CONTROL AND OPTIMISATION OF SUPPLY NETWORKS

B. Erik Ydstie*, D. Coffey† and Mark Read‡
Carnegie Mellon
Department of Chemical Engineering
Pittsburgh PA 15213

Abstract: In this paper we apply concepts from dynamics and control to study structural issues that arise in the management of highly distributed supply chains. The supply chain system is represented by a graph using variables that capture seven important aspects of the supply chain management problem: the topology, transportation (more generally rate of activity), shipping/receiving and market conditions, assembly/disassembly, storage of assets, signal processing (forecasting) and performance evaluation. We use the manufacture of silicon to motivate a model for supply chain analysis which can be represented as a large scale system of differential algebraic equations. We demonstrate existence, uniqueness and stability of solutions when the routing policies are monotone. These conditions are necessary and sufficient. We also develop optimality properties that arise directly from the system structure using a decentralized control policy. We finally connect the modeling approach with a previous study on centralized control of multi-echelon, supply chain system with batch processing units. In the centralized approach we develop optimisation based schedules for routing and flow control.

Keywords: Distributed control, process networks, supply chain management, silicon, stability, self-optimisation, optimal control.

* To whom all correspondence should be addressed: Carnegie Mellon, Department of Chemical Engineering, 5000 Forbes Avenue, Pittsburgh PA 15213, e-mail ydstie@andrew.cmu.edu.
† The DOW Chemical Co., 2301 N. Brazoport Blvd, B-1226, Freeport TX 77541.
‡ ELKEM Metals, PO Box 266, Pittsburgh PA 15230-0266.

Introduction

The use of Information Technology (IT) tools to improve the resource allocation, flow of materials and diffusion of knowledge within factories, companies and entire enterprises has been on the increase during the last decade. Enterprise Resource Planning (ERP) systems, supplied by companies like SAP, i2, Oracle, J.D. Edwards and others integrate business processes, streamline production systems and provide company wide access to information related to critical work processes [17]. Such systems can be used to track inventory levels, identify bottle necks, smooth flows and evaluate performance. Impressive gains have been reported in a great variety of industries, including the computer industry (hardware and software), discrete parts manufacture and commodity chemicals [13, 6, 15, 18, 2].

The application of ERP tools has made it apparent that it does not suffice to focus on the internal processes alone. Upsets are often created by factors beyond the control of a single company. This led to the development of Advanced Planning Systems (APS) that link the database capabilities of the ERP system to market forecasts and process models. Such tools enable a company to evaluate scenarios and respond to changes in the market place by applying passive feedback processes. However, it is clear that improved agility and better performance can be achieved by application of active feedback and advanced tools from process systems theory like, distributed control and optimisation on-line [12], [19].

In our context a supply chain is thought of as a "network of organizations that are involved, through upstream and downstream linkages, in different processes and products [4]." Stadtler and Kilger [17] define Supply Chain Management as "the task of integrating organisational units along a supply chain and coordinating materials, information and financial flows in order to fulfill (ultimate) customer demands with the aim of improving competitiveness of a supply chain as a whole." The SCM perspective therefore includes the idea of two or more legally separate partners working together towards a common goal within a business sector.

The current paper was motivated by the organizers of the conference. We were asked to provide a "process controls perspective" on Supply Chain Management (SCM). A process controls perspective normally includes three elements: a dynamic model represented by conservation laws, a performance objective and finally, a feedback mechanism that achieves the stated objective using variables that may represent filtered or predicted measures of performance and other variables of interest.

The model we put forward for SCM is developed on the basis that the assets are conserved in the sense that they do not change properties during transportation or storage. In a production process the characteristica of the asset may change. However, there remains an invariance which may be thought of as the total " mass" or "energy". Moreover, the value of the asset should increase as it is transported and transformed. These properties, we show, lead to convexity and a variational principle.

Our research groups have developed two approaches for generating approximate solutions to SCM problems. These are reviewed very briefly in the paper. One approach is based on distributed decision making and solves optimisation problems using decentralized feedback mechanisms that set activity rates using value added as the decision variable. If there exist two parallel activities then the activity with lower cost is chosen. The approach can be guaranteed to be stable provided a monotonicity condition is satisfied [5]. The other method is based on centralized decision making and uses finite horizon predictive control to determine feedback controls using a rolling horizon objective [10] [11].

Our long term goal is to develop optimal control policies that distribute computational load and are decentralized so that one can add and take away processing or storage facilities and change connectivity without sacrificing stability and optimality. Such properties ensure scaleability and allow the design of flexible supply chain processes without excessive reconfiguration costs.

The paper is organized as follows. In the next section we describe an example to highlight some issues in supply chain management. In the following section we develop an abstract model motivated by our recent studies of thermodynamic networks. This model is suitable for studying stability and performance of decentralized control strategies. We then develop the main theorems of the paper. These include results on optimality, uniqueness and stability of decentralized control of supply networks. These results show that particular classes of feedback control laws for supply networks automatically optimise system performance. We then review a centralized approach which developes an explicit solution to the optimisation problem. We finish with some conclusions.

Motivating Example

Silicon is prized for its semi-conductor properties, its abilities to form polymer chains, and its excellent alloying properties with materials like aluminum and steel. A by-product of the process, called microsilica, is used as a binder to form high strength cement. The main market segments and annual consumption rates of silicon are respectively (year 2001)

Electronics and Photovoltaics: 42,000 MT
Silicon based polymers: 401,000 MT
Aluminum alloys: 573,000 MT

Silicon manufacture is an integral part of major supply chains for consumer electronics, the construction industry, as a major consumer of steel and silicone fillers and, the auto, beverage can and aircraft industries, as major consumers of aluminum. Almost all steel being produced includes some silicon. With the computer revolution, silicon became one of the most important and well studied industrial materials. The bridge linking Sweden and Denmark, many high rise buildings and off-shore oil rigs use microsilica enhanced cement.

Silicon for micro-electronics is currently produced with improbably low impurity levels of 1 ppt (parts per trillion). Impurity levels for photovoltaics, silicone polymers and fumed silica fall somewhere in the range 1-100 ppm [14]. The supply chain leading to the manufacture of materials with such a high degree of purity and varied applications is necessarily quite complex and involves a variety of technologies and a large number of customer-supplier relationships in diverse geographical locations.

Figure 1 shows the basic elements of a typical manufacturing facility for metallurgical grade silicon (Si_{MG}). The Si_{MG} typically has a purity about 99.5%, depending on application area. The main ones are chemicals, electronic and primary and secondary aluminum. The facility takes in the raw materials at receiving points by rail or ship. The raw materials are held in storage bins, mixed and processed in electro-chemical reactors according to the scheme [14]

$$SiO_2 + (1+x)C \rightleftharpoons xSi + (1-x)SiO + (1+x)CO$$

The parameter x is an operational parameter which determines the trade off between silicon and microsilica production. The multi-product nature of the silicon process provides trade-off among the use of raw materials, energy and the allocation of product portfolio. A typical manufacturing site has several silicon producing reactors that need to be scheduled and allocated to specific market sectors.

Metallurgical grade silicon is generally produced where electricity costs are low and suitable quartz and carbon materials can be shipped in at a reasonable cost. Quartz is abundantly available in the earth's crust almost everywhere and easy to transport. However, electricity prices, labor costs and carbon quality vary considerably. Charcoal is an excellent source of carbon, but, it is expensive to transport since it has poor structural integrity. These and other factors show that the topology of the supply network is very important. For example, Dow Corning's recently acquired silicon capacity in Brazil.

The supply chain for the manufacture of polysilicon for micro-electronics through the Bremanger Silgrain plant in Norway represents about 80% of all silicon used for micro-electronics in Japan (Figure 2). There are terminals for receiving quartz for shipment in Galicia, Spain and terminals for

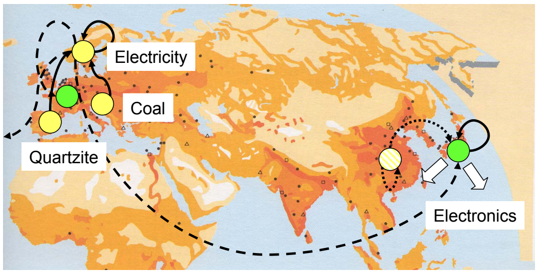

Figure 2: The supply chain for micro-electronics in Japan is based on raw materials from Europe. It is clear from the picture that China can represent a challenge due to cheaper labor and closeness to the market.

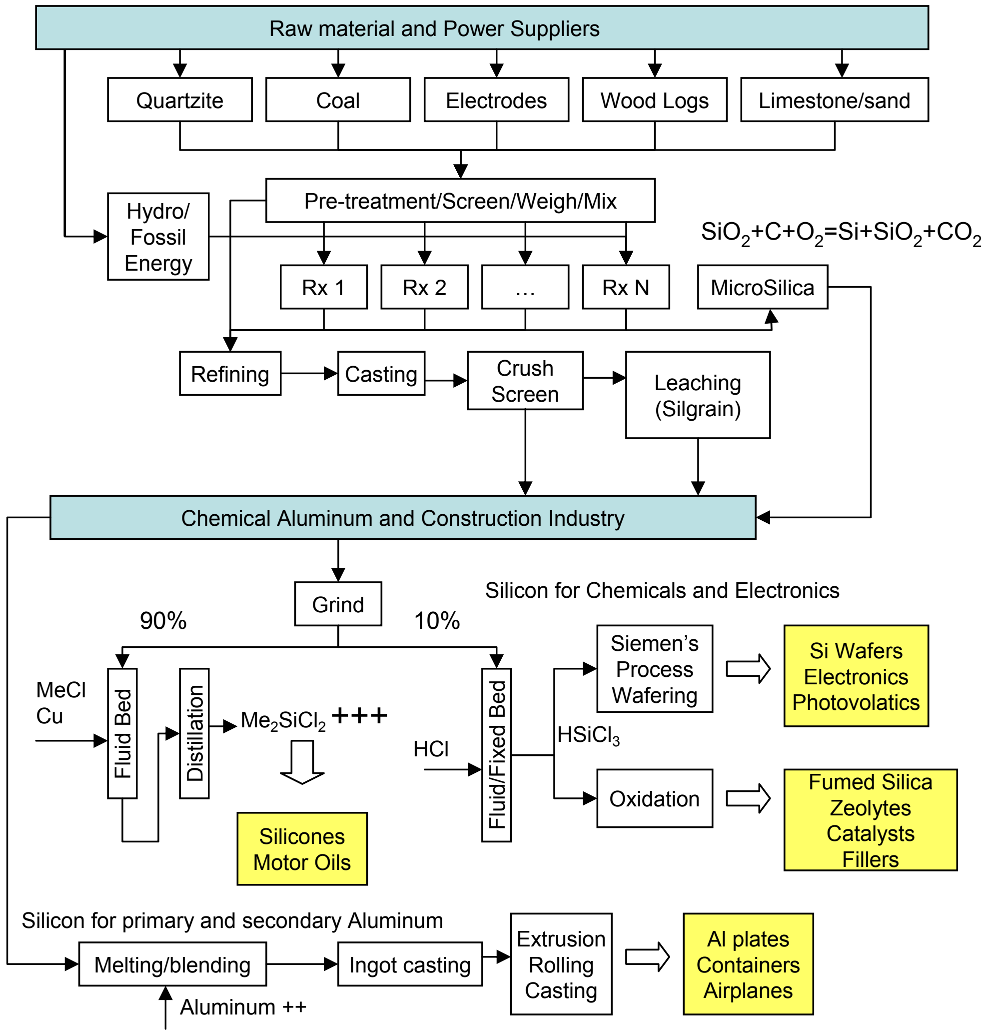

Figure 1: The major components of the supply chain for silicones, PolySilicon for micro-electronics and silicon based aluminum alloys

coal in Germany. There are production facilities for silicon in Thamshavn, Norway. This product is transported to the Silgrain factory, producing 30kt, 99.5% pure Si per year, in Bremanger, Norway. The Silgrain is shipped to terminal points in Japan where customers like Mitsubishi and Kawasaki purify the Silicon via Trichlorosilane (TCS) distillation. The TCS is deposited as Si with less than 1ppb impurities in the Siemens CVD process. The resulting product, PolySilicon is sent to crystallization, wafering and production of consumer electronics by companies like Kyocera, Sharp and Sony. During manufacture about 15% reject material is generated. This material produces the source material for the photovoltaics industry.

The main tenets of the silicon process technology have changed little over the last century [14]. However, better reactor and electrode technology has led to larger, more energy efficient designs. The silicon industry in the West has benefitted greatly from the application of statistical process control data analysis and on-line monitoring of inventory using ERP systems.

Large production volumes imply that a policy of using safety/holding stock to decouple the supply chain and absorb unanticipated shocks in the market has limited value. Stocks of commodities generally account for a very small percentage of total annual production rate. Over or under production and swings in the market therefore give significant changes in the spot-price for silicon. Long term swings are compensated for either by shutdowns or capacity expansion. In some areas short term stock build up can be used effectively to buffer seasonal variations in electricity prices.

The US import price for Silicon has been falling steadily in recent years (Figure 3). This is part of a general trend in the commodity businesses [16]. A commensurate reduction in energy, rawmaterial and per unit labor cost has not been observed. In fact labor and rawmaterial prices have increased. The business strategy clearly depends strongly on being able to forecast such future trends and capitalize on cycles in demand and price. For example Elkem's annual reports showed losses in their silicon and ferrosilicon business sectors in the early 90's. In the late 90's the same divisions reported profits on the same salevolumes and lower prices for the products. These positive results have been due to a number of factors, many of which fall under

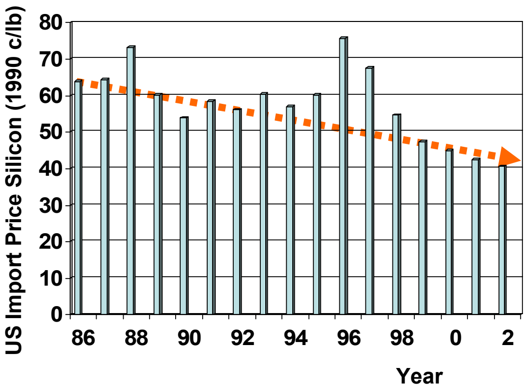

Figure 3: The import price for silicon has been falling steadily.

the heading of supply chain management. Alliances have been formed to forge good customer-supplier relationships. Vertical integration and takeovers have led to re-structuring and a realignment of the silicon industry. These kinds of factors are sketched out in general terms in Figure 4.

There exist many different accounting measures that can be used to evaluate the performance of a silicon business unit. These include profit margin, operating profit, Return on Assets (and related measures like ROI/ROCE), Price Earnings ratio and "shareholder value" [7]. The financier, Warren Buffet, argues that the performance should be evaluated using an underlying and potentially subjective measure he calls the "intrinsic value" of the company. The intrinsic value is defined to be the "discounted value of cash that can be taken out of a business during its remaining life [3]."

In an ideal setting all resource allocations should be made so that the intrinsic value is maximized. This will enhance the competitive position and allow the company to turn a profit and grow over time. However, in order to do this it is necessary to evaluate the sensitivity of the intrinsic value of the business unit with respect to each and every decision variable. This is difficult and maybe impossible to do in all but a very imprecise manner.

The very brief discussion above illustrates that there are seven distinct factors that need to be taken into to account when we develop a conceptual model of a supply chain in the commodity chemicals industry. These include

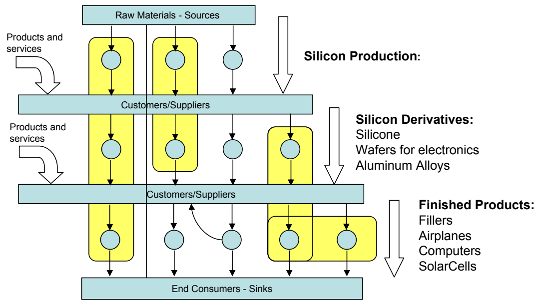

Figure 4: Alliances and purchases in the recent years have led to an integrated supply chain for silicon based products. Dow Corning for example has a fully integrated supply chain for production of silicones. If ALCOA's takeover bid for Elkem were to succeed then ALCOA will have a fully integrated line for aluminum-silicon alloys and products therof. Elkem, by the recent purchase of SAPA, already has a fully integrated line for production of aluminum profiles and a wide range of other aluminum products.

1. Topology.
2. Transportation costs.
3. Shipping and receiving.
4. Production via assembly and disassembly of parts and chemicals.
5. Storage and holding.
6. Forecasting and signal processing.
7. Performance evaluation.

The market conditions set the boundary conditions for the supply chain. In addition one needs to consider intangibles, including legal position, know-how, intellectual property and how well this is protected, customer-supplier relationships and the quality of workforce and management.

The Supply Chain as a Process Network

The objectives of this and the next few sections are twofold. First we want to formalize the objects introduced above using tools from mathematical systems theory. Second, we want to develop feedback controls (business systems) that allow us to direct the flow of assets so that the system constraints are satisfied and the performance is optimised. We will discuss two types of controls. One is based on decentralized decision making. The other is based on centralized decision making.

We take the point of view that a control and optimisation model for SCM can be developed by tracking the flow and storage of assets and assigning a value to stored assets and, costs to each and every activity. This type of analysis, called activity based analysis, develops the specific costs and benefits generated by an activity, product line or business segment, based on the physical processes and resource requirements [7].

An asset, being an item of value to a company at a given location and instant in time, can be characterized by the triplet $\mathcal{A} = \{v, c, z\}$. The function v ([units]) represents the inventory, c ([value/unit]) represents the value of one unit of a specific asset and finally, z represents the specification. The specification identifies an asset by its name, SKU-number, chemical composition or some other identifier which should be unique. All these variables are functions of time and geographical coordinates. In financial terms the number vc may be thought of as the book value of the inventory of a specific asset. We will use a dynamic interpretation of this quantity and assign a value to each and every asset in the system and track the evolution of these values as they move through, and are transformed by, the production system.

The flow of an asset represents an activity. Activities are characterized by a pair $\mathcal{F} = \{f, z\}$. The scalar f gives the activity rate [units/time] and z, as before gives the specification of the asset in question. Let c_0 represent the value of an item before an activity starts and let c_1 represent the value after it is finished. The number

$$w = c_1 - c_0$$

then represents the per unit value of an activity. In order to capture the idea that there is a positive cost associated with every activity, we choose sign so that asset flow is positive in the direction of increasing value and negative in the direction of decreasing added value. The number

$$\pi_{\mathrm{H}} = fw \tag{1}$$

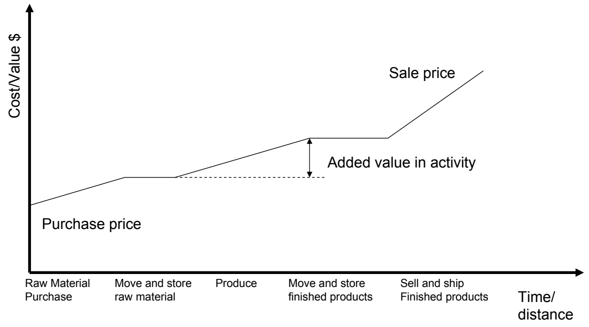

Figure 5: The generation of value of a unit passing through the production process.

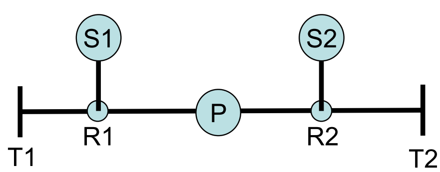

Figure 6: Graph of a simple production facility corresponding to value generation shown in Figure 5.

then represents the rate of value generation.

The added value from performing an activity may be negative. Going back to the Silicon example, we see that the internal value of quartzite, on site in Norway, is equal to the value of quartzite in Galicia plus the activity costs associated with shipping and handling in order to deliver the quartzite to Norway. However, bringing quartzite back to Galicia adds negative value since it is worth no more than locally purchased quartzite. Thus the added value in going from destination a to destination b is the negative of the added value of going from b to a, assuming, of course that the market conditions have not changed in the meantime. Cyclical activity does not generate value! Figure 5 illustrated some of these points. A more detailed assessment is given by Taylor and Brunt [18].

The specifications of an item sold has to match the specifications given by the purchaser in order to effectuate a transaction. Consistency must also hold true for internal suppliers and customers and, between companies in a supply chain.

Assumption 1. *The value of an asset is unique, inventory is a conserved property and all activities are consistent.*

The first part of the assumption means that the value of an item is independent of path used to produce the item. In thermodynamics we say that such functions are state functions. The second part implies that we can describe the system dynamics in terms of conservation laws. The third assumption ensures that packets and streams satisfy the specifications given by internal and external customers.

A graph is characterized by a set of vertices and a set of edges. In light of the discussion above, we find it useful to introduce more structure and represent the topology of the supply chain using a graph with five classes of elements. These represent production, storage, terminals, routing points and activities. The activities are represented by edges while the other four elements are represented by vertices. More formally we have the graph

$$\mathcal{G} = (\mathcal{P}, \mathcal{S}, \mathcal{T}, \mathcal{R}, \mathcal{H})$$

Elements $p_i \in \mathcal{P}, i = 1, ..., n_p$ represent the production sites where we assemble or disassemble chemical constituents or parts into pre-cursors or products. Elements $s_i \in \mathcal{S}, i = 1, ..., n_s$ denote the storage facilities. Elements $t_i \in \mathcal{T}, i = 1, ..., n_t$ denote terminals for receiving and shipping. Elements $r_i \in \mathcal{R}, i = 1, ..., n_r$ represent points where material, energy, money and data can be routed in different directions. Finally, $h_i \in \mathcal{H}, i = 1, ..., n_h$ denote the activities.

Example 1. Consider for the graph of the production facility shown in Figure 6. There is a terminal where materials are received from the supplier. There are storage locations for raw materials and products, an assembly plant and a shipping terminal. All nodes are connected by vertices that represent activities, in this case material flow. More vertices and edges can be added to represent flow of services, orders, information, capital and energy. If there is no flow then there is no activity. There are two routing points in this figure. At routing point 1 decisions are made about sending raw materials to

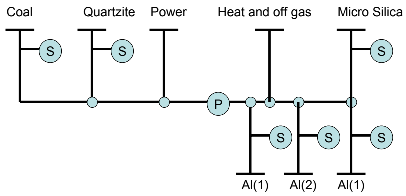

Figure 7: Graph of a silicon production facility with one reactor serving four markets.

storage (Storage 1) or production. At routing point 2 decisions are made about sending finished products to the plant warehouse (Storage 2) or shipping. Note that there is no storage allowed at routing points, production facilities or terminals. □

Example 2. Figure 7 shows a graph of a silicon production facility with one reactor which serves four different market segments, primary aluminum, secondary aluminum and micro silica. In this case the reactor needs to be scheduled so that different products are produced at different times. □

Example 3. Figure 8 shows a production facility with five reactors serving a single silicon market and a microsilica market. In this case there are five different paths (the five reactors) that lead to the same product and the production rates need to be balanced in order to meet demand and maximize value generation. □

The added value along a path consisting of several segments is given by the formula

$$w = \sum_{\text{Segments}} w_i \tag{2}$$

where w_i represents the added value of each subactivity. This number, which may be positive, zero or negative, does not depend on the path taken in accordance with Assumption 1. In fact, for a cyclical activity we have

$$0 = \sum_{\text{Loop}} w_i \tag{3}$$

The rate of value generation for a network can be written

$$H = \sum_{\text{Activities}} \pi_H \tag{4}$$

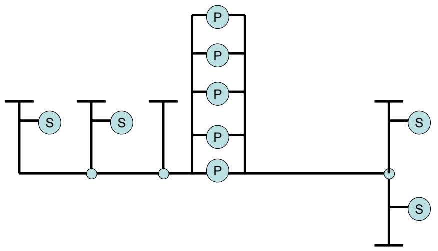

Figure 8: Graph of silicon production facility with five reactors serving two markets.

where the summation has to be carried out over all activities. We now describe the different nodes more detailed.

Delays and discontinuities in activities are represented by hyperbolic, partial differential equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial f}{\partial x} = 0$$

This equation models delays as a pipeline. The function $\rho(x,t)$ now denotes the "density of an activity" at point x and time t while $f(x,t)$ is the local activity rate. The theory behind such pipelines is given in [20]. Using this definition we write the added value along the vertix, representing an activity, so that

$$\pi_H = \int_0^1 f \mu dx \tag{5}$$

where x represents how far the activity has progressed with 0 representing the initiation and 1 representing the completion of an activity. The variable

$$\mu = \frac{\partial w}{\partial x} \tag{6}$$

defines differential added value along an edge. We can then re-write equation (5) so that

$$\pi_H = \int_{w_0}^{w_1} f dw \tag{7}$$

In circuit theory, this quantity is referred to as the "co-content".

The conservation principle of Assumption 1 states that asset flow through nodes (routing points, terminals and production plants and accumulation is storage) is conserved. We write this as

$$0 = \sum_{\text{Connections}} f_i \tag{8}$$

The summation is carried out so that the index ranges over all activities connected to the corresponding node in the network.

For a routing point we get in addition

$$z_i = z_j, \qquad \text{for all } i, j$$

The index ranges of all connections, indicating that routing and transportation should not change specification.

A terminal has no capacity for storage. However, material may be stored at adjacent storage facilities until it is routed for distribution or further processing. Applying the conservation law to the terminal gives

$$0 = f_1 + f_2 \tag{9}$$

where f_1 denotes the flow into the terminal and f_2 the flow out with direction chosen as before. The value of the material being received or shipped is then given by

$$\pi_{\mathrm{T}} = fc \tag{10}$$

This corresponds to a COD/FOB delivery system with π_{T} being positive for sales and negative for purchases. The number

$$T = \sum_{\text{Terminals}} \pi_T \tag{11}$$

gives the difference between rate of revenue from sales and expense for purchased services, energy and materials. We call this number *the net rate of revenue*. Note that the consistency condition has to be satisfied when we connect customers and suppliers to form the supply chain across a terminal. In terms of the variable above we write this so that

$$z_1 = z_2$$

Transformation of assets via assembly or disassembly takes place at production sites. Like terminal and routing points, there is no capacity for storage in the production facility and production takes place instantaneously and without delay. We model production in accordance with the rules used to describe chemical process systems. These include conservation of mass and energy and dissipation via the second law. The conservation principle is now written

$$f_2 = f_1$$

But, we no longer have consistency since the assembled part has a different specification so that

$$z_2 \neq z_1$$

Example 4. For the silicon process the production site is modeled as node with a flow in and a flow out with property vectors

$$z_1 = \begin{pmatrix} Si \\ SiO \\ CO \end{pmatrix}, \qquad z_2 = \begin{pmatrix} SiO_2 \\ C \end{pmatrix}$$

However, this description is not detailed enough. Different products have different specifications according to crush size, shape, purity and alloying materials that may have been added. Specification therefore includes more information than can be expressed by composition alone. □

The conservation principle states that the rate of change in storage is given by the differential equation

$$f = \frac{dv}{dt}, \qquad v(0) = v_0$$

where v_0 the amount stored initially. We also have no change in specification so that

$$z_1 = z_2$$

where index the index depends on flow direction. Negative f denotes flow out of storage whereas positive f denotes flow into storage. This type of storage, referred to as "tanque pulmon", represents a capacitor in an electrical network. The value of assets at a given storage location is given by the formula

$$\pi_S = vc$$

where c is calculated along the path from any terminal by using equation (2).

In the same way as for equation (7) we have

$$\pi_S = \int_{w_1}^{w_2} v\,dw \tag{12}$$

Liabilities are represented as inventory with negative value.

Example 5. Perea et al. [12, 10] developed an optimisation model for a supply chain that (in a minimal sense) captures the dynamics of Unilever's European distribution network for a family of consumer products. The model has the capacity of representing demand amplification as reviewed in [17]. The model was based on the idea that there are two important flows in the supply chain: the flow of information (orders) and the flow of material (products). These flows affect the inventory levels and the order levels associated with each storage location. The simplest model consisted of a single chain with a raw material supplier, production center producing products A and B, plant storage, distribution centers, retailers and customers. The information received at one location consists of orders from the locations it serves. There is no delay in the order flow. There is a delay associated with shipping. The objective is to minimize cost while achieving a certain customer satisfaction (fill rate).

Based on these assumptions we write the material balance

$$\frac{dI_k}{dt} = \begin{cases} y_{k'k} - \sum_{k''} y_{kk''} \\ up_k^s - \sum_{k''} y_{kk''} \end{cases}$$

and the order balance

$$\frac{dO_{kk''}}{dt} = \begin{cases} u_{k'k} - y_{kk''} \\ d_{kk''} - y_{kk''} \end{cases}$$

where k denotes a node, k' its supplier and k'' its customer.

Furthermore,

$$\begin{aligned} I_k &= \text{Inventory at node } k \\ O_{kk''} &= \text{Orders to be processed at node } k \text{ for delivery at node } k'' \\ y_{kk''} &= \text{Material delivery rate from } k \text{ to downstream node } k'' \\ y_{k'k} &= \text{Material delivery rate to } k \text{ from supplying node } k' \\ y_k &= \text{Shipping rate from } k \\ up_k^s &= \text{Material delivery rate to } k \text{ from production site} \\ u_{kk''} &= \text{Ordering rate to } k \text{ from downstream node } k'' \\ d_{k''k} &= \text{Ordering rate to } k \text{ from the consumer } k'' \end{aligned}$$

This system can clearly be represented using the model developed here by attaching a storage location to each routing point. We then make corresponding assignments for flows, storage, terminals and productions sites. □

A dynamic system which satisfies the constraints described above is called a *supply network*. It is quite easy to show that the topology of a supply network can be written as a DAE system

$$\begin{aligned} \frac{d\mathbf{v}}{dt} &= G_0\mathbf{f}, \qquad \mathbf{v}(0) = \mathbf{v}_0 \\ 0 &= G_1\mathbf{f} \\ G_2\mathbf{w} &= 0 \end{aligned} \tag{13}$$

where $\mathbf{f}$, $\mathbf{w}$ and $\mathbf{v}$ are concatenated and ordered vectors of inventories, flows and activity costs. The entities $G_i, i = 0, 1, 2$ are rectangular matrices that express the topology of the system, flow to storage, the conservation laws and continuity of the cost. The system is said to be at steady state if $d\mathbf{v}/dt = 0$.

Associated with the supply network we have topological measures as defined above

$$\begin{aligned} n_n &= \text{number of routing points} \\ n_h &= \text{number of activities} \\ n_s &= \text{number of storage locations} \\ n_t &= \text{number of terminals} \\ n_p &= \text{number of production centers} \end{aligned}$$

The equations above are underspecified since there are fewer constraints than needed to specify the activities uniquely. The remaining degrees of freedom (flows) are specified using feedback control mechanisms.

The control problem we consider is the one of finding feedback mechanisms that give the optimal allocation of resources in order to maximize the performance of a business unit, a company or a supply chain. In the next section we therefore discuss very briefly how such performance can be characterized in financial terms.

Financial Analysis

The instantaneous cash performance of a business is evaluated by taking difference between revenues and expenses. The net rate of revenue is not useful for developing a business strategy, however. It varies considerably from one instant to the next since it only considers current performance and it also ignores the value of asset accumulation. To overcome "jumpiness" it is common to integrate over a period of time and define the profit P during the period $\{t_0, t_1\}$ so that

$$P = \int_{t_0}^{t_1} T dt$$

We then get classic accounting measures. For example, if $\{t_0, t_1\}$ represents a quarter then P gives the quarterly net income from operations. We can now average and get smoothed estimates of rates. For example

$$\bar{T} = \frac{P}{t_1 - t_0}$$

represents the average rate of profit during the period.

However, as argued in [7] and [3], focusing on accounting measures gives a myopic view of the value of a business since it does not include the time value of future business performance and holdings. There has therefore been a tendency to move away from the use of measures based purely on profit towards the use of measures that include value creation.

To advance this idea we express the accounting (book) value of a business unit by the formula

$$V = \sum_{\text{Storage Locations}} \pi_{S,i}$$

where the index i ranges over all stored assets. With this number as initial condition we integrate the expected value of future earnings using a discount factor to give

$$\Pi(t) = \int_0^{\tau} \sigma^s T(t+s) ds + V(t), \qquad 0 \le \tau \le \infty$$

The number Π then represents the estimated value of the company or business unit taking into account discounted cash flow into the future using the discount factor σ. However, future values of T are not known so these must be replaced by optimal forecasts using current and past information. More formally, we express this as a conditional expectation

$$\bar{T}(t+s) = E\{T(t+s)|F(t)\}, \qquad s > 0 \qquad (14)$$

where the filtration $F(t)$ represents all information available at time t to estimate the future cash flow [8]. The variable $\bar{\Pi}(t)$ then provides an estimate of the intrinsic value of the company based on the best use of current information.

If $\bar{\Pi}_i(t)$ represents the estimated value of business unit i in a supply chain, then the measure

$$\bar{\Pi}(t) = \sum_{\text{Business Units}} \bar{\Pi}_i(t)$$

gives the value of all business units in a given supply chain. SCM systems should be designed to optimise Π according Mr. Buffett.

Remarks:

1. The accuracy of the predictions depends on how much information one has available at a given time. More information is available for company insiders, especially with regards to the development of new technology and negotiation of contracts. Such developments are therefore guarded until they are consummated or protected.

2. The importance of evaluating expression (14) as accurately as possible cannot be over estimated since it impacts the business strategy and may motivate significant changes in business operation.

3. The accounting measures are readily available and are therefore not so interesting for SCM since the sharing of such information cannot lead to competitive advantage. It seems to us, that in order to achieve good cooperation along a supply chain, companies need to share information about expected future developments, new technology and how these are going to impact future earnings and price structures. This information then needs to updated as new data becomes available so that companies along a cooperating supply chain can more quickly re-adjust their behavior to respond to changes in market conditions and new technology. The sharing of such information between companies is generally not encouraged, and raise numerous questions about security, risks and the legality of sharing sensitive market data among companies.

Network Operators

In the previous section we saw how estimated and filtered properties are used in the evaluation of business performance. These filtered variables smooth flow. We also saw how filtering was used to represent forecasts. In this section we formalize these ideas and introduce the concept of a network operator.

Consider for example flow averaging so that

$$\bar{f} = \frac{1}{t_1 - t_0} \int_{t_0}^{t_1} f dt$$

and cost averaging

$$\bar{w} = \frac{1}{t_1 - t_0} \int_{t_0}^{t_1} w dt$$

These operators are linear and equations (2) and (8) hold for the averaged quantities. A range of other operators have the same property. Examples include expectation, time-averaging with finite, moving window, discounting with an exponential, linear filters, Fourier transform, multiplication with constant matrices and vectors, forecasts based on past information and sampling to mention a few.

We denote operators of the type above by the symbol Γ and give subscript according to which variable we operate with respect to. The transformed variables are given an overbar so that

$$\bar{v} = \Gamma_v v, \qquad \bar{f} = \Gamma_f f, \qquad \bar{w} = \Gamma_w w$$

Such operators are called *network operators* provided equations (2) and (8) hold for the transformed variables. In the following we may drop the overbar notation when no confusion should arise.

Activity Costs

In order to develop production schedules that balance system load we need to evaluate the rate of value generation and its sensitivity with respect to changes in the activity rate. In the simplest case this may be a linear function with a downward trend to reflect discounts for larger volumes. More generally we find that the cost may vary in a nonlinear way. There may for example be a minimum start up cost before any activity can start, beyond this the activity cost may vary in stepwise

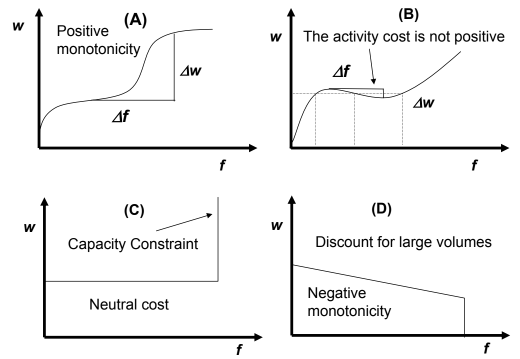

Figure 9: Examples of cost/value functions for activities.

manner until at some point the activity saturates. Other examples include the barrier function, which describes capacity constraints, gradient directions that result from optimisation of convex cost functions and more generally any cost which is monotonic in the sense that higher added value gives incentive higher activity rates.

Let Δ be the difference operator so that for any variable z we have $\Delta z = z_2 - z_1$.

Definition 1. *An activity is said to have* monotonic *rate if for any* f_1 *and* f_2

$$\Delta f \Delta w \leq 0.$$

The definition gives the example of a negative monotonic rate. A similar definition can be given for positive rates. If the cost does not depend on rate then it is said to be neutral. Neutral rates need to be augmented by capacity limiters to ensure boundedness of the solutions. Thus a neutral rate can only be supported in a finite region. Figure 9 shows examples of (positive) monotonic activity rate and one example of an activity which is not monotonic.

Remarks:

1. The cost schedule plays the role of constitutive equations in transport problems and resistance in electrical circuits.

2. A wide variety of nonlinear constraints can be modeled. An element modelled as an ideal diode gives a capacity constraint.

3. Monotonicity is related to uniqueness of solutions. In fact, the solutions to a supply chain model of the type developed here, are unique if and only of the routing policies are monotone.

4. The activity costs are used to solve load balancing and resource allocation problems since they show how the cost varies with respect to production volume. Without such costs load balancing is not a well posed problem.

Business Systems

The model described in equation (13) represents a single business unit or a group of businesses working to satisfy a demand when all free terminals are connected so that there is at least one continuous path from every terminal connecting raw material suppliers and customers. All activities, including financial ones, should be part of at least one such path in order to be considered part of the supply network. We need to have consistency at all connections, a price forecast must be given for all sources and products. We need internal activity costs so that we can evaluate the relative merits of different activities along the supply chain as well as the relative merits of parallel activities representing different contracts, technologies and work practices. Once the system dynamics and the cost structure is available we can develop control systems to balance the load so that all system requirements and constraints are met and the activity costs are minimized. In SCM we are therefore faced with a stochastic optimal control problem:

$$\min_f \bar{\Pi}(t) \qquad \text{with:}$$

$$\begin{aligned}
\bar{\Pi}(t) &= \int_t^{t+\tau} \sigma^{t-s} E\{T(t+s)|F(t)\} ds \\
\frac{dv}{dt} &= G_0 f, \qquad v(0) = v_0 \\
0 &= G_1 f \\
G_2 w &= 0
\end{aligned}$$

We distinguish between two approaches of implementing SCM systems that use feedback to approximate solutions to the stochastic control problem described above. In our context we refer to these as Decentralized Supply Chain Management (DSCM) and Centralized Supply Chain Management (CSCM).

In DSCM the idea is to distribute the decision making as much as possible. In this paper we consider policies that depend only on local information so that

$$f = \hat{f}(w, t) \qquad (15)$$

The main advantage is that we decompose the system so that information is processed and used for feedback adjustment throughout the system without needing a central coordinator.

In CSCM all information is gathered and analyzed before any decision is made. The main advantage here is that the total system response can be considered and optimised. Examples of centralized policies include those policies that are based on predictive control with use of all state information.

The tendency in the last decade has been to distribute the decision making process as much as possible in an effort to make the business units autonomous and responsive. One example is the use 6sigma and related business systems that incorporate a number of ideas from feedback control theory. However, computational tools for large scale systems, process systems engineering, computing and IT tools have been developed that allow us to access more and more process information centrally. In practice the well run supply chain will combine elements of both approaches. The current and open challenge is how to combine DSCM and CSCM perspectives in such a way that the advantages of both approaches can be retained without leading conflicts of interest.

Decentralized SCM

In this section we consider SCM where the control policies are functions of added value as given by equation (15). The functions $\hat{f}$ corresponds to the cost functions given in the section on activity costs. We show that choosing the activity rates this way we balance parallel activities so that the rate of value generation is maximized.

Consider for the moment a supply network represented by a graph with no terminals or storage locations

$$\mathcal{G} = (\mathcal{P}, \mathcal{R}, \mathcal{H})$$

Let $n_n = n_p + n_r$ represent the number of nodes. The balances (8) then places n_n constraints on the activity rates so that only $n_f = n_h - n_n + 1$ activities may be set independently. The remaining rates, f_j, may be found by linear relations

$$f_j = \sum_{i=1}^{n_f} g_{ij} f_i, \qquad j = 1, ..., n_h \tag{16}$$

The rectangular $(n_f \times n_h)$ matrix $G = \{g_{ij}\}$ describes the interconnection behavior of the system. This matrix is called the "tie set schedule" in circuit theory. Furthermore, for each independent activity we can define one cycle in the network that does not contain any other independent activity. For each of the cyclical activities we use equation (3) and write

$$0 = \sum_{j=1}^{n_f} g_{ij} w_j, \qquad i = 1, ..., n_f \tag{17}$$

By multiplying through with w_j in equation (16) we get

$$w_j f_j = \sum_{i=1}^{n_f} g_{ij} w_j f_i, \qquad j = 1, ..., n_h$$

We now sum over the n_h activities to give

$$\sum_{j=1}^{n_h} f_j w_j = \sum_{j=1}^{n_h} \sum_{i=1}^{n_f} g_{ij} w_j f_i$$

Using equation (17) gives the result

$$\sum_{j=1}^{n_h} f_j w_j = 0$$

The result generalizes readily to graphs with terminals when we use bond conventions to create temporary connections between the terminals outside the system. We get

$$\sum_{\text{Terminals}} fc = \sum_{\text{Activities}} fw$$

Thus, at each instant in time, the flows are balanced among the elements of the network and the terminals. It follows that we can write

$$T = H$$

and maximizing H is therefore the same as maximizing T.

Consider now a supply chain represented by a graph which can be in different states, indicated by (a) and (b). By different states, we mean that the activity rates, routing policies and the production rates may be different, either as a result of looking at the same system at different times, or as a result of comparing the performance of two different businesses making the same products using the similar technologies.

By combining equations (16) and (17) again, now using different states, we have the following quasi-cost theorem

$$\sum_{\text{Terminals}} f_a c_b = \sum_{\text{Activities}} f_a w_b \tag{18}$$

In circuit theory this equation is referred to as Tellegen's theorem [9]. In thermodynamics we have a similar concept called virtual work and in field theory the result corresponds to the fact that the inner product between a solenoidal field and an irrotational field is zero. The usefulness and generality of this result stems from the fact that it is topological in nature; it does not depend on how the activities are generated.

We now state a useful generalization of equation (18) using projected variables.

Lemma 1. *Let Γ_f and Γ_w represent network operators and let $\bar{f} = \Gamma_f f$ and $\bar{w} = \Gamma_w w$ represent the transformed variables. We have*

$$\sum_{\text{Terminals}} \bar{f}_a \bar{c}_b = \sum_{\text{Activities}} \bar{f}_a \bar{w}_b$$

Proof. Follows by the same reasoning as illustrated above. □

Briefly, what the result states is that vector spaces composed of activity rates and activity costs are orthogonal. The important distinction here is that we use the filtered variables which may represent averages, forecasts or results form using network operators.

Identical techniques are used to derive a result for systems with storage. We get

$$\sum_{\text{Terminals}} \bar{f}_a \bar{w}_b = \sum_{\text{Storage}} \bar{f}_a \bar{w}_b + \sum_{\text{Activities}} \bar{f}_a \bar{w}_b$$

The following result states that monotone policies maximize the rate of generation of value of a network with respect to finite perturbations in the activity rates.

Theorem 1. *Consider a steady state, supply network (defined by equation (13)) with decentralized control policies (defined by equation (15)). The rate of value generation, (represented by the variable H in equation (4)), is stationary. Furthermore, if the transportation policies are non-positive, then H is minimized.*

Proof. Let f^* and w^* denote the solution to the DSCM using policies (15) and let f denote a perturbed flow. We want to show that any finite perturbation in flow leads to a lower rate of value generation.

Let furthermore Γ_f be the identity operator and let Γ_w take the difference between w and w^*. From Lemma 1, we have

$$\sum_{\text{Terminals}} f^*(c^* - c) = \sum_{\text{Activities}} f^*(w^* - w) = 0$$

Consider deviations in activity rates that leave the terminal prices fixed. The left hand side then vanishes and we see that any first order variations leave the activity costs invariant. It follows that H is stationary.

We now consider DSCM with monotone policies. Using Definition (1) so that

$$\int_w^{w^*} (f^* - f)dw \leq 0$$

Re-arranging gives

$$\begin{aligned}\int_w^{w^*} f^* dw &\leq \int_w^{w^*} f dw \\ &\leq \int^{w^*} f dw - \int^{w} f dw\end{aligned}$$

Using equation (7) with appropriate lower limit of integration gives

$$f^*(w^* - w) \leq \pi_H(w^*) - \pi_H(w)$$

By summing over all activities we get

$$\sum_{i=1}^{n_h} f_i^*(w_i^* - w_i) \leq \sum_{i=1}^{n_h} \pi_H(w^*)_i - \sum_{i=1}^{n_h} \pi_H(w)_i$$

Using Lemma 1 we then get

$$\sum_{i=1}^{n_t} f_i^*(c_i^* - c_i) \leq \sum_{i=1}^{n_h} \pi_H(w^*)_i - \sum_{i=1}^{n_h} \pi_H(w)_i$$

By setting $c_i = c_i^*$, which corresponds to keeping the terminal prices fixed, we set the left hand side to zero. This gives the result since

$$\sum_{i=1}^{n_h} \pi_H(w^*)_i \geq \sum_{i=1}^{n_h} \pi_H(w)_i$$

It follows that the rate of value generation is maximised with respect to finite variations in activity rates.

□

A number of similar results for DSCM can be developed using different types of feedback policies. The results generalize quite readily to networks with storage and unsteady state as well.

We now want to consider more general performance objectives and define "quasi-costs" by introducing dual variables via the Legendre-Fenchel transform. In order to develop this idea we introduce a generalized cost Π_G. This may for example that represent the intrinsic value of the business unit or supply chain. But, it could represent any other measure as long as it is convex in $\mathbf{f}$, the vector of all activity rates.

Assumption 2. *The generalized cost $\Pi_G = \hat{\Pi}_G(\mathbf{f})$ is convex.*

We now define the dual Π_G^* and a new set of variables $\mathbf{w}$ so that

$$\Pi_G^*(\mathbf{w}) = \sup_{\mathbf{f} \in F_0} (\Pi_G(\mathbf{f}) - \mathbf{w}^T \mathbf{f})$$

The vector $\mathbf{w} \in W^*$ define the sensitivities of Π_G with respect to all activity rates in the network that satisfy the constraints given by equation (13). The mapping from $\mathbf{f}$ to $\mathbf{w}$ is involutive due to convexity [1]. Furthermore, if $\Pi_G(\mathbf{f})$ is twice differentiable with respect to $\mathbf{f}$ then we have

$$\mathbf{w} = \frac{\partial \Pi_G}{\partial \mathbf{f}}$$

and

$$M = \left\{ \frac{\partial^2 \Pi_G}{\partial f_i \partial f_j} \right\}$$

It now follows that the variables $\mathbf{w}$ corresponds to a set of generalized activity costs. In the DSCM problem we now find that the optimality result just developed still holds when we use feedback laws using the generalized sensitivities. This development, the details of which will be reported in an extended version of this paper, shows that the network is self optimising and maximises a unique and well defined objective function without central coordination. We also note that if the cost relations are positive then we solve a minimisation problem.

The following result gives uniqueness of solutions

Theorem 2. *The solution $f = f^*$ is unique and stable if and only if the routing policies are monotone.*

Proof. This follows the same development as in [5]. □

We now get to the main result of this section. It is the following

Theorem 3. *Consider two identical supply networks with monotone rates and decentralized feedback as given in equation (15). The initial conditions are different but cost structure and the boundary conditions are the same. Over time the solutions converge so that* $lim_{t\to\infty}\|f_1 - f_2\| = 0$

Proof. Follows the same lines as in [5] and [20]. □

The result shows that companies and supply chains that compete in the same markets have operating margins that converge over time. In order to remain competitive they need to lower cost, develop new technology and differentiate products.

Remarks:

1. The results of this section remain valid for systems with transportation delays. The theory for delayed pipelines and pipelines with dissipation is given in [20]. We also note that we can develop a dual result for fixed flows and varying prices.

2. By letting C denote capital expenditures we can, in analogy with a thermodynamic system, define a variable T_c so that

$$\frac{1}{T_c} = \frac{\partial \Pi}{\partial C}$$

   T_c then represents a generalized temperature. If T_c is low then capital expenditures have a significant effect on intrinsic value. If T_c is high then any additional capital infusion has marginal effect.

3. The parameter

$$L = \frac{f}{w}$$

   is called the rate activity turnover rate. Let the pairs $\{f_a, f_b\}$ and $\{w_a, w_b\}$ represent activity levels and added values for competing activities. The two activities are said to have the same performance if $L_a = L_b$. If $L_a > L_b$ then we have

$$\frac{f_a}{w_a} > \frac{f_b}{w_b}$$

   and it is clear that line activity (a) has higher performance.

4. We showed that DSCM has optimality properties and that we can solve optimal resource allocation problems on-line. For example, if the activity costs are neutral and all activity rates are capacity limited then the DSCM with linear feedback solves a linear program. The solution will be constrained at a point where one or more activity rates are capacity constrained and the activity costs are minimized. If the the activity rates are linear and strictly monotone, then the linear feedback laws solves a quadratic program. If this solution is constrained then the solution corresponds to a KKT point. These methods therefore minimic the minimization methods that have been used very effectively to solve large scale optimisation problems in our research groups here at Carnegie Mellon. In the present case we replace the optimiser with a differential equation which resembles the one used to describe transport in irreversible systems obeying the Onsager reciprocal relations. The maximum in our case then corresponds to a minimum in entropy production and is found by dynamic simulation of a distributed system rather than by equation solving.

5. More complex systems can be devised by considering elements with nonlinearities that include, for example hysteresis to mimic the asymmetry of start up and shut down of production units.

Centralized SCM

In this section we tie the theory developed in the previous section together with the application of optimisation methods to find optimal routing policies. We will in particular describe how we use the *rolling horizon* approach to solve a resource allocation problem in a multi-echelon supply chain. This work and results are described more fully in [10] and [12].

The model of the system is based on a generalization of the model described in Example 5. There is a supplier, a production plant, a plant warehouse, distribution centers and retailers. The system model, which can be written in the form of equation (13), is based on making the following assumptions:

1. The distribution network consists of independent distribution nodes.
2. Each node handles as many products as the total number of products in the network.
3. There are multiple products that are produced sequentially in time.
4. Nodes receive and place orders from and to a number of neighboring nodes.
5. Customer orders accumulate for a period of one day before they are passed on.
6. There are storage costs and transportation times.
7. The production plant is modeled as a single stage, single unit, multi-product batch plant.
8. Plants purchase from two sources, one with a fast response-time and a high cost, another with a slow response-time and a lower cost.
9. The sampling time is two hours.
10. Future demands are estimated accurately for the duration of the prediction horizon.

The main objective of the study was to implement a centralized policy that would schedule the batch plant and route material through the supply chain in order to maximize the profit defined so that

$$\text{Profit} = \text{Revenue - Cost}$$

over a finite horizon τ using discount factor $\sigma = 1$. The reason why we set the discount factor to one was that the prediction horizon in our study was quite short, ranging from 4 to 27 days. The plant cost was broken down into several terms. These included the production costs, the raw material purchase cost, the inventory holding cost and the transportation cost.

We formulate the problem described above as a multiperiod Mixed Integer Linear Problem. For a given initial condition, the MILP calculated functions that described the optimal routing over the prediction horizon, using GAMS and XPRESS MP. The policy was implemented using the rolling horizon approach, akin to model predictive control. We updated the controls once every 14 hours. This approach gave feedback and enables the algorithm to track changes in demand and market parameters.

Several case studies were evaluated using the algorithm. The case study reported here represented a system with three production plants, three different products and a distribution network consisting of four distribution centers, ten retailers and twenty customers. Each customer bought different amounts of products, but not all customers bought all products. The prediction horizon was 12 days with weekly updates for demand forecasts. This gives an MILP model with 1296 binary variables, 85,898 continuous variables ande 59,150 constraints.

Remarks:

1. It was found for this problem that the centralized control policies using the MILP formulation gave better system performance than a heuristic feedback policy, giving about 15% higher profit with slightly lower revenues. The improvement is the result of a better balance between costs incurred in the distribution network and the plant schedule.
2. One important point discovered by Perea was that a too short a prediction horizon (less than four days in this case) led to a myopic point of view. This led to a selling off inventory and shutting down the production line. In the (very) short run this is indeed optimal. However, in the extended perspective (more than four days in this case) this gives serious losses since the production line is shut down.

3. We found that the policies converged for longer horizons and that extending the prediction horizon beyond a certain horizon left the policy invariant. These results are well known in linear control, but we believe this is the first time such behavior has been observed in the context of a switched system.

A number of other cases are reported in the thesis by Perea [10].

Summary and Conclusions

In this paper we have developed an abstract model for supply chain management that connects the optimisation based approaches with the network-based approaches. The model captures seven essential features of the supply chain problem, the topology, transportation, shipping/receiving, assembly, disassembly, storage, signal processing and performance evaluation. The resulting model can be described as a large scale DAE system with dynamic states which are represented by inventories and flows determined by feedback mechanisms analogous to constitutive relations in transport problems.

The degrees of freedom are represented by a fixed number of activity rates. We introduced the concept of monotone acitivity costs. Monotone cost guarantees the existence, uniqueness and stability of the initial value problem represented by our model of the supply network. We furthermore showed that if the costs are are monotone, then different supply networks (with the same topology, routing policies and boundary conditions) converge to the same solution, leading to more competitive markets. Everything else being equal, this gives advantages to producers that have low activity costs. We introduced the notion of a temperature for use to allocate capital expenditures. Finally, we reviewed an application of a centralized, rolling horizon approach to a simple routing problem. We showed that choosing the prediction horizon too short gave divergence of the cost.

Acknowledgements: The authors want to acknowledge the help from Dr Edgar Perea-Lopez in setting up the rolling horizon problem for the supply chain system. The reviewers did an outstanding job in going through the first manuscript. I wish I could name them here. R. Kaiser of Elkem gave many helpful comments and brought the use of intrinsic value to our intention. BEY is also grateful for substantial support from the Elkem foundation.

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