{c} Y_{ijh} \\ \sum_{k \in g} V_{jk} \geq S_{ij} B_{ij} \qquad \forall g \in h \\ \dfrac{B_{ij}}{Pr_i} \geq \dfrac{T_{ij}}{M_h} \\ CQ_{ij} = \sum_{\substack{k=N_j^{OLD}+1,N_j \\ k \in h}} CB_{jk} - \sum_{\substack{k=1,N_j^{OLD} \\ k \notin h}} CS_{jk} \end{array} \right] \forall i, \forall j \quad (3)$$

The first constraint in the disjunction corresponds to the sum of the unit sizes included in the group g that must be large enough to produce a batch of product i. $B_{ij}$ is the batch size of product i in stage j, and $S_{ij}$ the size factor. The second constraint is the limiting time $TL_i$ of product i, meaning that the time between two consecutive batches of product i must be greater than the operation time of the stage $T_{ij}$, divided by the number of groups in option h, $M_h$. This expression is modified considering that the same production rate $Pr_i$ must be satisfied in all the stages to avoid accumulation of material in the intermediate storage tanks. Finally the last equation defines the cost of this alternative $CQ_{ij}$. In the first summation, $CB_{jk}$ is the buying cost of unit k that is determined using the same expression of previous works. In the second summation, $CS_{jk}$ is the selling value of the useless units that is a constant defined a priori for each old unit.

Another discrete decision is to allocate new storage tanks between batch stages. There are j=1,…,N-1 positions. Position j is between batch stage j and j+1. The sale of useless tanks is also considered. The $HT_j$ available options in position j depend on the number $NT_j$ of tanks available in that position. Only one new tank can be allocated at position j. For example, if $NT_j$=1, there are $HT_j$= 4 options: 1) Two tanks, the old and the new, 2) The old tank (no tank is added), 3) The new tank (the old tank is sold), 4) No tanks (the old tank is sold).

The following disjunction consider all the available options in the position j, where the Boolean variable $YT_{ij,ht}$ is true in only one term of the disjunction:

$$\bigvee_{ht \in HT_j} \left[ \begin{array}{c} YT_{ij,ht} \\ \dfrac{1}{\theta} \leq \dfrac{B_{ij}}{B_{i,j+1}} \leq \theta \\ \sum_{kt \in ht} VT_{j,kt} \geq ST_{ij} ( B_{ij} + B_{i,j+1} ) \\ TQ_{ij} = \begin{pmatrix} TB_{j,NT_j+1} \\ NT_j + 1 \in ht \end{pmatrix} - \sum_{\substack{kt=1,NT_j \\ kt \notin ht}} TS_{j,kt} \end{array} \right] \begin{array}{c} \forall i \\ j=1,N-1 \end{array} \quad (4)$$

This is the general form of the terms of this disjunction. The option with no tanks is different from (4). For example, the first constraint relates the batches of consecutives stages (Ravemark, 1995). If a tank exists at position j, then the batch sizes downstream and upstream of the tank can be different. θ is the maximum ratio allowed between consecutive batch sizes. In the option with no tanks both batch sizes (up and downstream) must be equal. The second constraint determines the size $VT_{j,kt}$ of the storage tank kt at position j, where $ST_{ij}$ is the size factor for product i and position j. In the term with no tanks this constraint is not included. The last constraint is the cost of the disjunction, $TQ_{ij}$. In the first term, $TB_{j,NT+1}$ is the cost of the new tank, which is considered only when it is included in the option ht. In the last term, $TS_{j,kt}$ is the value of the useless storage tanks that are sold.

The objective function of the problem is:

$$Max\ z = \sum_{i=1}^{P} p_i\ Q_i - \sum_{j=1}^{N} CE_j - \sum_{j=1}^{N-1} CET_j \qquad (5)$$

where $p_i$ is the net profit per unit and $Q_i$ is the production of product i. $CE_j$ and $CET_j$ are the value of the units in the batch stage j, that result from the following constraints:

$$CE_j \geq CQ_{ij} \qquad \forall i; \forall j=1,N \qquad (6)$$

$$CET_j \geq TQ_{ij} \qquad \forall i; \forall j=1,N\text{-}1 \qquad (7)$$

The batch units included in a stage must be used for all the products, so the options considered are reduced through the following constraint:

$$\sum_{\substack{g,h \\ k\in g \\ g\in h}} Y_{ijh} = \sum_{\substack{g,h \\ k\in g \\ g\in h}} Y_{ii,jh} \qquad \forall j, \forall k, \forall i \neq ii \qquad (8)$$

$N_{ij}$ is the number of batches of product i in stage j. $Q_i$ must not overcome the amount processed in each stage:

$$Q_i \leq N_{ij} B_{ij} \qquad \forall i, \forall j \qquad (9)$$

All the products must be produced in the available time horizon H:

$$\sum_i \frac{Q_i}{Pr_i} \leq H \qquad (10)$$

To solve this model, the disjunctions are transformed into mixed integer constraints using the relaxation by convex hull (Balas, 1979). Transformations are introduced to avoid non-convex term in Eq. 9 (Vaselenak et al., 1987). Then, the first term in the objective function is concave. To overcome this problem, the same authors have proved that the negative exponential functions in that term can be approximated by a system of piecewise linear underestimators. This approximation overestimates the objective function so it can be employed to find the global solution of this model.

Finally a MINLP is obtained that has been solved using the OA/ER/AP method.

*Table 1. Example data*

| | $ST_{ij}$ | | | |
|---|---|---|---|---|
| | Product A | Product b | $KT_j$ | $Ct_j$ |
| Position 1 | 1 | 1 | 10,000 | 10 |

*Table 2. Results of the example*

| | Yoo et al. (1999) | | Our approach | |
|---|---|---|---|---|
| Product | A | B | A | B |
| $Q_i$/1000 | 2,000 | 4,000 | 2,000 | 4,000 |
| | New units | | New units | |
| Stage 1 | 1,000 | | - | |
| | (u2,u3) | (u2)-(u3) | (u2) | (u2) |
| Stage 2 | - | | - | |
| | (u2) | (u2) | (u3) | (u3) |
| Storage Tank Position 1 | - | | - | 3,600 |
| | Sold units | | Sold units | |
| Stage 1 | u1 | | u1 | |
| Stage 2 | u1, u3 | | u1, u2 | |
| Profit ($) | 752,000 | | 759,400 | |

**Examples**

Several examples have been solved with this formulation. The results obtained depend on the size factors and the costs of the intermediate storage tanks that have not been considered in previous approaches of the retrofit problem. The values have been selected to show the potential applications of this approach. Here, the example 5 of Yoo et al. has been selected. Table 1 presents the data added to example 5 of Yoo et al. (1999), where $KT_j$ and $ct_j$ are the parameters required to determine the cost of the storage tanks.

Table 2 shows the result of this problem, where u2 means unit 2. In both formulations old units are sold. Units between brackets operate in phase. Figure 2 shows the solution for the Yoo et al. (1999) approach. Units in grey are new. Figure 3 corresponds to the optimal solution with the proposed formulation. A storage tank is allocated between both batch stages. Although the optimal profits are very similar, both plants look very different.

**Computational performance**

Table 3 show the CPU time for the 5 examples presented by Yoo et al. (1999) that have been solved using both approaches. The first column corresponds to the number of discrete variables before the linearization of the objective function. The following two columns shows the CPU times for these examples using the formulation by Montagna et al. (2001), that also considered storage tanks, and for this approach, respectively. All the times were obtained using a PC with a Pentium Celeron processor of 650 Mhz. With the present formulation a considerable reduction has been obtained in the CPU time required to solve the MINLP.

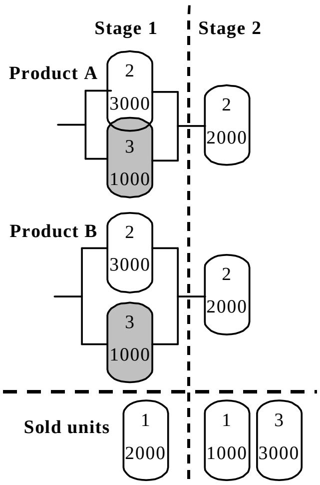


*Figure 2. Solution with the Yoo et al. approach*

*Table 3.Computational performance*

| | Discrete variables | CPU time Montagna et al. (2001) | CPU time for this approach |
|---|---|---|---|
| Ex. 1 | 60 | 3 | 3 |
| Ex. 2 | 128 | 60 | 33 |
| Ex. 3 | 120 | 21 | 15 |
| Ex. 4 | 134 | 97 | 28 |
| Ex. 5 | 208 | 144 | 28 |

## Conclusions

A disjunctive model has been presented to solve the retrofit of multiproduct batch plants. This formulation considers all the feasible options for this kind of problems. Disjunctions have been formulated for the discrete decisions about the structure of the new and old units. The sale of useless units was included. Intermediate storage tanks were also considered with a similar formulation.

The problem solution has been obtained by transforming the problem into a Mixed Integer Nonlinear Program (MINLP) using the convex hull relaxation of a disjunctive set.

Using this approach several advantages can be obtained. The first and more remarkable is that the problem formulation is easier to generate, and the model is more understandable than the previous proposed by Yoo et al. (1999) and Montagna et al. (2001). The inclusion of intermediate storage tank produces better optimal solutions with lower costs. Besides, important CPU time reduction is obtained with this approach compared to the previous one.

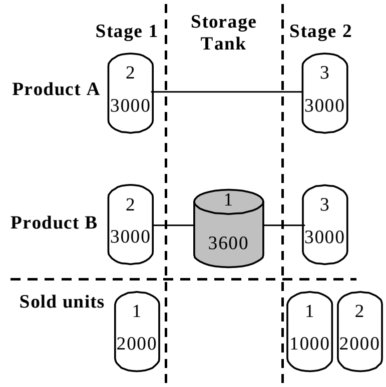


*Figure 3. Solution with the proposed approach*

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