

# SENSOR NETWORK REALLOCATION AND UPGRADE FOR EFFICIENT FAULT DIAGNOSIS

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## *Abstract*

Fault diagnosis is a pre-requisite for ensuring safe, efficient and optimal operation of chemical process plants. The success of any diagnosis strategy depends critically on the sensors measuring the process variables. With potentially many sensor locations, sensor placement can be optimized based on criteria like cost, reliability etc. We present formulations to perform sensor reallocation and upgrade of an existing sensor network to ensure comprehensive fault diagnosis. These formulations are based on minimizing cost required to achieve the desired reliability, and maximizing reliability for a given cost. A combined formulation which maximizes reliability and, among the various solutions, selects the one with minimum cost is also presented. The cause-effect information for these formulations is obtained from a semi-quantitative representation of the process which might be useful for performing sensor location for industrial processes. The utility of the proposed approach is demonstrated through a retrofit analysis of the Tennessee Eastman case study.

## *Keywords*

Sensor Location, Fault Diagnosis, Reliability Maximization, Cost Minimization, Reallocation and Upgrade.

## **Introduction**

Observing fault symptoms, which are propagated to some or all process variables, is an important step in process fault diagnosis. Thus the success of any diagnosis strategy depends critically on the choice of variables used to monitor the process and therefore on the location of the sensors measuring these variables.

There has been some work for designing sensor location for fault diagnosis by few researchers including Lambert (1977) and Bhushan and Rengaswamy (2002a, 2002b). Bagajewicz and Sanchez (2000) have proposed a model for upgrade and reallocation of sensors at minimum cost to achieve maximum precision of selected parameters.

This work extends the sensor location optimization formulations presented by Bhushan and Rengaswamy (2002a, 2002b) to the problem of upgrading and reallocating existing instrumentation so as to maximize system reliability (from fault diagnosis perspective) for given resources or achieve the desired reliability at minimum cost.

## **Optimization Formulations**

The basic principle of this approach is to maximize overall system reliability or in other words, the probability

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that a fault will be detected. The only way a fault can occur without being detected is that the fault occurs and simultaneously the affected sensors fail. The probability of such an event, called as unobservability,  $U_i$  of the fault is

expressed as  $f_i \prod_{j=1}^n \prod_{k=1}^{l_j} s_{jk}^{d_{ij} x_{jk}}$ , where  $f_i$  is the fault

probability,  $s_{jk}$  is the probability of failure of the sensor of the  $k^{\text{th}}$  type measuring variable  $j$ ,  $d_{ij}$  is a binary variable equal to 1 if fault  $i$  affects variable  $j$  and zero otherwise (which can be obtained from the bipartite matrix  $\mathbf{D}$ ),  $x_{jk}$  is the number of  $k^{\text{th}}$  type (of  $l_j$  different kinds) sensors measuring variable  $j$ . Generation of the matrix  $\mathbf{D}$  is discussed in Bhushan and Rengaswamy (2002b).

The reliability of detecting a fault is thus inversely proportional to the unobservability of the fault. Linear integer programming formulations for different cases of upgrade and reallocation are presented below.

#### Formulation I: Minimum Cost Instrument Upgrade

$$\min \sum_{j=1}^n \sum_{k=1}^{l_j} c_{jk} q_{jk}$$

$$U \geq \ln(U_i), \quad i = 1, \dots, m$$

$$U \leq U^*$$

$$x_{jk} = q_{jk} + x_{jk}^*, \quad k = 1, \dots, l_j \text{ \& } j \in SS$$

$$x_{jk}, q_{jk} \in \mathbf{Z}^+$$

where  $c_{jk}, q_{jk}$ , are cost and number of  $k$  type sensors measuring variable  $j$  that are upgraded,  $SS$  is the set of possible locations of sensors,  $x_{jk}^*$  is the number of  $k$  type sensors currently measuring variable  $j$  and  $U^*$  is the threshold unobservability.

#### Formulation II: Maximum Reliability Instrument upgrade

$$\min U$$

$$U \geq \ln(U_i), \quad i = 1, \dots, m$$

$$\sum_{j=1}^n \sum_{k=1}^{l_j} c_{jk} q_{jk} \leq C^*$$

$$x_{jk} = q_{jk} + x_{jk}^*, \quad k = 1, \dots, l_j \text{ \& } j \in SS$$

$$x_{jk}, q_{jk} \in \mathbf{Z}^+$$

where  $C^*$  is the total available resources (cost) for sensor location. Similar models can be developed for reallocating sensors (transferring sensors from one variable to other) in an existing network. One drawback of the above formulations is that the maximum reliability model will

give the most reliable sensor network, but in general, will not give the least cost network. The following formulation for sensor upgrade and reallocation ensures that among all those solutions that maximize the primary objective (reliability), those that also satisfy the secondary objective (minimum cost) are selected.

#### Formulation III: One-step formulation for instrument upgrade and reallocation

$$\min U - \alpha x_s$$

$$\sum_{j=1}^n \sum_{k=1}^{l_j} c_{jk} q_{jk} + \sum_{t \in M_t} \sum_{r \in M_r} \sum_{k=1}^{l_t} h_{t,r,k} u_{t,r,k} + x_s = C^*$$

$$U \geq \ln(U_i), \quad i = 1, \dots, m$$

$$\left. \begin{aligned} x_{jk} &= q_{jk} + x_{jk}^* - \sum_{r \in M_r} u_{j,r,k}, \quad j \in M_t, \notin M_r \\ x_{jk} &= q_{jk} + x_{jk}^* + \sum_{t \in M_t} u_{t,j,k}, \quad j \in M_r, \notin M_t \\ x_{jk} &= q_{jk} + x_{jk}^* + \sum_{t \in M_t} u_{t,j,k} - \sum_{r \in M_r} u_{j,r,k}, \\ &\quad j \in M_r \cap M_t \\ x_{jk} &= q_{jk} + x_{jk}^*, \quad j \notin M_r \cup M_t \\ \sum_{r \in M_r} u_{j,r,k} &\leq x_{jk}^*, \quad j \in M_t \end{aligned} \right\} k = 1, \dots, l_j$$

$$q_{jk}, u_{t,r,k} \in \mathbf{Z}^+, x_s \geq 0$$

where  $u_{t,r,k}$  and  $h_{t,r,k}$  are the number and cost of  $k^{\text{th}}$  type sensors reallocated from variable  $t$  to  $r$ ,  $x_{jk}^*$  is the number of  $k$  type sensors currently measuring variable  $j$ ,  $M_t$  is the set of variables whose measurements may be reallocated,  $M_r$  is the set of corresponding variables to which sensors may be reallocated,  $C^*$  is the available resource,  $x_s$  is the unutilized resource and  $\alpha$  is an appropriately chosen weighting factor which ensures that the multiple objectives are attained (Bhushan and Rengaswamy, 2002a).

#### Case study- Tennessee Eastman process

The Tennessee Eastman (TE) flowsheet, originally presented by Downs and Vogel (1993) as a process control challenge problem is chosen for case study. A Signed Digraph with gains based process representation (Bhushan and Rengaswamy 2002b) is used to generate cause effect information of the process from the modified process model suggested by Ricker and Lee (1995).

The process (Fig 1) consists of five major unit operations: an exothermic two-phase reactor, product condenser, flash separator, a reboiled stripper, and a recycle compressor. Two products, G and H and a by-product F are produced from reactants A, C, D and E along with an inert component B. Possible locations of sensors, sensor failure probabilities, sensor costs, faults considered

and probabilities of their occurrences are listed in Table 1. Techniques for generation of fault sets, reduction in computational effort etc. are discussed in Bhushan and Rengaswamy (2002b). Unobservability value for the base case is  $-2$  and the above strategies to increase system reliability are applied to the process flowsheet. Cost of

reallocating pressure, flow and concentration sensors are 10, 60 and 160 units respectively. In this case study, only the same type of sensor as the existing one is considered for upgrade. Value of  $\alpha$  chosen for the one-step formulation is  $10^{-5}$  (Bhushan and Rengaswamy, 2002b).

Table 1 Sensor and Fault Data for TE Process

Sensor Data (* denotes Existing sensors)																		Fault data			
Tag	Var	$\log s_j$	Cost	Tag	Var	$\log s_j$	Cost	Tag	Var	$\log s_j$	Cost	Tag	Var	$\log s_j$	Cost	Tag	Var	$\log s_j$	Cost	Var	$\log f_j$
S1*	P <sub>r</sub>	-3	100	S14	y <sub>A,7</sub>	-3	800	S27	y <sub>F,8</sub>	-3	800	S40	x <sub>F,10</sub>	-3	700	S53*	V <sub>is</sub> <sup>s</sup>	-2	150	F <sub>1</sub>	-2
S2*	P <sub>s</sub>	-3	100	S15	y <sub>B,7</sub>	-3	800	S28	y <sub>G,8</sub>	-3	800	S41	x <sub>G,10</sub>	-3	700	S54	C <sub>ls</sub>	-4	100	F <sub>2</sub>	-2
S3*	P <sub>m</sub>	-3	100	S16	y <sub>C,7</sub>	-3	800	S29	y <sub>H,8</sub>	-3	800	S42	x <sub>H,10</sub>	-3	700	S55	CV <sub>ls</sub>	-4	100	F <sub>3</sub>	-2
S4*	F <sub>6</sub>	-3	300	S17	y <sub>D,7</sub>	-3	800	S30*	y <sub>A,9</sub>	-3	800	S43*	x <sub>G,11</sub>	-3	700	S56*	V <sub>lp</sub> <sup>s</sup>	-2	150	F <sub>4</sub>	-2
S5*	y <sub>A,6</sub>	-3	800	S18	y <sub>E,7</sub>	-3	800	S31*	y <sub>B,9</sub>	-3	800	S44*	x <sub>H,11</sub>	-3	700	S57	C <sub>lp</sub>	-4	100	F <sub>8</sub>	-2
S6*	y <sub>B,6</sub>	-3	800	S19	y <sub>F,7</sub>	-3	800	S32*	y <sub>C,9</sub>	-3	800	S45	x <sub>D,r</sub>	-3	700	S58	CV <sub>lp</sub>	-4	100	F <sub>9</sub>	-2
S7*	y <sub>C,6</sub>	-3	800	S20	y <sub>G,7</sub>	-3	800	S33*	y <sub>D,9</sub>	-3	800	S46	x <sub>E,r</sub>	-3	700	S59	F <sub>10</sub>	-3	200	F <sub>10</sub> <sup>p</sup>	-2
S8*	y <sub>D,6</sub>	-3	800	S21	y <sub>H,7</sub>	-3	800	S34*	y <sub>E,9</sub>	-3	800	S47	x <sub>F,r</sub>	-3	700	S60	F <sub>11</sub>	-3	200	F <sub>11</sub>	-2
S9*	y <sub>E,6</sub>	-3	800	S22	y <sub>A,8</sub>	-3	800	S35*	y <sub>F,9</sub>	-3	800	S48	x <sub>G,r</sub>	-3	700	S61	T <sub>s</sub>	-2	500	T <sub>cr</sub>	-1
S10*	y <sub>F,6</sub>	-3	800	S23	y <sub>B,8</sub>	-3	800	S36*	y <sub>G,9</sub>	-3	800	S49	x <sub>H,r</sub>	-3	700					T <sub>cs</sub>	-1
S11	y <sub>G,6</sub>	-3	800	S24	y <sub>C,8</sub>	-3	800	S37*	y <sub>H,9</sub>	-3	800	S50*	V <sub>lr</sub> <sup>s</sup>	-2	150					C <sub>d</sub>	-2
S12	y <sub>H,6</sub>	-3	800	S25	y <sub>D,8</sub>	-3	800	S38	x <sub>D,10</sub>	-3	700	S51	C <sub>lr</sub>	-4	100						
S13	F <sub>7</sub>	-3	300	S26	y <sub>E,8</sub>	-3	800	S39	x <sub>F,10</sub>	-3	700	S52	CV <sub>lr</sub>	-4	100						

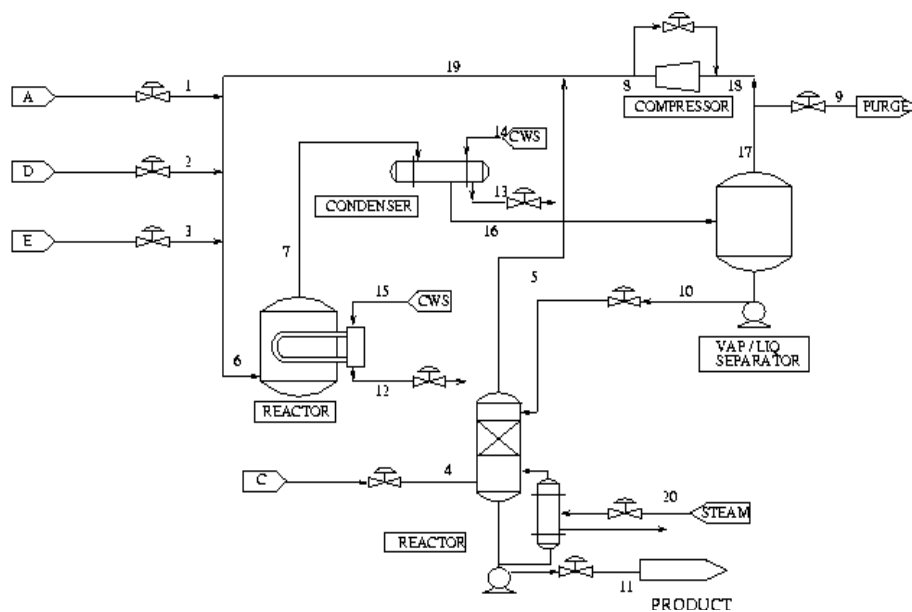


Figure 1 TE Flow sheet

## Results

Solutions of the above described optimization formulations using an optimization package- LINDO are tabulated in Tables 2 (Formulation I), 3 (Formulation II) and 4 (Formulation III).

Table 2 Minimum Cost Instrument Upgrade

U, U*	Cost	New sensors
-5	400	S13, S55
-8	900	S2,S13(2), S58(2)
-11	3000	S2(2),S8,S9,S13(3),S55(3)
-14	4200	S2(3),S5,S8,S9,S13(4),S55(3)

Table 3 Maximum Reliability Instrument Upgrade

C*	Cost	U	New sensors
850	800	-6	S2,S13(2), S58
1350	1300	-8	S2,S4,S13(2), S55(3)
3400	3000	-11	S2(2),S8,S9,S13(3),S55(3)
4300	4300	-14	S2(3),S5,S8,S9,S13(4),S55(4)

Table 4 One-step Upgrade and Reallocation

C*	Cost	U	New sensors	Sensors Reallocated	
				From	To
200	160	-5	S54	S4	S13
				S4	S13
500	470	-6	S13,S58	S1	S2
				S4	S13
600	570	-8	S13,S55,S58	S1	S2
				S1,S3	S2
1400	1300	-11	S13(2),S57 S58(2)	S4	S13
				S30	S5
				S34	S9
				S1,S3	S2
2000	1860	-14	S2,S13(3) S54(3)	S1,S3	S2
				S4	S13
				S30	S5
				S33	S8
				S34	S9

Comparison of Tables 2 and 3 reveals that it is often possible to achieve the desired reliability at a cost lower than what is obtained by solving the maximum reliability problem. However, reallocating existing sensors lowers the cost further. The cost suggested also happens to be minimum for attaining the desired reliability. The solutions to the combined formulation also indicate which sensor could be upgraded and which need to be reallocated (Table 3).

## Conclusions

The utility of optimization formulations for upgrade and reallocation of an existing sensor network to enhance system reliability from the perspective of fault diagnosis was demonstrated through the Tennessee Eastman case study. The disadvantage of single objective models can be overcome by the use of a suitably posed one-step multi-objective problem.

## Nomenclature

- $c_{j,k}$  = Cost of upgrading k type sensor measuring variable j  
 $C^*$  = Available resource  
 $C_d^-$  = Catalyst deactivation  
 $C$  = Controller  
 $CV$  = Control valve  
 $d_{ij}$ ,  $D$  =  $i,j$ <sup>th</sup> element of bipartite matrix  $D$   
 $f_i$  = Prob. of occurrence of fault i  
 $F_i$  = Molar flow rate of stream i  
 $h_{t,r,k}$  = Cost of moving k type sensor from variable t to r  
 $l_j$  = Number of types of sensor measuring j  
 $m,n$  = Number of faults, and measurable variables  
 $P$  = Pressure  
 $q_j$  = Number of new sensors chosen to measure j  
 $S_{j,k}$  = Prob. of failure of sensor of type k measuring j  
 $T$  = Temperature  
 $u_{t,r,k}$  = Number of k type sensors moved from variable t to r  
 $U, U_i$  = Unobservability of system and fault i  
 $V_l$  = Liquid volume  
 $x_{jk}, x_{jk}^*$  = Number of k type sensors measuring j post and prior to upgrade  
 $x_{i,j}$  = Mole fraction of component i in liquid stream j  
 $x_s$  = Unutilized resource  
 $y_{i,j}$  = Mole fraction of component i in vapour stream j  
 $\alpha$  = weighting factor  
 $r,s,p$  = Reactor, separator, stripper respectively

## References

- Bagajewicz, M.J., Sanchez, M.C. (2000). Reallocation and upgrade of instrumentation in process plants. *Comp. Chem. Engg.*, **24**, 1945.  
 Bhushan, M., Rengaswamy, R. (2002a). Comprehensive design of sensor network based on various diagnosability and reliability criteria: I Framework. *I & EC Res.*, **41**, 1826.  
 Bhushan, M., Rengaswamy, R. (2002b). Comprehensive design of sensor network based on various diagnosability and reliability criteria: II Applications. *I & EC Res.*, **41**, 1840.  
 Downs, J.J., Vogel, E.F. (1993). A Plant-wide industrial process control problem. *Comp. Chem. Engg.*, **17**, 245.  
 Lambert, H.E. (1977). Fault trees for locating sensors in process systems. *Chem. Engg. Prog.* Aug, 81.  
 Ricker, N.L., Lee J.E. (1995). Nonlinear modeling and state estimation for the Tennessee Eastman Challenge Process. *Comp. Chem. Engg.*, **19**, 983.