ECONOMIC LOT SCHEDULING UNDER PERFORMANCE DECAY

Alessandro Alle and Jose M. Pinto*
University of Sao Paulo
Sao Paulo, SP 05508-900, Brazil

Lazaros G. Papageorgiou
University College London
London WC1E 7JE, UK

The aim of this work is to propose a mathematical programming model for the Economic Lot Scheduling Problem (ELSP) with performance decay. The main difficulties related to this model are the nonlinearities and/or nonconvexities associated with the dynamic behavior of the system with time. Firstly, the problem is formulated as a Mixed Integer Non Linear Program (MINLP), which is found to be nonconvex. The model is then transformed into a Mixed Integer Linear Programming (MILP) model through the discretization of cycle time. Therefore, the globally optimal schedule can be obtained. A case study demonstrates the applicability of the MILP model and its potential benefits in comparison with a hierarchical approach.

Keywords

Economic Lot Scheduling Problem, Mixed-Integer Programming, Performance Decay.

Introduction

The Economic Lot Scheduling Problem (ELSP) is a well-known problem in the Operations Research Literature (Drexl and Kimms, 1997). It can be described in the following way: given a set of products that have continuous fixed demand over an infinite planning horizon, schedule them in a cyclic way in the plant such that the overall production cost, composed of setup and holding costs, is minimum.

Sahinidis and Grossmann (1990) proposed an MINLP model for multiple lines with sequence dependent setups. Pinto and Grossmann (1994) studied the case of multistage plants with intermediate inventory. Recently, Oh and Karimi (2001) proposed a mathematical programming model as well as a heuristic solution procedure for solving the ELSP with fixed planning horizon for a single machine with sequence dependent setups.

To our knowledge, there is no work in the literature that considers the ELSP subject to performance decay. Jain and Grossmann (1998) developed a mathematical programming model for scheduling the cyclic operation of multiproduct parallel units with performance decay. However, this model did not include inventory costs. This same gap exists in the MINLP model proposed by Alle et al. (2002), which is aimed at scheduling production and cleaning operations for multiproduct serial plants. These works cannot be strictly classified as ELSP studies because they lack one of its main features, which is the consideration of holding costs.

The aim of this work is to propose a mathematical programming model for the ELSP with performance decay. The main difficulties related to the proposed model are the nonlinearities and/or nonconvexities associated with the dynamic behavior of the system along cycle time. The model is firstly posed as a nonconvex MINLP model which is further discretized, thus giving rise to an MILP model, which can provide an approximation of the global optimum without the risk of being trapped into local optimal solutions.

* To whom all correspondence should be addressed

Problem Statement

The plant is composed of one single stage manufacturing *NP* products. For every product, *i*, a fixed demand rate, d_i, should be satisfied. The plant allows only one run of each product per cycle.

Process yield, h_i, is assumed to decay linearly with time as follows:

$$h_i(t) = a_i - b_i t \qquad \forall i \tag{1}$$

As performance decreases, the unit requires cleaning to restore productivity to its initial state. The decision of stopping for maintenance interferes with the production schedule. Therefore, it is important to consider both production and cleaning scheduling decisions simultaneously. The overall objective of the planning model is to minimize overall cost (sum of raw material, cleaning and holding costs) over cycle time. Cleaning or setup operations take place when the unit is setup for a different product campaign. Sequence dependent setup times and costs are incurred whenever the line changes from one product to another.

Simultaneous cleaning and production scheduling involves trade-offs between raw material consumption, cleaning and inventory costs. A policy of frequent stops may be adopted to minimize raw material consumption because the unit operates at larger yields. An added advantage is that shorter runs mean lower inventory costs. On the other hand, if the objective is to minimize cleaning costs, longer cycles are necessary which implies larger inventory costs. In addition, raw material consumption increases due to operation at lower yields. Therefore, it is evident that cleaning decisions should be considered in the ELSP with performance decay. Overall, this problem can be stated as follows:

> *Determine* product sequence (Z_{ij}), start times (TS_i), processing times (TP_i), cycle time (Tc) and product amounts (W_i) for *given* demand rates (d_i), decay functions ($h_i(t)$), raw material (Cf_i), setup (Ctr_{ij}), inventory ($Cinvf_i$,) costs, setup times (t_{ij}), and feeding rates (G_i) *so as to* minimize overall cost (*OC*) over *Tc*.

Mathematical Formulation

Defining G_i as the feed rate of product *i*, the amount of product *i* produced, W_i, is given as follows:

$$W_i = \int_0^{TP_i} h_i G_i dt \Rightarrow a_i G_i TP_i - \frac{1}{2} b_i G_i TP_i^2 \qquad \forall i \tag{2}$$

where TP_i is the processing time of product *i* during the cycle. The continuous demand, d_i, over the cycle time, *Tc*, must be satisfied:

$$W_i = d_i Tc \qquad \forall i \tag{3}$$

The inventory behavior of product *i*, $Invf_i$, during the cycle is shown in Fig. 1.

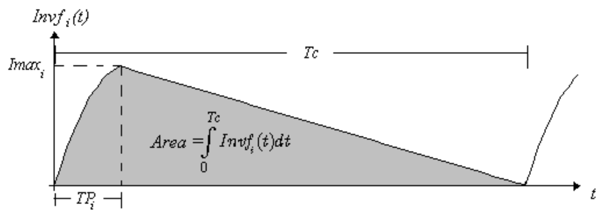

Figure 1. Inventory under performance decay.

The cost of holding product *i* during the cycle, CH_i is proportional to the area below the curve in Fig. 1:

$$CH_i = Cinvf_i \int_0^{Tc} Invf_i(t)dt \qquad \forall i \tag{4}$$

where

$$Invf_i(t) = W_i(t) - d_i t \qquad \forall i \qquad \text{if } 0 \le t \le TP_i \tag{5}$$

$$Invf_i(t) = W_i(TP_i) - d_i t \qquad \forall i \quad \text{if } TP_i < t \le Tc \tag{6}$$

The overall inventory cost, *CH*, is given as follows:

$$CH = \sum_i Cinvf_i \left(\frac{1}{2}(a_i G_i - d_i) TP_i^2 - \frac{1}{6} b_i G_i TP_i^3 + \frac{1}{2} d_i (Tc - TP_i)^2 \right) \tag{7}$$

The cost of raw material, *CF*, is given by:

$$CF = \sum_i Cf_i G_i TP_i \tag{8}$$

Variable Z_{ij} defines product ordering as follows (e.g. Alle and Pinto, 2002): $Z_{ij} = 1$ if product *i* precedes product *j*; *0*, otherwise. Thus, the overall setup cost, *CT*, is given by:

$$CT = \sum_i \sum_j Ctr_{ij} Z_{ij} \tag{9}$$

Only one product succeeds and precedes the other:

$$\sum_i Z_{ij} = 1 \quad \forall j \text{ and } \sum_j Z_{ij} = 1 \quad \forall i \tag{10}$$

As shown in Fig. 2, there are two cases for the inventory level profile $Invf_i(t)$. In case A, the maximum inventory level occurs at the end of production of *i*, TP_i. After that, inventory is depleted at continuous demand rate, d_i. In case B, the maximum inventory level $Imax_i$ occurs at (tn_i, IM_i), where $tn_i = (a_i G_i - d_i)/G_i b_i$ and $IM_i = (a_i G_i - d_i) tn_i - 0.5 b_i G_i tn_i^2$, which is the null-derivative point of $Invf_i(t)$.

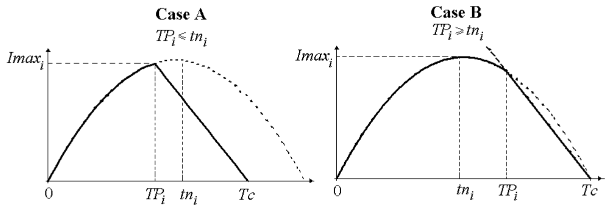

Figure 2. Inventory profiles.

In order to determine the maximum inventory level, $Imax_i$, a new binary variable, Y_i, is introduced to indicate whether case A occurs together with the following constraints:

$$-TP_i^{up} Y_i + e \le TP_i - tn_i \le TP_i^{up}(1-Y_i) \qquad \forall i \qquad (11)$$

$$Imax_i \ge IM_i(1-Y_i) \qquad \forall i \qquad (12)$$

$$Imax_i \ge d_i(Tc - TP_i) - Imax_i^{up}(1-Y_i) \qquad \forall i \qquad (13)$$

Storage capacity, $Imax_i^{up}$, must not be exceeded:

$$Imax_i \le Imax_i^{up} \qquad \forall i \qquad (14)$$

The cycle time must be greater than the summation of processing and transition times:

$$Tc \ge \sum_i TP_i + \sum_i \sum_j t_{ij} Z_{ij} \qquad (15)$$

As the schedule is cyclic, product 1 is arbitrarily chosen as the first one to enter the production line:

$$TS_1 = \sum_j t_{j1} Z_{j1} \qquad (16)$$

A product only starts to be processed after previous processing and setup. Thus,

$$TS_j - (TS_i + TP_i + t_{ij}) \ge -(1 - Z_{ij}) Tc^{up} \quad \forall i \ne j, j > 1$$
$$TS_j - (TS_i + TP_i + t_{ij}) \le (1 - Z_{ij}) Tc^{up} \quad \forall i \ne j, j > 1 \qquad (17)$$

It must be pointed that (17) prevents solutions from presenting subcycles (Alle and Pinto, 2002). The model (M1) of ELSP with performance decay can be stated as:

$$\min OC = \frac{CT + CH + CF}{Tc} \qquad (18)$$

subject to

Cost definition constraints (7), (8) and (9)
Sequencing constraints (10)
Mass balance constraints (2) and (3)
Maximum inventory constraints (11)-(13)
Cycle timing constraints (15)
Timing constraints (16) and (17).

Model M1 has a nonconvex objective function and nonconvex constraints, (2) and (7). In the next section, model M1 is reformulated as an MILP by assuming discrete values for the cycle time.

Model Reformulation

Substituting (3) into (2) and solving the resulting quadratic equation, it follows that

$$TP_i = \frac{a_i}{b_i}\left(1 - \sqrt{1 - 2\frac{b_i d_i\, Tc}{a_i^2 G_i}}\right) \qquad \forall i \qquad (19)$$

Substituting (19), (7) and (8) into (18) yields:

$$OC = \frac{CT + B2}{Tc} - \frac{\sum_i B3_i \sqrt{1 - C_i Tc}}{Tc} + \frac{2}{3}\sum_i D_i \sqrt{1 - C_i Tc} - D + ETc \qquad (20)$$

where $B2 = \sum_i \left(\frac{a_i^3 G_i Cinvf_i}{3b_i^2} + Cf_i G_i\right)$, $B3_i = \frac{a_i^3 G_i Cinvf_i}{3b_i^2} + Cf_i G_i$, $D = \sum_i D_i$, $C_i = 2\frac{b_i\, d_i}{a_i^2 G_i}$, $D_i = \frac{a_i\, d_i Cinvf_i}{b_i}$ and $E = \sum_i \frac{b_i d_i Cinvf_i}{2}$.

Objective function (20) is nonconvex and has two variables: *CT* and *Tc*, which can be discretized as follows:

$$Tc = \sum_l Tc_l\, X_l \text{ and } \sum_l X_l = 1 \qquad (21)$$

$$TP_i = \sum_l TP_{il} X_l \qquad \forall i \qquad (22)$$

where Tc_l are discrete values and TP_{il} are given from direct substitution of Tc_l into (19). Defining variable $\overline{CTX}_l$ as the product between *CT* and binary variable X_l, which can be determined by

$$0 \le \overline{CTX}_l \le CT^{up} X_l \qquad \forall l \qquad (23)$$

$$CT = \sum_l \overline{CTX}_l \qquad (24)$$

the MILP model (M2) can finally be stated as:

$$\min OC = \sum_l \frac{\overline{CTX}_l}{Tc_l} + \sum_l \frac{B2}{Tc_l} X_l - \sum_l \frac{\sum_i B3_i \sqrt{1 - C_i Tc_l}}{Tc_l} X_l$$
$$+ \frac{2}{3}\sum_l \sum_i D_i \sqrt{1 - C_i Tc_l}\, X_l - D + E \sum_l Tc_l\, X_l \qquad (25)$$

subject to

Cost definition constraints (9), (23) and (24).
Sequencing constraints (10)
Maximum inventory constraints (11)-(13)
Cycle timing constraints (15), (21) and (22)
Timing constraints (16) and (17)

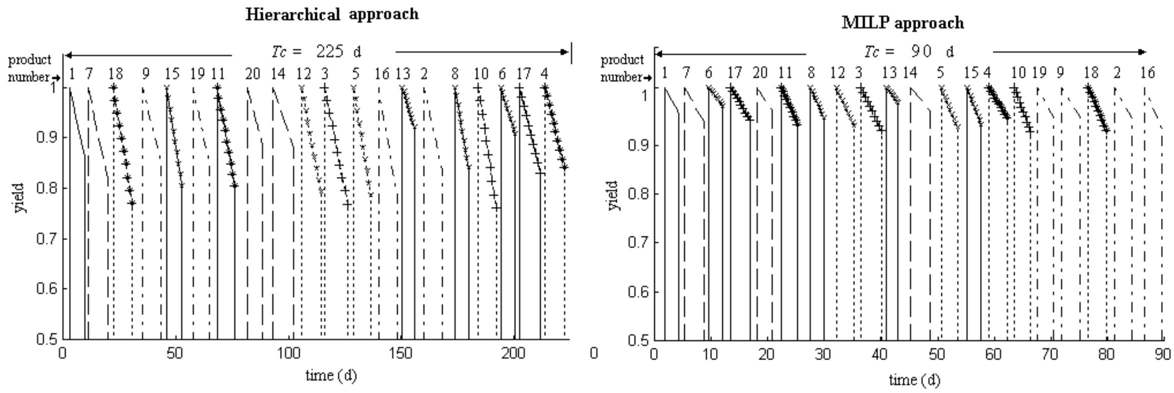

Figure 3. Schedules for the hierarchical and MILP approaches.

Example

In order to show the effectiveness of the proposed approach, this illustrative example shows a comparison between the proposed MILP and a hierarchical procedure for scheduling a unit that processes 20 products. The hierarchical approach comprises two steps. At the first step, the product sequence is obtained by minimizing the overall setup cost. The second step determines cycle and campaign time of products for this sequence.

Problem data was randomly generated as shown in Table 1, where U[*a*,*b*] represents a uniformly distributed random variable in the range [*a*,*b*]. The MILP model was implemented in GAMS and solved in 9.4 s in a PC Pentium III 500 MHz, 512 Mb RAM with the MILP solver CPLEX 7.0. Fig. 3 and table 2 compare the schedules obtained from both approaches.

Table 1. Parameters for the case study.

d_i = U[15,25] ton/d	G_i = U[410,680] ton/d
$Cinv_i$ = 0.5.U[1,3] ton/d	Cf_i = 100.U[1,3] ton/d
$a_i = 1$	b_i = 0.01.U[1,3] 1/d
$Ctr_{ij} = Ctr_i$.U[1,5] \$ where $Ctr_i = 10^4$.U[1,3] \$	
$t_{ij} = t_i$.U[1,5] \$ where t_i = 0.4.U[1,3] \$	
$Imax_i = IM_i$.U[0.5,1.5] ton	
Tc_l = 75 + (360-75).(l-1)/19 where l = 1…20	

Table 2. Costs for Hierarchical and MILP approaches.

Costs	Hierarchical	MILP	Benefit
CF/Tc (10^3 \$/h)	84.4	79.0	6.8%
CT/Tc (10^3 \$/h)	2.1	8.2	-74.2%
CH/Tc (10^3 \$/h)	45.1	18.1	150%
OC (10^3 \$/h)	131.7	105.3	20.0%

Fig. 3 shows that approaches yield different production sequences. The hierarchical approach has the minimum setup cost whereas the MILP schedule has shorter cycle time (90 d vs 225 d). As a consequence, the MILP approach gives higher process yields as well as lower inventory levels. In fact, Table 2 shows that the MILP has 74.2% larger setup cost. However, it presents 6.8% less raw material and 150% less inventory costs, which yields 20.0% lower overall cost per unit of cycle time.

Conclusions

In this work, the problem of the ELSP with performance decay has been studied. The overall problem was formulated as a nonconvex MINLP model that was transformed into an MILP model. The advantages of the proposed simultaneous approach were shown by comparison with a hierarchical approach.

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