A Conditional Value-at-Risk Framework For Optimal Microgrid Day-Ahead Market Bidding

Robert Herding[a] , Emma Ross[b], Wayne R. Jones[b],Vassilis M. Charitopoulos[a], Lazaros G. Papageorgiou[a,*]

[a] The Sargent Centre for Process Systems Engineering, Department of Chemical Engineering, UCL (University College London), Torrington Place, London WC1E 7JE, UK

[b] Shell plc, Shell Centre, London SE1 7NA, UK

Abstract

Microgrids are decentralised systems supplying local demands with reliable, low-emission energy from internal distributed energy resources (DER), while adding extra flexibility to the main grid. Meanwhile, the deregulation of electricity markets drives operators of energy systems to develop market strategies to maintain their economic competitiveness. This work investigates the bidding problem of a microgrid consisting of a battery, power generator, photovoltaic (PV) system and a commercial electricity demand. The daily operational task is to determine hourly bidding curves for the day-ahead electricity market that are feasible under present market rules and optimal for recourse schedules of microgrid resources after market clearing. A mixed integer linear programming (MILP) model is formulated using stochastic programming with uncertainty in electricity price and PV power. Particularly, the proposed optimisation model optimally selects both price and quantity values of individual bidding curve points subject to a limit on the total number of utilised bid points per curve enforced by market rules. Conditional value-at-risk (CVaR) is applied as a risk-measure to guard the microgrid from undesirable losses of its bidding decisions. Tradeoffs between expected total cost and CVaR are found using the ε-constraint method. A computational study is conducted to show the applicability of the optimisation framework.

Keywords

Microgrid bidding, Day-ahead market, Conditional value-at-risk.

The recent 2021 Texas power outage demonstrates that traditional energy consumers, who rely entirely on the main grid, are exposed to grid outages through extreme weather events and rising price instabilities. The latter is expected to increase further due to an expansion of non-dispatchable renewables in the grid generation mix. Microgrids offer great potential to soften such challenging effects for its integrated energy consumers. A microgrid is a local network of loads and distributed energy resources controllable as a single entity. It can operate either grid-connected or decoupled from the main grid known as island mode. Microgrid deployments in the power sector are growing rapidly providing resilient and mostly green energy to internal energy loads through its locally deployed DERs (Feng et al., 2018; Silvente et al., 2018).

Bidding problems of energy systems in short-term energy markets are widely studied in research, utilising the flexibility of energy systems to buy (sell) electricity during periods of low (high) prices (Fleten and Pettersen, 2005; Liu et al., 2016; Krishnamurthy et al., 2018; Leo et al., 2021). Before gate closure on a current day, bidding decisions need to be submitted for each trading block of the following day. After gate closure the electricity market is cleared, market prices reveal and quantities from accepted bids become committed, i.e. they must physically settle at the corresponding trading block of the day ahead. In general, a bid is a pair of price and quantity, expressing the willingness to buy (sell) an energy quantity below (above) or equal to a particular price threshold. A bidding curve can be expressed as a collection of multiple price-quantity points submitted for the same trading block. Among other market rules, the cardinality of bid points per curve is restricted, for instance to a maximum of ten bid points in the Electric Reliability Council of Texas (ERCOT) or the California Independent System Operator (CAISO) day-ahead electricity market. Typically, bidding approaches respect this limit heuristically by fixing a selection of price values for each bidding curve a-priori to the bidding optimisation problem (Ottesen et al., 2016).

In contrast, this work integrates the selection of bid prices in the stochastic optimisation model to optimally determine both bid price and bid quantity values of bidding curves simultaneously. Overall, the microgrid bidding problem is formulated as MILP using a two-stage stochastic programming

* Corresponding author: Lazaros G. Papageorgiou (E-mail: `l.papageorgiou@ucl.ac.uk`).

approach with day-ahead market bidding as first stage decisions and uncertainty considered for the electricity price and PV power (Birge and Louveaux, 2011). Moreover, the exposure to financial risks from bidding curves is taken into account (Fleten and Pettersen, 2005; Zhao et al., 2020). Financial risk may be quantified for instance by an expectation of not meeting a monetary target (Barbaro and Bagajewicz, 2004) or an expected deviation from the mean profit (Shao and Zavala, 2019). Particularly, this work applies CVaR as a risk measure to compromise expected cost against the risk of undesirable losses (Verderame and Floudas, 2010; Zhang et al., 2016). The multi-objective problem is solved through a solution procedure based on the ε-constraint method.

Mathematical Model

The microgrid bidding problem is formulated as a multi-objective, two-stage stochastic MILP model. First stage decisions are day-ahead market bidding curves, second stage decisions are DER schedules over the day ahead and electricity trades in the real-time market. The set of constraints consists of two main bodies. One is related to DERs of the microgrid system, the other represents technical constraints from electricity market rules enforced on bidding decisions. In the mathematical model, index $t \in T$ represents hourly time intervals whose time horizon is the day ahead; index $s \in S$ jointly represents scenarios for day-ahead market electricity price and PV power.

DER Constraints

The following constraints are related to DERs of the microgrid. Battery storage (E^B_{st}) in scenario s at time t is the battery storage from the previous time point adding charged electricity using charge rate $Q^{B,c}_{st}$ with efficiency η^c and subtracting discharged electricity using discharge rate $Q^{B,d}_{st}$ with efficiency η^d over time step length δ.

$$E^B_{st} = E^B_{t_0} + \delta \cdot (\eta^c \cdot Q^{B,c}_{st} - 1/\eta^d \cdot Q^{B,d}_{st}), \quad \forall s, t = 1 \tag{1}$$

$$E^B_{st} = E^B_{s,t-1} + \delta \cdot (\eta^c \cdot Q^{B,c}_{st} - 1/\eta^d \cdot Q^{B,d}_{st}), \quad \forall s, t > 1 \tag{2}$$

where $E^B_{t_0}$ is the initial battery storage at the beginning of the time horizon. Moreover, the operational range of the battery storage is limited by upper limit $E^{B,max}$ and lower limit $E^{B,min}$ at any time t of scenario s.

$$E^{B,min} \leq E^B_{st} \leq E^{B,max}, \quad \forall s, t \tag{3}$$

Additionally, the battery storage at the end of the time horizon in scenario s must be greater or equal to the initial battery storage.

$$E^B_{t_0} \leq E^B_{s,t=|T|}, \quad \forall s \tag{4}$$

Furthermore, battery charging ($Q^{B,c}_{st}$) and discharging ($Q^{B,d}_{st}$) at time t of scenario s cannot exceed battery's maximum (dis)charge rate R^B.

$$Q^{B,c}_{st} \leq R^B, \quad \forall s, t \tag{5}$$

$$Q^{B,d}_{st} \leq R^B, \quad \forall s, t \tag{6}$$

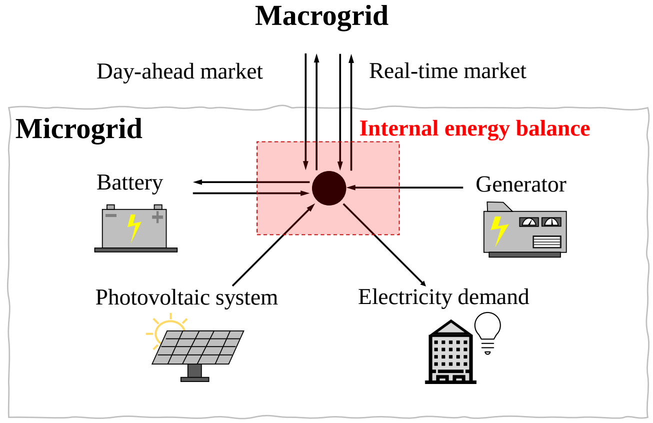

Figure 1. Microgrid overview and internal energy balance of electricity supply and demand.

To curb long-term degradation of the battery, its daily total amount of (dis)charged electricity must not exceed a multiple L^B of its maximum battery storage capacity in scenario s.

$$\delta \cdot \sum_t Q^{B,c}_{st} \leq L^B \cdot E^{B,max}, \quad \forall s \tag{7}$$

$$\delta \cdot \sum_t Q^{B,d}_{st} \leq L^B \cdot E^{B,max}, \quad \forall s \tag{8}$$

Furthermore, generator power (Q^G_{st}) cannot exceed its capacity limitation R^G at any time t of scenario s.

$$Q^G_{st} \leq R^G, \quad \forall s, t \tag{9}$$

Microgrid energy balance of internal electricity supply and usage around a central node must hold at any time t of scenario s. This energy balance is illustrated in Fig. 1. Internal electricity supply at time t of scenario s consists of battery discharge, generator power, PV power (Q^{PV}_{st}) and electricity bought in the day-ahead market ($Q^{DA,b}_{st}$) and real-time market ($Q^{RT,b}_{st}$). Internal electricity usage at time t of scenario s consists of battery charge, known electricity demand (Q^D_t) and electricity sold to the day-ahead market ($Q^{DA,se}_{st}$) and real-time market ($Q^{RT,se}_{st}$).

$$\begin{aligned} &Q^{B,d}_{st} + Q^G_{st} + Q^{PV}_{st} + Q^{DA,b}_{st} + Q^{RT,b}_{st} \\ &\quad = Q^{B,c}_{st} + Q^D_t + Q^{DA,se}_{st} + Q^{RT,se}_{st}, \quad \forall s, t \end{aligned} \tag{10}$$

The examined microgrid is assumed to act as a price-taker in energy markets, whose bids do not influence the electricity market price. The following constraints are related to technical requirements from market rules enforced on bidding decisions submitted by the microgrid on a daily base before market closure. They were motivated by the ERCOT day-ahead market.

Monotonicity Constraints

Bidding curves for buying (selling) electricity have to be monotonically decreasing (increasing). Hence, the buy (sell) quantity of the bidding curve at day-ahead market price of scenario s at time t has to be equal or larger than the quantity of its closest but higher (lower) price of scenario s'.

$$Q_{st}^{DA,b} = \Delta Q_{st}^{DA,b}, \quad \forall s \in S_t^P, t : O_{st} = O_t^{max} \tag{11}$$

$$Q_{st}^{DA,b} = Q_{s't}^{DA,b} + \Delta Q_{st}^{DA,b},$$
$$\forall (s,s') \in S_t^P, t : O_{s't} = O_{st} + 1 \land 1 < O_{s't} \leq O_t^{max} \tag{12}$$

$$Q_{st}^{DA,se} = \Delta Q_{st}^{DA,se}, \quad \forall s \in S_t^P, t : O_{st} = 1 \tag{13}$$

$$Q_{st}^{DA,se} = Q_{s't}^{DA,se} + \Delta Q_{st}^{DA,se},$$
$$\forall (s,s') \in S_t^P, t : O_{st} = O_{s't} + 1 \land 1 < O_{st} \leq O_t^{max} \tag{14}$$

where $O_{st} \in \mathbb{Z}^+$ represents the order of scenario s from low to high day-ahead market electricity prices at time t, $O_t^{max} = \max_s (O_{st})$ represents its maximum at time t, positive slack variables $\Delta Q_{st}^{DA,b}$ and $\Delta Q_{st}^{DA,se}$ represent the incremental quantity increase of the bidding curve at time t for buying and selling, respectively, at day-ahead market electricity price of scenario s. Subset S_t^P represents scenarios with distinct day-ahead market electricity price values at time t, *i.e.* $C_{st}^{DA} \neq C_{s' \neq s,t}^{DA}$ for $s, s' \in S_t^P$ at time t.

Nonanticipativity Constraints

To respect nonanticipativity of uncertainty across the time horizon, a certain realisation of an electricity price at time t must imply a certain accepted quantity after market clearing based on submitted bidding curves for this hour (Birge and Louveaux, 2011). Therefore, buy (sell) quantities at day-ahead market electricity prices of scenario s and s' have to be equal if prices of both scenarios are the same at time t.

$$Q_{st}^{DA,b} = Q_{s't}^{DA,b}, \quad \forall s \in S_t^P, s' > s, t : C_{st}^{DA} = C_{s't}^{DA} \tag{15}$$

$$Q_{st}^{DA,se} = Q_{s't}^{DA,se}, \quad \forall s \in S_t^P, s' > s, t : C_{st}^{DA} = C_{s't}^{DA} \tag{16}$$

where C_{st}^{DA} represents the day-ahead market electricity price at time t of scenario s.

Price-quantity Point Limit Constraints

If the bid quantity at day-ahead market electricity price of scenario s of the bidding curve for buying (selling) at time t increases incrementally by $\Delta Q_{st}^{DA,b}$ ($\Delta Q_{st}^{DA,se}$), it represents an active price-quantity point and implies binary variable $Y_{st}^{DA,b}$ ($Y_{st}^{DA,se}$) to be equal one. Any incremental bid quantity increase within a bidding curve must exceed a minimum threshold (ΔQ^{min}).

$$\Delta Q^{min} \cdot Y_{st}^{DA,b} \leq \Delta Q_{st}^{DA,b} \leq M^{Q,b} \cdot Y_{st}^{DA,b}, \quad \forall s \in S_t^P, t \tag{17}$$

$$\Delta Q^{min} \cdot Y_{st}^{DA,se} \leq \Delta Q_{st}^{DA,se} \leq M^{Q,se} \cdot Y_{st}^{DA,se}, \quad \forall s \in S_t^P, t \tag{18}$$

where $M^{Q,b}$ and $M^{Q,se}$ are sufficiently large values and binary variable $Y_{st}^{DA,b}$ ($Y_{st}^{DA,se}$) equals one if the bid quantity at day-ahead market electricity price of scenario s increases incrementally and forms an active price-quantity point of the bidding curve for buying (selling) at time t. Conversely, we label a bid point as inactive if its bid quantity does not increase incrementally and binary variable $Y_{st}^{DA,b}$ ($Y_{st}^{DA,se}$) equals zero. Figure 2 further illustrates the mathematical construction of

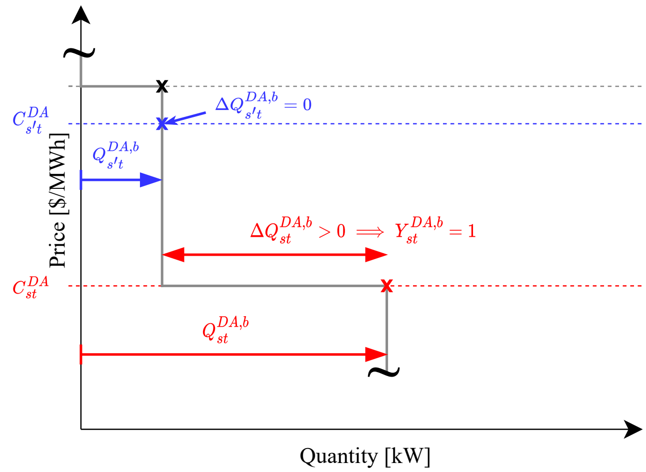

Figure 2. Mathematical construction of bid points for a bidding curve for buying electricity at time t.

an inactive and active price-quantity point for a buy bidding curve. Generally, inactive price-quantity points contain no additional information to construct or evaluate bidding curves once a bidding solution has been obtained. Consequently, bidding curves as first stage decisions can be expressed by $B_t^b = \{(Q_{st}^{DA,b}, C_{st}^{DA}) \mid s \in S_t^P \land Y_{st}^{DA,b} = 1\}$ for buying and $B_t^{se} = \{(Q_{st}^{DA,se}, C_{st}^{DA}) \mid s \in S_t^P \land Y_{st}^{DA,se} = 1\}$ for selling, using only its active price-quantity points.
Furthermore, to comply with restrictions on the number of points per bidding curve, the sum of active price-quantity points of the bidding curve for buying (selling) electricity at time t is not allowed to exceed N points per curve.

$$\sum_{s \in S_t^P} Y_{st}^{DA,b} \leq N, \quad \forall t \tag{19}$$

$$\sum_{s \in S_t^P} Y_{st}^{DA,se} \leq N, \quad \forall t \tag{20}$$

where the value of N is determined by electricity market rules.

Simultaneous Buy and Sell Bid Constraints

We assume that buy and sell bidding curves can be submitted together at the same time t as long as their quantity commitments after market clearing for buying and selling electricity at a certain electricity price are mutually exclusive. For this purpose, auxiliary variables $P_t^{up,b}$ and $P_t^{lo,se}$ are introduced to represent the highest price value of active price-quantity points of the bidding curve for buying and the lowest price value of active price-quantity points of the bidding curve for selling at time t, respectively. Their values are determined by the following logical constraints.

$$C_{st}^{DA} \cdot Y_{st}^{DA,b} \leq P_t^{up,b}, \quad \forall s \in S_t^P, t \tag{21}$$

$$P_t^{lo,se} \leq C_{st}^{DA} + M^P \cdot (1 - Y_{st}^{DA,se}), \quad \forall s \in S_t^P, t \tag{22}$$

where M^P is a sufficiently large value. In case all price-quantity points of the buy (sell) bidding curve at time t are inactive, these logical constraints are non-binding.

Due to the monotonicity property enforced in Eqs. (11)-(14) it is sufficient that the highest price value of active price-quantity points of the bidding curve for buying is lower than the lowest price value of active price-quantity points of the bidding curve for selling at time t to prevent both buy and sell bids to be accepted simultaneously at a certain electricity price.

$$P_t^{up,b} \leq P_t^{lo,se} + \varepsilon, \quad \forall t \tag{23}$$

where a small positive number ε ensures strict inequality.

Expected Total Cost Objective Term

Two objectives are to be minimised, the expected total cost and CVaR. The expected total cost (ϕ^{TC}) is determined by:

$$\phi^{TC} = \sum_s \pi_s \cdot TC_s \tag{24}$$

where π_s is the probability and TC_s the total cost associated with scenario s. It is notable that the examined microgrid bidding problem has no scenario-independent cost term. The total cost (TC_s) for scenario s is the summation of battery degradation cost (O1), generator fuel cost (O2), net cost from day-ahead market trading (O3) and net cost from real-time market trading (O4).

$$\begin{aligned} TC_s = &\overbrace{\delta \cdot C^{B,degr} \cdot \sum_t (Q_{st}^{B,c} + Q_{st}^{B,d})}^{\text{O1, battery degradation cost}} \\ &+ \overbrace{\delta \cdot HR^G \cdot C^{gas} \cdot \sum_t Q_{st}^G}^{\text{O2, generator fuel cost}} \\ &+ \overbrace{\delta \cdot \sum_t (C_{st}^{DA} \cdot (Q_{st}^{DA,b} - Q_{st}^{DA,se}))}^{\text{O3, day-ahead market net cost}} \\ &+ \overbrace{\delta \cdot \sum_t (C_{st}^{RT,b} \cdot Q_{st}^{RT,b} - C_{st}^{RT,se} \cdot Q_{st}^{RT,se})}^{\text{O4, real-time market net cost}} \end{aligned} \tag{25}$$

where $C^{B,degr}$ is battery degradation cost per charge or discharge power, HR^G is generator heat rate, C^{gas} is natural gas price, $C_{st}^{RT,b}$ ($C_{st}^{RT,se}$) is the price for buying (selling) electricity in the real-time market at time t of scenario s. Particularly, real-time market prices for buying or selling electricity are assumed to be less favourable than prices for trading in the day-ahead market (Liu et al., 2016; Leo et al., 2021). Hence, we assume real-time market prices at time t of scenario s as:

$$C_{st}^{RT,b} = C_{st}^{DA} + \omega \cdot C_{st}^{DA}, \quad \forall s,t \tag{26}$$

$$C_{st}^{RT,se} = C_{st}^{DA} - \omega \cdot C_{st}^{DA}, \quad \forall s,t \tag{27}$$

where parameter ω represents the extend of premium charge that is assumed to be paid on electricity in the real-time market compared to the day-ahead market.

Conditional Value-at-Risk Objective Term

CVaR as a measure of financial risk is considered as a second objective term (Rockafellar and Uryasev, 2000). It requires to specify a loss function, which in this case is the total cost defined by Eq.(25). In its primal definition, CVaR for a continuous loss distribution represents the expected loss greater than or equal to the value-at-risk (VaR) for a given confidence value $\alpha \in (0,1)$, with VaR being the α-quantile of the loss distribution (Rockafellar and Uryasev, 2000). For a general loss distribution, not necessarily continuous and for instance discretised by scenarios, CVaR is a weighted average of VaR and an "upper" CVaR, the latter defined as the conditional expectation of loss strictly greater than VaR (Rockafellar and Uryasev, 2002). We refer to Rockafellar and Uryasev (2002) for further details. Based on their work, CVaR can be included into our stochastic optimisation problem by minimising the following special objective function:

$$\hat{\phi}^{CVaR} = \zeta + 1/(1-\alpha) \cdot \sum_s (\pi_s \cdot \max\{0, TC_s - \zeta\}) \tag{28}$$

where CVaR $= \min_{\zeta \in \mathbb{R}} (\hat{\phi}^{CVaR})$ and ζ being an optimisation variable that results in VaR in case its optimal solution is a singleton. Finally, a linear reformulation of the max-operator yields the following constraints.

$$\phi^{CVaR} = \zeta + 1/(1-\alpha) \cdot \sum_s (\pi_s \cdot \Delta TC_s) \tag{29}$$

$$TC_s - \zeta \leq \Delta TC_s, \quad \forall s \tag{30}$$

where $\Delta TC_s \geq 0$ represents the excess of total cost over ζ in scenario s.

Multi-objective Problem

Overall, the multi-objective problem (MOP) is given by:

$$\min_{x \in \Omega} \left(\phi^{TC}, \phi^{CVaR}\right) \tag{31}$$

where x represents the vector of all decision variables and Ω its feasible region defined as $\Omega = \{x \mid$ Eqs. (1)-(10) $\cap$ Eqs. (11)-(25) $\cap$ Eqs. (29)-(30)$\}$. Furthermore, we define single objective problem SOP-TC as a special case of MOP exclusively minimising ϕ^{TC} (hence, ignoring ϕ^{CVaR}) and single objective problem SOP-CVaR as a special case of MOP exclusively minimising ϕ^{CVaR} (hence, ignoring ϕ^{TC}).

Solution Procedure

The ε-constraint method is applied to transform the problem in Eq. (31) into a single-objective problem (ε-SOP) by converting the minimisation of ϕ^{CVaR} into inequality with upper bound ε.

$$\begin{aligned} &\min_{x \in \Omega} \left(\phi^{TC}\right) \\ &\text{s.t. } \phi^{CVaR} \leq \varepsilon \end{aligned} \tag{32}$$

Based on this method, a solution procedure is applied to generate $|J|$ bidding solutions with objective values $\left\{(\phi_j^{TC}, \phi_j^{CVaR}) \mid j \in J\right\}$ from which a decision maker would select a favourite solution depending on his risk awareness.

Step1: best case for total cost ($j = 1$)

1.1: $\phi_1^{TC} \leftarrow$ solve SOP-TC

1.2: $\phi_1^{CVaR} \leftarrow$ solve SOP-CVaR with $\phi^{TC} \leq \phi_1^{TC}$

Step2: best case for risk ($j = |J|$)

2.1: $\phi_{|J|}^{CVaR} \leftarrow$ solve SOP-CVaR

2.2: $\phi_{|J|}^{TC} \leftarrow$ solve SOP-TC with $\phi^{CVaR} \leq \phi_{|J|}^{CVaR}$

Step3: compromise solutions

For $j = 2$ to $|J| - 1$ do:

3.2: $\varepsilon_j = \phi_1^{CVaR} - (\phi_1^{CVaR} - \phi_{|J|}^{CVaR})/(|J| - 1) \cdot (j - 1)$

3.3: $(\phi_j^{TC}, \phi_j^{CVaR}) \leftarrow$ solve ε-SOP with $\varepsilon = \varepsilon_j$

Computational Study

The microgid bidding approach was demonstrated for a particular day of a microgrid operating in the Houston area, Texas. Input data such as DER technical parameters as well as time series data for PV power and electricity demand were provided by the industrial partner. Time series data for the day-ahead market electricity price were taken from the ERCOT market. The extend of premium applied on real-time market electicity price was assumed to be $\omega = 0.2$. PV power scenarios were generated using a seasonal autoregressive moving average (SARIMA) model, day-ahead market electricity price scenarios were received directly from the industrial partner. From their cartesian product, $|S| = 400$ scenarios were randomly drawn without replacement and applied in the stochastic optimisation. The presented solution procedure was performed with cardinality $|J| = 5$ yielding in five alternative bidding solutions for the microgrid with different emphasis between total costs and risk measured by CVaR.

All calculations were executed on an Intel Xeon Processor E5-1650 v3@3.5 GHz with 32 GB memory. Optimisation problems were modelled in GAMS 27.1.0 and solved using Gurobi 8.1.1. Termination criteria for each optimisation were 1800 s CPU time limit or two percent relative optimality gap. Stochastic optimisation problems were each comprised of 92 050 equations and 113 650 variables, from which 13 200 were binary. In the examined runs, their average relative optimality gap was 2.2 %. Furthermore, the total CPU time of the solution procedure was 4960 s on average for three investigated cases with $\alpha = 0.85$, 0.9 and 0.95. Overall, these results demonstrate the computational applicability of the proposed solution procedure to generate multiple alternative bidding solutions on a daily base for microgrid operations; and the applicability of the microgrid bidding model to find near optimal solutions in a reasonable time using standard optimisation solvers.

Furthermore, it can be seen in Fig. 3 that the risk of high losses measured by CVaR could be successfully reduced with relatively small additional expected total cost. For instance in the case of $\alpha = 0.95$, CVaR could be reduced by over 6 % with a cost increase of just below 1 %. Moreover, histograms in Figs. 4-6 reveal that a tighter constraint on the CVaR objective term effectively shifted probability from the right tail of cost distribution to the left towards lower cost values. For instance, cost between 12 \$ and 18 \$ in Fig. 5 was less frequent for bidding solutions of cases $j = 5$ and $j = 3$, compared to case $j = 1$ that exclusively prioritised minimising expected total cost. In addition, histograms such as in Fig. 5

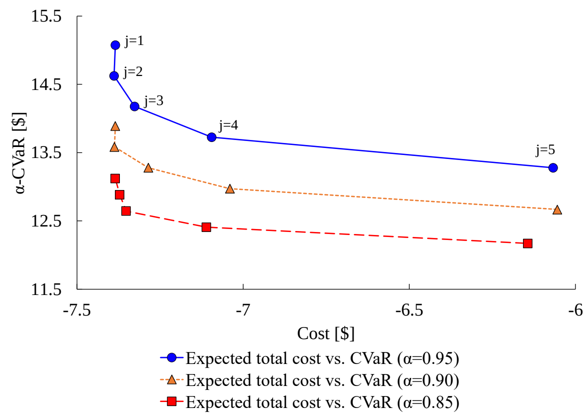

Figure 3. Tradeoff solutions between expected total cost and CVaR for different confidence values α.

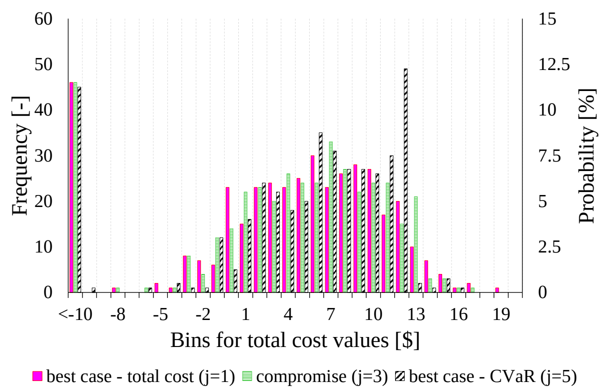

Figure 4. Histogram of total cost values over scenarios for confidence value $\alpha = 0.95$.

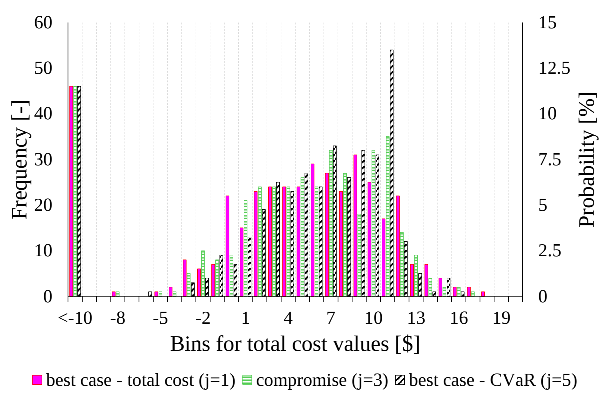

Figure 5. Histogram of total cost values over scenarios for confidence value $\alpha = 0.90$.

exhibit an accumulation of cost values between 10 \$ and 13 \$ for increasing risk awareness, which coincides with the solution value of ζ displayed in Table 1. Particularly, the optimisation had extra incentive through the CVaR objective term to determine bidding decisions that reduce total cost values larger than its incumbent solution for ζ, eventually compromising on the expected total cost. However, Fig. 3 and histograms in Figs. 4-6 demonstrate that to a certain extend high total cost values were unavoidable. For $\alpha = 0.95$, the best-case bidding solution for risk ($j = 5$) could not reduce CVaR further than 13.5 % compared to CVaR when expected total

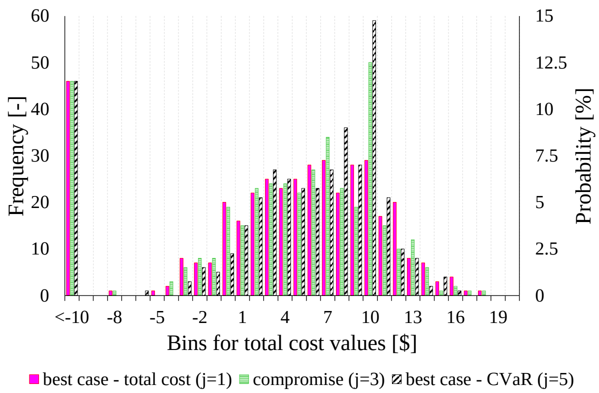

Figure 6. Histogram of total cost values over scenarios for confidence value $\alpha = 0.85$.

cost is exclusively optimised ($j = 1$). Particularly, in scenarios with extremely low PV power the microgrid may likely be forced to buy electricity even at unfavourable prices to serve its internal electricity demand. Additionally, in around 11 % of cases total cost values were in a range below -10 \$. These cases correlated with scenarios characterised by a high, spiky peak electricity price during which the microgrid took advantage of its flexibility to store electricity before selling it at the peak electricity price with profit.

Table 1. Solution values of variable ζ.

α [-]	ζ [\$]				
	j=1	j=2	j=3	j=4	j=5
0.85	11.09	10.84	10.82	10.75	10.81
0.9	12.16	12.01	11.69	11.50	11.69
0.95	13.25	13.37	13.14	12.78	12.34

Concluding Remarks

This work has presented a multi-objective, two-stage stochastic MILP formulation for a microgrid bidding in the day-ahead electricity market. Its overall goal is to minimise the expected total cost of the microgrid over the day ahead and simultaneously reduce the risk of undesirably high losses measured by CVaR. The proposed MILP model outputs bidding curves for buying and selling electricity as first stage decisions utilising multiple price-quantity bid points per curve while complying with market rules. The latter explicitly includes a limit on the number of utilised bidding points per curve. A solution procedure based on the ε-constraint method has been suggested solving a sequence of single-objective problems to find tradeoff solutions between expected total cost and CVaR. A computational study has demonstrated the applicability of the proposed solution framework to determine a sequence of near optimal bidding solutions with different emphasis between the two conflicting objective terms.

Acknowledgments

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