A COMPARISON OF THREE DIFFERENT MODELING APPROACHES FOR SOLVING MULTI-PRODUCT, MULTI-PURPOSE PLANT SCHEDULING

Per Jernström and Tapio Westerlund
Process Design Laboratory
Department of Chemical Engineering, Åbo Akademi University
Biskopsgatan 8, FIN-20500 Åbo, Finland

Abstract

In this paper we discuss the scheduling of multi-product, multi-purpose plants with blocking production and move-out times. We also compare three different scheduling models and evaluate them on basis of efficiency as well as flexibility. The models examined are; a continuous time disjunctive model, a state-task network (STN) based on a discrete time formulation and finally a continuous time resource-task network (RTN) model using time slots.

The examples presented in this paper are of a blocking type and with move-out times. A product can wait on a production stage, but then the stage is blocked. No other product can be processed or even pass. In other words there is no separate storage for intermediates. When the next stage begins, the previous is not immediately released for new tasks. This need for overlapping can be due to several reasons, such as conveyor belt configurations in flexible manufacturing systems (FMS) or simply pumping something from one vessel to another.

The scheduling models examined are all different approaches to a scheduling problem. In all models we concentrate on operational aspects, leaving matters like combined scheduling and process synthesis out. The models are not only evaluated on the basis of computational effectiveness. Instead we have tried to give an objective overview of the advantages and disadvantages of each model. The flexibility of the model is also accounted for. This includes issues such as handling different constraints, like known machine downtimes.

Keywords

Scheduling, State-Task Network, Resource-Task Network, Blocking flowshop scheduling

Introduction

Many new production facilities are designed with no intermediate storage. Another trend is to use equipment that is flexible and can produce different products. The production lanes are usually built for the automated movement of intermediates from one machine to another. The setup can be very efficient requiring minimal human interaction, but with a higher integration level, the scheduling becomes both more important as well as more difficult. Without proper scheduling it is likely that bottlenecks are formed in the production lanes.

The purpose with this paper is to benchmark different scheduling models. All models are used without any special solving methods, such as rolling horizons or decompositions. The setup used in this paper is an imaginary process in five stages producing two different products. The production process is of the No Intermediate Storage (NIS) type. The first stage is a pre-processing stage, common for all products. The second stage consists of two product dedicated machines and a multi-machine capable of producing both products. Third stage is a common one for all products. In the fourth stage there are product-dedicated machines, but no multi-machine. The last stage is a finalizing step for packaging. This is a single machine stage as in stages one and three.

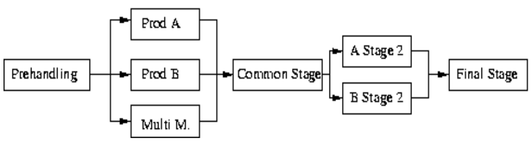

Figure1. Schematic overview of material flows.

Every machine is able to process only one product at a time. When a product is ready it can be moved to the next machine, but it can also wait on the machine. Abadi (1995) shows that this can produce better results than a zero Wait (ZW) strategy. It is not possible to remove a product from the material flow path. Each machine is reserved for a short period of time even after the following step in the flow path has started. This behavior makes the scheduling models a bit more complicated. The system is presented as a State-Task Network below.

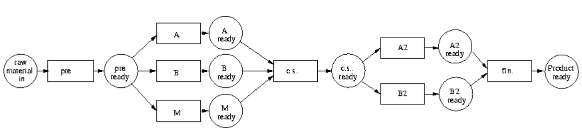

Figure 2. State-Task Network

A state is shown as a circle and a task as a square. The fact that a product can wait will, in some cases, yield a better performance, measured as units/time, compared to a ZW system.

Scheduling Models

All models used are mathematically mixed-integer linear ones that are solved using MILP methods. The thing that differs most in the models is the representation of time. The first model has a uniform discrete time scale, the second one a continuous time scale with time slots and the last one a continuous time without time slots.

Objective Function

The objective function may be formulated in a number of different ways. In the actual study the shortest possible completion time has been used as the objective, for all three models.

Discrete time model

The discrete time model uses a discrete time grid with binary variables for each grid.

$$\sum_{i \in I_j} \left(W_{ij,t} + \sum_{n=0}^{\theta} W_{ij+1,t-n} \right) \leq 1, \forall j,t \qquad (1)$$

$$\sum_{i \in I_j} \left(W_{ij+1,t} + \sum_{m=0}^{\tau} W_{ij,t-m} \right) \leq 1, \forall j,t \qquad (2)$$

The first sum sets the duration, τ, of each task. The second one defines the move-out time, θ. Figure 3 shows a graphical presentation of the variables θ and τ. Furthermore there is a storage term, S, for each equipment.

$$S_t = S_{t-1} + W_{j,t} - W_{j+1,t} \qquad (3)$$

The storage terms are all binary variables, as only one product can be processed at a time.

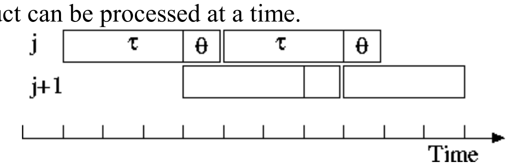

Figure 3. Gantt chart of two stages

The formulations are built based on material balances. The performance of this model is highly dependent on the number of grid points to be used. In the actual case, limit for practical usability comes already at two products in our setup. The bad performance is due to the large amount of binary variables in the formulation. This means that the model is very much dependent on the ratio between the total duration and discretization time. If this ratio becomes high, then the model is usually very inefficient. Using methods such as rolling horizons could make this formulation more efficient. Even though this model shows a slow performance in this particular example, it does not mean that it is useless. Machine downtimes are very easy to implement using this model. The disjunctive model below especially suffers a lot from introducing additional constraints. Another feature is that the optimization results can be used without post processing. One problem with the model is that the total time required for the production set has to be known in advance. This may be a problem, as too few time units make the problem infeasible and too many make it slow to calculate. The formulation becomes larger (more variables) with increased time scope, so keeping the headroom as small as possible is desirable.

Time slot model

This is a further development of the STN formulation used for the discrete time model. In this model there is a continuous time scale, but a predefined number of time slots. Every beginning and end of any task must coincide with these time slots.

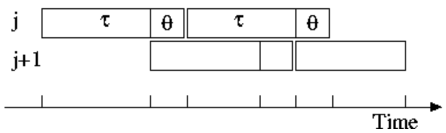

Figure 4. Time representation in time-slot model

The system is based on a RTN model as described by Schilling (1997). This is a further development of the STN model and like the STN, the RTN is also based on material balances. The model is significantly more complex than the discrete time model. Due to space limitations the model will not be presented here.

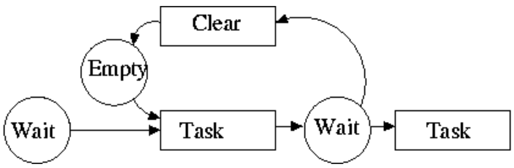

Figure 5. RTN-model of one task

This model performs better than the discrete time model, but the performance is, to a great extent, dependant upon the number of time slots. The time slots are not completely trouble free, as the amount has to be specified in advance. A problem modeled with too few time slots will not yield an optimal solution. Schilling (1997) points out that even if some time slots have a zero duration, when solved, it is no proof of an adequate amount of time slots. There is also a need to post-process the results in order to obtain correct times for each event.

Disjunctive Model

The disjunctive model is not based on material balances like the two other models. Instead it is a strictly time based model. According to Jain and Meeran (1999) the idea of using disjunctive representations in scheduling was proposed in the 1960's.

An event is either before another one or after it. The positive result of this is that the time is continuous without time slots. The downside is that there is no elegant handling of different material flow routes as with in the other two models. Different material flow routes can be handled, but the results from the optimization must be post processed in order to sort out the relevant variables from relaxed irrelevant ones. The option that made this model the fastest performer seems to be the strictly time based formulation without any time slots or time grids. Neither does the time-scope need to be known in advance.

The model is based on a before-after thinking on all machines. Assuming two products, *a* and *b*, either *a* is done before *b*, or vice versa.

$$T_{bj} \geq T_{aj+1} + \theta_{aj} \vee T_{aj} \geq T_{bj+1} + \theta_{bj} \qquad (4)$$

The disjunctive formulation above can be re-written in linear form using a Big-M reformulation.

$$\begin{aligned} &T_{bj} + M \cdot (1 - y_1) \geq T_{aj+1} + \theta_{aj} \\ &T_{aj} + M \cdot (1 - y_2) \geq T_{bj+1} + \theta_{bj} \\ &y_1 + y_2 = 1 \end{aligned} \qquad (5)$$

The binary variables, *y*, state that only one of the constraints can be true. The other one is relaxed by a big integer value, *M*. The binary variables are not only used for setting the sequence. They are also used to set the path for the material flow. For every equipment there will be $n * n - 1$ product pairs, where *n* is the number of products. There must also be constraints for the sequence within each product.

$$T_{j+1} + M \cdot (1 - x) \geq T_j + \tau_j \qquad (6)$$

The binary variable, *x*, equals 1 if both *j* and *j+1* are on the selected material flow path. If there is only one possible way to do a production step, the Big M relaxation is left out.

Conclusions

None of the scheduling models examined in this paper can be regarded as a high performance one. Still, the models were found to have very different performance. Using different methods such as rolling horizons or decompositions may overcome the limitations in the computational efficiency. Additionally, different objective functions may contribute to significant variations performance-wise. The chosen objective function, minimization of the total make span, is an objective function that is frequently used in the literature. Another objective function, like minimization of the total tardiness of products delivered, may performance-wise be totally different. Compared to models not accounting for move-out times, the models presented here are much heavier to calculate. This is especially true for the models based on material balances. The main reason for this is that the material balance is not capable of reserving two machines at the same time. All transitions are expected to happen instantly. On the other hand global optimal schedules are, in principle, possible to obtain with the considered models.

Table 1.Features of the different formulations

Model	STN	RTN	Disjunctive
Formulation	Discrete Time	Time slots	Continuous Time
Formulation base	Material balance	Material balance	Time base
Post processing of variables	No	Yes	Yes
Expands with production time	Yes	No	No
Process synthesis	Yes	Yes	No

The best choice among these three formulations is the disjunctive model, assuming process synthesis is not needed. The speed of this model is superior to the other two in the application presented.

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Nomenclature

$W_{ij,t}$ Discrete variable noting the start of task *i* on equipment *j* at time *t*, 1 if task starts, otherwise 0
S_t Discrete variable for equipment allocation at time *t*. 1 if allocated, 0 if not.
τ Time it takes to complete one production step
θ Time it takes to clear a production unit
T_{ij} Continuous variable noting the start of task *i* on equipment *j*.
M Big M, large integer for relaxation
y_x binary variable, 1 if statement true, 0 if false.
i Index referring to a task
j Index referring to equipment
t Index referring to time slot
a,b index referring to different products

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