

Oil Production Optimization Under Gas-Coning Conditions by Well-Cycling and Mixed-Integer Programming

Jose Santiago Rodriguez, Francisco Trespalacios (former employee), Abhishek Mishra, and Keith Zorn
ExxonMobil Upstream Research Company
Spring, TX USA
Email: jose.s.osorio@exxonmobil.com

Abstract—When oil is produced from a reservoir, a pressure gradient is introduced throughout the formation. Such pressure gradient can induce upward and downward movement of water and gas, respectively, near producing wells. These phenomena, known as coning, can significantly impact well productivity and overall reservoir efficiency. To mitigate coning during oil production, wells can be shut-in for periods of time to raise the gas-oil contact. This paper proposes an optimization model for oil production under gas-coning conditions to maximize cumulative oil subject to gas processing capacity constraints. The model combines nonlinear equations that predict the gas-oil ratio (GOR) with binary decisions on when to shut-in or bring back online a given well. The resulting model is a mixed-integer nonlinear program (MINLP) that is too challenging to solve for commercial solvers. In order to solve the problem we present a mixed-integer linear formulation obtained by piece-wise linearizing the nonlinear functions, thereby allowing the use of integer-programming algorithms and state-of-the-art mixed-integer linear programming (MILP) solvers.

I. INTRODUCTION

Well-cycling is an operational strategy to increase oil production in wells that cone gas or water [13]. Coning is an undesirable and unavoidable condition where gas cap gas or bottom water infiltrates the well and reduces oil production [6]. During gas coning, the gas-oil contact near a production well is drawn downwards as pressure near the well decreases. When sufficient oil has been produced for the gas-oil contact to reach the top of the producing interval, higher gas-oil ratios (GORs) are observed. Shutting-in the well allows pressure near the well to increase, the gas-oil contact to flatten, and results in increased oil rates for a period of time after the well is brought back into operation [11]. Similar behavior is observed for water and the water-oil contact.

There are potentially many wells in a field. The well cycling problem answers the question of how to schedule the shut-in periods for every well in an effective manner. An optimal schedule maximizes oil production while satisfying operational constraints. In general, it is undesirable to shut-in a well for long periods of time and not all wells need to be shut-in. Also, subsets of wells may not be shut-in together at the same time. Moreover, every well behaves differently, with different recovery rates and different operating conditions. Hence, to determine an optimal cycling schedule it is important to develop an optimization strategy that: predicts well rate behavior under coning effects, considers binary (yes/no)

decisions for when, and decides for how long to shut in a well given all combinations of wells in the field.

An extensive amount of research has been focused on developing models for predicting oil flow rates through wells [10], [8], [14], [22], [12], [1], [20]. Muskat [15] pioneered a model for predicting gas coning and GOR. His concepts were later extended to include gas and oil production variations [14]. Thereafter in 1954, Gilbert et. al. [8] developed one of the most popular correlations for estimating liquid flow rates through a choke. The correlation was also extended in subsequent studies and adaptations have been proposed in [17], [2], [16], [19]. More recently, a different line of research has applied artificial intelligence and machine learning techniques to predict oil rate for high GOR wells [12]. Also recently, the work of Togudu-Hosseini et. al. proposed using genetic algorithms for the estimation of gas-oil ratios [7].

While there has been significant effort to develop models for the prediction of flow rates in oil wells and a great variety of strategies have been proposed, the development of optimization models in this space remains considerably challenging [11], [21], [13], [4]. One of the main challenges for incorporating such predicting models within an optimization framework arises from the fact that most, if not all, are nonlinear, non-convex models [6]. Compared to related literature for prediction/forecasting, very few publications are available for well cycle optimization. This can be attributed to the fact that solving the well cycling problem requires global optimization algorithms, capable of handling binary (yes/no) decisions together with nonlinear expressions that describe coning phenomena.

In this paper, we present a mixed-integer, piecewise-linear model for solving the well cycling problem. The model considers Gilbert-like expressions for the prediction of oil rates together with correlations for well GOR. Since those correlations are nonlinear, we introduce additional binary variables to discretize time and avoid nonlinearity. We also use a standard piecewise linearization technique to discretize GOR and approximate bilinear terms that arise in the calculation of gas rates.

The main contribution of this work is the inclusion of Gilbert-like GOR expressions within a mixed integer framework. Doing so allows the use of powerful, commercial MILP solvers such as CPLEX [5] and Gurobi [9] for solving

the well cycling problem. A second contribution of this work is the demonstration of the numerical tractability of the approach that, despite combinatorial complexity with thousands of binary variables, solves in a matter of minutes.

The paper is structured as follows: Section II presents the mathematical symbols used throughout the document. In Section III we present two mixed-integer optimization formulations for the well cycling problem. This is followed by a case study and numerical results in Section IV. Finally, Section V closes the paper with concluding remarks and directions of future work.

II. NOMENCLATURE

The following are the key mathematical symbols used throughout the paper.

Sets

- \mathbb{W} is the set of wells (index w)
- \mathbb{T} is the set of time periods (index t)
- \mathbb{M}_w is the set of operating modes for well w (index m)
- $\mathbb{H}_w \subseteq \mathbb{M}_w$ is the set of GOR healing modes for well w
- $\mathbb{G}_w \subseteq \mathbb{M}_w$ is the set of GOR growth modes for well w
- $\mathbb{A}_{w,m} \subseteq \mathbb{M}_w$ is the set of modes which mode m can switch to for well w
- $\mathbb{Q}_{w,m}$ is the set of potential active times for mode m for well w (index q)
- \mathbb{P}_w is the set of discretization points for GOR piece-wise linearization for well w (index p)

Parameters

- $\bar{y}_{w,m}$ is a binary parameter that indicates if mode m for well w is active at the beginning of the planning horizon
- \bar{a}_w is the number of time periods that well w has been active at the start of planning
- D_w is a slope growth constant for well w
- C_w is an intercept growth constant for well w
- B_w is the healing rate constant for well w
- R_w is a healing constant for well w
- ν_w is a oil constant for well w
- P_w is the head pressure for well w
- CHK_w is the choke level for well w
- α_w is a constant for oil rate calculation for well w
- WCT_w is the water cut for well w
- γ_w is a constant for oil rate calculation for well w
- T_w^{Hmax} is the maximum number of consecutive time periods well w can be active on healing
- T_w^{Gmax} is the maximum number of consecutive time periods well w can be active on growth
- T_w^{Hmin} is the minimum number of consecutive time periods well w must be active on healing
- T_w^{Gmin} is the minimum number of consecutive time periods well w must be active on growth
- $\hat{x}_{w,m,p}^{\text{OIL}} \in \mathbb{R}^+$ is the oil discretization value at discrete point p for well w in mode m
- $\hat{x}_{w,m,p}^{\text{GAS}} \in \mathbb{R}^+$ is the gas discretization value at discrete point p for well w in mode m

$\hat{x}_{w,m,p}^{\text{GOR}} \in \mathbb{R}^+$ is the gas-oil ratio discretization value at time t for well w in mode m

Variables

- $y_{w,m,t} \in \{0,1\}$ indicates if well w is active on mode m at time t
- $z_{w,m,t} \in \{0,1\}$ indicates if well w switched to mode m at time t
- $v_{w,m,t,q} \in \{0,1\}$ indicates if well w at time t has been active in mode m for q consecutive time periods
- $z_{w,t} \in [0,1]$ indicates if there is a mode switch on well w at time t
- $a_{w,t} \in \mathbb{Z}^+$ is the total active time of well w at time t since last mode switch
- $x_{w,t}^{\text{GOR}_0} \in \mathbb{R}^+$ is the initial gas-oil ratio at time t for well w
- $x_{w,t}^{\text{GOR}} \in \mathbb{R}^+$ is the gas-oil ratio at time t for well w
- $x_{w,t}^{\text{OIL}} \in \mathbb{R}^+$ is the oil produced at time t for well w
- $x_{w,t}^{\text{GAS}} \in \mathbb{R}^+$ is the gas produced at time t for well w
- $\tilde{x}_{w,t}^{\text{OIL}} \in \mathbb{R}^+$ is the piece-wise linear approximation of oil at time t for well w
- $\tilde{x}_{w,t}^{\text{GAS}} \in \mathbb{R}^+$ is the piece-wise linear approximation of gas at time t for well w
- $u_{w,t,p} \in [0,1]$ is the piece-wise linear weight at time t for well w at point p
- $s_{w,t,p} \in \{0,1\}$ is the piece-wise linear selector at time t for well w at point p

III. PROBLEM FORMULATION

A. Well Cycling Models

We begin this work by presenting three models for estimating GOR, oil, and gas rates in vertical wells. The GOR models are exponential and logarithmic correlations that predict GOR values when wells are online or offline. For wells that cone gas, GOR increases monotonically with time on production. We call this operating mode GOR growth mode. Figure 1 illustrates an example of a GOR growth curve.

The shape of the curve is a function of a number of operational parameters directly related to intrinsic well characteristics including water cut, well head pressure, bottom hole pressure, and choke level among many others. The shape is also a function of operational variables such as online time and initial GOR. Equation (1) describes the GOR growth mode. There, the coefficient D_w and C_w are constants that can be calibrated from field operational data. $x_w^{\text{GOR}_0}$ is the GOR at the time the well is brought online and τ is the time since the well was brought online.

$$r_w^{\text{GOR}}(\tau, x_w^{\text{GOR}_0}) = (D_w x_w^{\text{GOR}_0} + C_w) \ln(\tau + 1) + x_w^{\text{GOR}_0} \quad (1)$$

Alternatively, when a gas coning well is taken offline, GOR decreases monotonically with shut-in time. We call this operating mode GOR healing mode. Equation (2) models the GOR values in GOR healing mode. Note that, similar to Equation (1), the GOR in GOR healing mode from Equation (2) is a function of the operational variables $x_w^{\text{GOR}_0}$ and τ ,

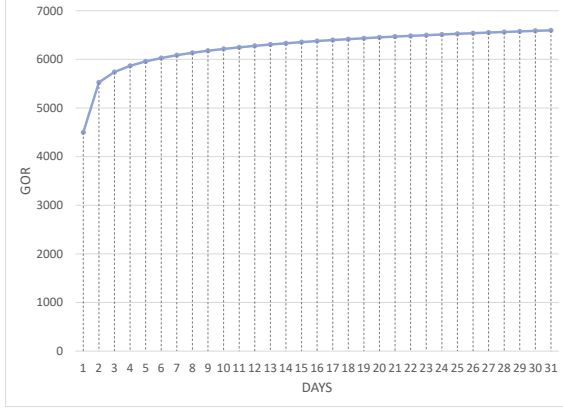


Fig. 1: Typical GOR behavior after opening a well that cones gas

and operational parameters (constant) B_w and R_w . Similarly, in Equation (2), $x_w^{\text{GOR}_0}$ represents the GOR at the time the well was shut-down, and τ the time since last shut-down. Figure 2 illustrates an example of a healing GOR curve.

$$r_w^{\text{GOR}}(\tau, x_w^{\text{GOR}_0}) = x_w^{\text{GOR}_0} e^{(-B_w \tau)} + R_w (1 - e^{(-B_w \tau)}) \quad (2)$$

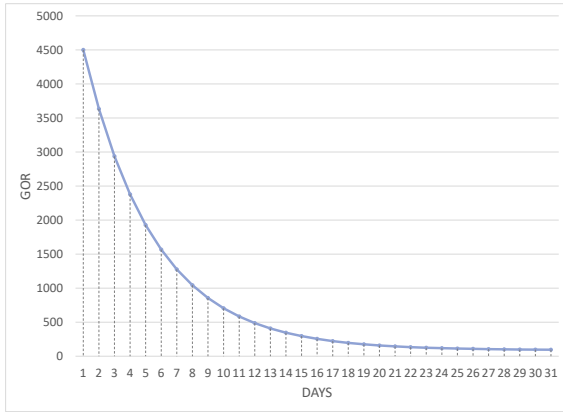


Fig. 2: Typical GOR behavior after shutting-in a well that cones gas

Focusing now on oil production, we propose using a Gilbert-like surrogate model to determine the oil flow rate when a well is online (i.e. growth). Based on Gilbert's formula [8], the oil flow rate is as a function of wellhead pressure, choke size, water cut, and GOR.

$$r_w^{\text{OIL}}(x_w^{\text{GOR}}) = \nu_w \frac{P_w C H K_w^{\alpha_w} (1 - WCT_w)}{(x_w^{\text{GOR}})^{\gamma_w}} \quad (3)$$

Similar to the GOR models, the constants ν_w , α_w , and γ_w are calibrated using field data. Finally, gas rate can be

computed as the product between GOR and oil rate.

$$r_w^{\text{GAS}}(x_w^{\text{GOR}}) = [r_w^{\text{OIL}}(x_w^{\text{GOR}})] x_w^{\text{GOR}} \quad (4)$$

It is important to note that, under the assumption of constant water cut, oil and gas rates can be calculated using the GOR of a well. Hence, when determining an optimal well-cycling schedule, monitoring GOR values is very important. The decisions on when to shut-in a well and how long to keep it offline are driven by GOR values. Intuitively, one may shut-in a well when it exhibits high GOR values. However, when dealing with multiple wells, each with its own scheduling constraints, and for a planning horizon of several days, the decision of which wells to shut-in becomes nontrivial and numerical optimization is required to optimize production. In the next section we describe a mixed-integer-nonlinear optimization model for optimal well-cycle scheduling.

Note that a well can have multiple operating modes. That is, a well can be modeled with a discrete number of curves (1)-(3) with different values for D_w , C_w , B_w , R_w , ν_w , γ_w , α_w depending on operating conditions. For example, a well can have three different growth curves for low, medium, and high choke levels. Each curve can be calibrated separately based on field data. In the next section, we refer to these different operating curves as modes and we let the optimization model determine which operating mode for a given shut-in cycle is optimal for the well-cycling schedule.

B. Mixed-Integer Nonlinear Formulation

The primary decisions in the optimization problem are when to shut-in and bring back online each well over the planning horizon T . These decisions are modeled mainly by the binary variables $y_{w,m,t}$ and $z_{w,m,t}$. When well w is active on a given mode m at time period t , $y_{w,m,t}$ takes a value of 1. Similarly, when well w switches to a given mode m at time period t , the variable $z_{w,m,t}$ takes a value of 1. Equations (5)-(7) enforce that only one mode can be active at a given time for each well and that each well can only switch to one mode at a time.

$$\sum_{m \in \mathbb{M}_w} y_{w,m,t} = 1 \quad (5)$$

$$\sum_{m \in \mathbb{M}_w} z_{w,m,t} = z_{w,t} \quad (6)$$

$$z_{w,t} \leq 1 \quad (7)$$

$$\forall w \in \mathbb{W}, t \in \mathbb{T}$$

When a well is active in a given mode, it must stay active until there is a switch to an allowed mode. Switches of mode can only happen when a mode goes from inactive to active. This logic is enforced by Equations (8)-(10).

$$\sum_{m' \in \mathbb{A}_{w,m}} z_{w,m',t} + y_{w,m,t} \geq y_{w,m,t-1} \quad (8)$$

$$z_{w,m,t} \leq 2 - y_{w,m,t-1} - y_{w,m,t} \quad (9)$$

$$z_{w,m,t} \leq y_{w,m,t} \quad (10)$$

$$\forall w \in \mathbb{W}, t \in \mathbb{T}, m \in \mathbb{M}_w$$

Minimum and maximum shut-in and online times can be imposed with standard periodicity constraints presented in Equations (11)-(14).

$$\sum_{m \in \mathbb{G}_w} z_{w,m,t} \leq \sum_{m \in \mathbb{G}_w} y_{w,m,t'} \quad (11)$$

$$\forall w \in \mathbb{W}, t \in \mathbb{T}, t' \geq t + T_w^{\text{Gmin}}$$

$$\sum_{m \in \mathbb{H}_w} z_{w,m,t} \leq \sum_{m \in \mathbb{H}_w} y_{w,m,t'} \quad (12)$$

$$\forall w \in \mathbb{W}, t \in \mathbb{T}, t' \geq t + T_w^{\text{Hmin}}$$

$$\sum_{m \in \mathbb{H}_w} \sum_{t' \leq t + T_w^{\text{Gmax}}} y_{w,m,t'} \geq 1 \quad (13)$$

$$\sum_{m \in \mathbb{G}_w} \sum_{t' \leq t + T_w^{\text{Hmax}}} y_{w,m,t'} \geq 1 \quad (14)$$

$$\forall w \in \mathbb{W}, t \in \mathbb{T} \setminus \{0\}$$

Every time a switch of mode occurs at a given well, the appropriate surrogate model from Section III-A must be used for calculating the GOR, oil rate and gas rate. Variables $z_{w,m,t}$ and $y_{w,m,t}$ can be used to select the mode equations within the optimization model. First, the $z_{w,t}$ variables are used to update $x_{w,t}^{\text{GOR}_0}$ which represents the initial GOR at the time of the switch, note that this artificially introduced variable $x_{w,t}^{\text{GOR}_0}$ remains constant while a given mode is active.

$$\left[\begin{array}{l} z_{w,t} = 1 \\ x_{w,t}^{\text{GOR}_0} = x_{w,t-1}^{\text{GOR}} \end{array} \right] \vee \left[\begin{array}{l} z_{w,t} = 0 \\ x_{w,t}^{\text{GOR}_0} = x_{w,t-1}^{\text{GOR}_0} \end{array} \right] \quad (15)$$

Note also that $z_{w,t}$ indicates if there was a switch in mode which is enforced with Equation (16)

$$z_{w,t} \geq z_{w,m,t} \quad \forall w \in \mathbb{W}, t \in \mathbb{T}, m \in \mathbb{M}_w \quad (16)$$

Second, the $z_{w,t}$ variables can also be used to update the mode activation time variable.

$$\left[\begin{array}{l} z_{w,t} = 1 \\ a_{w,t} = 0 \end{array} \right] \vee \left[\begin{array}{l} z_{w,t} = 0 \\ a_{w,t} = a_{w,t-1} + 1 \end{array} \right] \quad (17)$$

Third, the $y_{w,m,t}$ variables indicate the appropriate surrogate model to use to calculate GOR, oil, and gas values at a each point in time for every well.

$$\forall m \in \mathbb{M}_w \left[\begin{array}{l} y_{w,m,t} = 1 \\ x_{w,t}^{\text{GOR}} = r_{w,m}^{\text{GOR}}(a_{w,t}, x_{w,t}^{\text{GOR}_0}) \\ x_{w,t}^{\text{OIL}} = r_{w,m}^{\text{Oil}}(x_{w,t}^{\text{GOR}}) \\ x_{w,t}^{\text{GAS}} = r_{w,m}^{\text{Gas}}(x_{w,t}^{\text{GOR}}) \end{array} \right] \quad \forall w \in \mathbb{W}, t \in \mathbb{T} \quad (18)$$

Equation (19) imposes a limit to the total gas production below a given capacity threshold. Equation (20) determines the total oil produced in the system.

$$\sum_{w \in \mathbb{W}} x_{w,t}^{\text{GAS}} \leq \bar{F}_t \quad \forall t \in \mathbb{T} \quad (19)$$

$$\sum_{t \in \mathbb{T}} \sum_{w \in \mathbb{W}} x_{w,t}^{\text{OIL}} = x^{\text{TOTAL.OIL}} \quad (20)$$

The last set of equations in the formulation are the initial conditions imposed at the first time period of the planning horizon.

$$a_{w,0} = \bar{a}_w \quad (21)$$

$$x_{w,0}^{\text{GOR}_0} = \bar{x}_w^{\text{GOR}_0} \quad \forall w \in \mathbb{W} \quad (22)$$

$$y_{w,m,0} = \bar{y}_{w,m} \quad \forall w \in \mathbb{W}, m \in \mathbb{M}_w \quad (23)$$

To conclude this section, we summarize the optimization formulation in Problem P-MINLP.

$$\begin{aligned} \min \quad & x^{\text{TOTAL.OIL}} \\ \text{s.t.} \quad & \text{Equation(5) – Equation(23)} \end{aligned} \quad (\text{P-MINLP})$$

C. Mixed-Integer Piece-wise Linear Formulation

It is important to note that because of nonlinear GOR expressions and the oil and gas rate calculations in (18), the model can be significantly challenging to solve. Hence, we propose a linear version of the model. Although the GOR expressions in (1) and (2) are nonlinear, they can be linearized with the help of time indicator variables $v_{w,m,t,q}$. Instead of keeping track of active time with variable $a_{w,t}$ in Equation (17), index q can be used to determine the active time for a given mode. By introducing binary variables $v_{w,m,t,q}$, we discretize time and model the rate expressions for GOR as linear functions of GOR^0 .

$$\forall q \in \mathbb{Q}_{w,m} \left[\begin{array}{l} v_{w,m,t,q} = 1 \\ x_{w,t}^{\text{GOR}} = r_{w,m}^{\text{GOR}}(q, x_{w,t}^{\text{GOR}_0}) \\ x_{w,t}^{\text{OIL}} = r_{w,m}^{\text{Oil}}(x_{w,t}^{\text{GOR}}) \\ x_{w,t}^{\text{GAS}} = r_{w,m}^{\text{Gas}}(x_{w,t}^{\text{GOR}}) \end{array} \right] \quad \forall w \in \mathbb{W}, t \in \mathbb{T}, m \in \mathbb{M} \quad (24)$$

The discretization of time is achievable by imposing Equations (25)-(28).

$$\sum_{q \in \mathbb{Q}_{w,m}} v_{w,m,t,q} = y_{w,m,t} \quad (25)$$

$$v_{w,m,t,0} = z_{w,m,t} \quad (26)$$

$$\forall w \in \mathbb{W}, m \in \mathbb{M}, t \in \mathbb{T} \setminus \{0\}$$

$$v_{w,m,t,q} \leq v_{w,m,t-1,q-1} + (1 - y_{w,m,t}) \quad (27)$$

$$v_{w,m,t,q} \geq v_{w,m,t-1,q-1} - (1 - y_{w,m,t}) \quad (28)$$

$$\forall w \in \mathbb{W}, t \in \mathbb{T} \setminus \{0\}, \mathbb{Q}_{w,m} \setminus \{0\}$$

Unfortunately, nonlinearity in the oil and gas rates cannot be eliminated by simply discretizing time. For those expressions, we propose using a piecewise linearization of GOR. Note that once GOR is discretized with the piecewise linearization (29)-(35), both oil and gas rates can be expressed in a linear fashion as function of the piecewise linear GOR (with the oil discrete points can be computed as $\hat{x}_{w,m,t}^{\text{OIL}} = \nu_w \frac{P_w C H K_w^{\alpha_w} (1 - WCT_w)}{(\hat{x}_{w,m,t}^{\text{GOR}})^{\gamma_w}}$, and the gas discrete points as $\hat{x}_{w,m,t}^{\text{GAS}} = \hat{x}_{w,m,t}^{\text{OIL}} \hat{x}_{w,m,t}^{\text{GOR}}$).

$$\sum_{p \in \mathbb{P}} u_{w,t,p} \hat{x}_{w,p}^{\text{GOR}} = x_{w,t}^{\text{GOR}} \quad \forall w \in \mathbb{W}, \forall t \in \mathbb{T} \quad (29)$$

$$\sum_{p \in \mathbb{P}} u_{w,t,p} = 1 \quad \forall w \in \mathbb{W}, t \in \mathbb{T} \quad (30)$$

$$\sum_{p \in \mathbb{P}} s_{w,t,p} = 1 \quad \forall w \in \mathbb{W}, t \in \mathbb{T} \quad (31)$$

$$u_{w,t,p} \geq 0 \quad \forall w \in \mathbb{W}, t \in \mathbb{T}, p \in \mathbb{P} \quad (32)$$

$$u_{w,t,0} \geq s_{w,t,1} \quad \forall w \in \mathbb{W}, t \in \mathbb{T} \quad (33)$$

$$u_{w,t,p} \geq s_{w,t,p} + s_{w,t,p+1} \quad \forall w \in \mathbb{W}, t \in \mathbb{T}, p \in \mathbb{P} \setminus N \quad (34)$$

$$u_{w,t,N} \geq s_{w,t,N} \quad \forall w \in \mathbb{W}, t \in \mathbb{T} \quad (35)$$

The selection of the appropriate piece-wise segment is ensured with constraints (36) and (37).

$$\hat{x}_{w,m,t}^{\text{OIL}} = \begin{cases} \sum_{p \in \mathbb{P}} u_{w,t,p} \hat{x}_{w,m,p}^{\text{OIL}} & m \in \mathbb{G}_w \\ 0 & \text{otherwise} \end{cases} \quad (36)$$

$$\hat{x}_{w,m,t}^{\text{GAS}} = \begin{cases} \sum_{p \in \mathbb{P}} u_{w,t,p} \hat{x}_{w,m,p}^{\text{GAS}} & m \in \mathbb{G}_w \\ 0 & \text{otherwise} \end{cases} \quad (37)$$

$$\forall w \in \mathbb{W}, m \in \mathbb{M}, t \in \mathbb{T}$$

After piecewise linearization, Equation (18) is replaced by Equation (38).

$$\forall q \in \mathbb{Q}_{w,m} \left[\begin{array}{l} v_{w,m,t,q} = 1 \\ x_{w,t}^{\text{GOR}} = r_{w,m}^{\text{GOR}}(q, x_{w,t}^{\text{GOR}_0}) \\ x_{w,t}^{\text{OIL}} = \hat{x}_{w,m,t}^{\text{OIL}} \\ x_{w,t}^{\text{GAS}} = \hat{x}_{w,m,t}^{\text{GAS}} \end{array} \right] \quad \forall w \in \mathbb{W}, t \in \mathbb{T}, m \in \mathbb{M} \quad (38)$$

The linear formulation is summarized in Problem P-MILP.

$$\begin{aligned} \min \quad & x^{\text{TOTAL-OIL}} \\ \text{s.t.} \quad & \text{Equation(5) – Equation(15)} \quad (\text{P-MILP}) \\ & \text{Equation(19) – Equation(38)} \end{aligned}$$

IV. NUMERICAL RESULTS

This section documents numerical results for a case study with synthetic data. The system consists of four wells. Associated with the wells are three operational modes: one GOR healing mode, and two GOR growth modes. The healing mode follows the surrogate model presented in Equation (2). The first GOR growth mode follows the surrogate in Equation (1)* and the second follows a constant rate $r_w^{\text{GOR}}(t, x_{w,t}^{\text{GOR}_0}) = x_{w,t}^{\text{GOR}_0}$ applicable when $x_{w,t}^{\text{GOR}_0} \geq -C_w/D_w$.

The optimization horizon was set to 30 days and time was discretized on a daily basis. For all wells the maximum healing and growth times was set to 15 days (i.e. T_w^{Hmax} , T_w^{Gmax}). Note that the 15 days in growth may include switching between GOR growth mode in (1) to constant mode. The 15 days in healing do not allow switches since only one healing mode was considered. Note, however, that the MILP formulation is flexible to enabling multiple healing modes (e.g. see Equations (13) and (14)).

In order to avoid symmetry and improve performance within branch and bound iterations, a non-uniform GOR discretization was implemented following (39) and (40). This approach is particularly relevant given the implicit symmetry of piecewise segments at the end of the logarithmic and exponential curves.

$$\Delta \hat{x}_{w,p}^{\text{GOR}} = \frac{(\text{Max GOR}_w - \text{Min GOR}_w)}{2^{|\mathbb{P}_w|} - 1} (2^{p-1}) \quad (39)$$

$$\hat{x}_{w,p}^{\text{GOR}} = \hat{x}_{w,p-1}^{\text{GOR}} + \Delta \hat{x}_{w,p}^{\text{GOR}} \quad (40)$$

(P-MINLP) and (P-MILP) were implemented in AIMMS [3] and BARON [18] and CPLEX [5] were used to solve the MINLP and MILP, respectively. BARON failed to find a feasible solution in a two hour time limit for the MINLP. For the MILP formulation, however, CPLEX found an optimal solution in 59.41 seconds. The optimality gap was 0.09% and the size of the problem was 7278 variables (5603 integer variables), and 25444 constraints.

Figures 3 and 4 present the oil rates and GORs of the four different wells for a planning horizon of 30 days with

*The equation was modified slightly to input time in days such that $r_w^{\text{GOR}}(t, x_{w,t}^{\text{GOR}_0}) = (D_w x_{w,t}^{\text{GOR}_0} + C_w) \ln(24t + 1) + x_{w,t}^{\text{GOR}_0}$

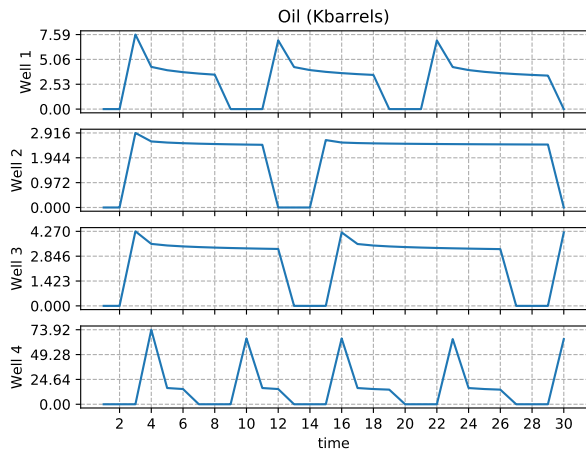


Fig. 3: Oil Rates for the Set of Four Cycled Wells.

all wells starting in healing mode. For the solution of this numerical example we observe the optimizer chooses to cycle well 4 with a higher frequency since it can produce significantly more oil than the other three wells.

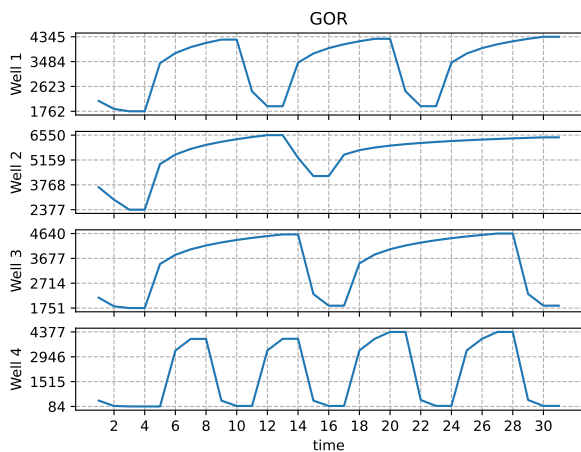


Fig. 4: Gas-Oil Ratios for a Set of Four Cycled Wells.

V. SUMMARY AND CONCLUSIONS

This paper investigated mathematical optimization of a well-cycling schedule for wells with a high GOR. Surrogate models for GOR, oil, and gas rates were introduced and discretized using the piecewise linearization strategy in Section III. The resulting optimization model was a mixed integer linear programming problem. The performance of the model was evaluated on a system with four wells and a scheduling horizon of 30 days. The primary contributions of this work are:

- The introduction of exponential and logarithmic surrogate models for predicting GOR, oil and gas rates.
- The inclusion of growth and healing models within a mixed-integer programming framework.

- The demonstration of numerical tractability of a MILP formulation discretization of time and the piecewise linearization of bilinear terms

Key areas of future work are the inclusion of uncertainty in the surrogate models as well as the dependencies across multiple wells. Here, we modeled the GOR of each well independently from the operating conditions of surrounding wells. Incorporating dependency across wells introduces coupling in the model and results in a more challenging optimization. In both aspects of future work, the challenge is to define more representative surrogate models while maintaining optimization tractability. For validation, the solution variables $y_{w,m,t}$ and $z_{w,m,t}$ obtained with CPLEX were fixed in the MINLP and the problem was solved using BARON. The solution in both cases was the same ensuring piecewise linearization was an adequate approximation.

REFERENCES

- [1] Toward prediction of petroleum reservoir fluids properties: A rigorous model for estimation of solution gas-oil ratio. *Journal of natural gas science and engineering*, 29:506–516, 2016.
- [2] F.E Ashford. An evaluation of critical multiphase flow performance through wellhead chokes. *Journal of petroleum technology*, 26(8):843–850, 1974.
- [3] Johannes Bisschop. *Aimms – optimization modeling*, 2003.
- [4] E Camponogara and A.M de Conto. Lift-gas allocation under precedence constraints: Milp formulation and computational analysis. 6(3):544–551, 2009.
- [5] IBM ILOG Cplex. V12. 10: Users manual for cplex. *International Business Machines Corporation*, 46(53):157, 2009.
- [6] Bjarne Foss. Process control in conventional oil and gas fields challenges and opportunities. *Control engineering practice*, 20(10):1058–1064, 2012.
- [7] Hamzeh Ghorbani, David A Wood, Jamshid Moghadasi, Abouzar Choubineh, Peyman Abdizadeh, and Nima Mohamadian. Predicting liquid flow-rate performance through wellhead chokes with genetic and solver optimizers: an oil field case study. *Journal of petroleum exploration and production technology*, 9(2):1355–1373, 2018.
- [8] W.E. Gilbert. Rigorous modeling of solution gasoil ratios for a wide ranges of reservoir fluid properties. *Drilling and Production Practice*, 13:126–157, 1954.
- [9] Gurobi Optimization, LLC. *Gurobi Optimizer Reference Manual*, 2021.
- [10] Svern Anton Halvorsen, Are Mjaavatten, and Robert Aasheim. Improved near-well approximation for prediction of the gas/oil production ratio from oil-rim reservoirs. *Computational geosciences*, 16(2):231–246, 2011.
- [11] Agus Hasan, Bjarne Foss, and Svein Sagatun. Optimization of oil production under gas coning conditions. *Journal of petroleum science & engineering*, 105:26–33, 2013.
- [12] Ahmed Farid Ibrahim, Redha Al-Dhaif, Salaheldin Elkatatny, and Dhafer Al Shehri. Applications of artificial intelligence to predict oil rate for high gasoil ratio and water-cut wells. *ACS omega*, 6(30):19484–19493, 2021.
- [13] M. J Kromah and R. A Dawe. Reduction of oil and gas coning effects by production cycling and horizontal wells. *Petroleum science and technology*, 26(3):353–367, 2008.
- [14] Are Mjaavatten, Robert Aasheim, Steinar Saelid, and Oddvar Grnning. A model for gas coning and rate-dependent gas/oil ratio in an oil-rim reservoir. *SPE reservoir evaluation & engineering*, 11(5):842–847, 2008.
- [15] M. (Morris) Muskat. *The flow of homogeneous fluids through porous media*. International series in physics. McGraw-Hill Book Company, inc., New York, 1st ed. edition, 1937.
- [16] N. C. J Ros. An analysis of critical simultaneous gas/liquid flow through a restriction and its application to flowmetering. *Applied Scientific Research*, 9(1):374–388, 1960.

- [17] M Safar Beiranvand, P Mohammadmoradi, B Aminshahidy, B Fazelabdolabadi, and S Aghahoseini. New multiphase choke correlations for a high flow rate iranian oil field. *Mechanical sciences (Gttingen)*, 3(1):43–47, 2012.
- [18] M. Tawarmalani and N. V. Sahinidis. A polyhedral branch-and-cut approach to global optimization. *Mathematical Programming*, 103:225–249, 2005.
- [19] Nour ElHouda Tellache, Meriem Waffa Hassen, Mohamed Otmanine, and Mohamed Khodja. Improved multiphase flow rate models for chokes in the algerian hmd oil field. *Arabian journal for science and engineering (2011)*, 46(7):6817–6833, 2020.
- [20] Ashutosh Tewari, Kuang-Hung Liu, and Dimitri Papageorgiou. Information-theoretic sensor planning for large-scale production surveillance via deep reinforcement learning. *Computers & Chemical Engineering*, 141:106988, 2020.
- [21] R.A Wattenbarger and C.H Urbanczyk. Optimization of well rates under gas coning conditions. *SPE advanced technology series*, 2(2):61–68, 1994.
- [22] Sohrab Zendehboudi, Ali Elkamel, Ioannis Chatzis, Mohammad Ali Ahmadi, Alireza Bahadori, and Ali Lohi. Estimation of breakthrough time for water coning in fractured systems: Experimental study and connectionist modeling. *AIChE journal*, 60(5):1905–1919, 2014.

VI. APPENDIX

A. MINLP Formulation

$$\begin{aligned}
\min \quad & \sum_{t \in \mathbb{T}} \sum_{w \in \mathbb{W}} x_{w,t}^{\text{OIL}} \\
\text{s.t.} \quad & \sum_{m \in \mathbb{M}_w} y_{w,m,t} = 1 && \forall w \in \mathbb{W}, t \in \mathbb{T} \\
& \sum_{m \in \mathbb{M}_w} z_{w,m,t} = z_{w,t} && \forall w \in \mathbb{W}, t \in \mathbb{T} \\
& \sum_{m' \in \mathbb{A}_{w,m}} z_{w,m',t} + y_{w,m,t} \geq y_{w,m,t-1} && \forall w \in \mathbb{W}, t \in \mathbb{T}, m \in \mathbb{M}_w \\
& \sum_{m \in \mathbb{G}_w} z_{w,m,t} \leq \sum_{m \in \mathbb{G}_w} y_{w,m,t'} && \forall w \in \mathbb{W}, t \in \mathbb{T}, t' \geq t + T_w^{\text{Gmin}} \\
& \sum_{m \in \mathbb{H}_w} z_{w,m,t} \leq \sum_{m \in \mathbb{H}_w} y_{w,m,t'} && \forall w \in \mathbb{W}, t \in \mathbb{T}, t' \geq t + T_w^{\text{Hmin}} \\
& \sum_{h \in \mathbb{H}_w} \sum_{t' \leq t + T_w^{\text{Gmax}}} y_{w,h,t'} \geq 1 && \forall w \in \mathbb{W}, t \in \mathbb{T} \setminus \{0\} \\
& \sum_{g \in \mathbb{G}_w} \sum_{t' \leq t + T_w^{\text{Hmax}}} y_{w,g,t'} \geq 1 && \forall w \in \mathbb{W}, t \in \mathbb{T} \setminus \{0\} \\
& \sum_{w \in \mathbb{W}} x_{w,t}^{\text{GAS}} \leq \bar{F}_t && \forall t \in \mathbb{T} \\
& z_{w,m,t} \leq y_{w,m,t} && \forall w \in \mathbb{W}, t \in \mathbb{T}, m \in \mathbb{M}_w \\
& z_{w,m,t} \leq 2 - y_{w,m,t-1} - y_{w,m,t} && \forall w \in \mathbb{W}, t \in \mathbb{T}, m \in \mathbb{M}_w \\
& \left[\begin{array}{l} z_{w,t} = 1 \\ x_{w,t}^{\text{GOR}_0} = x_{w,t-1}^{\text{GOR}} \\ a_{w,t} = 0 \end{array} \right] \vee \left[\begin{array}{l} z_{w,t} = 0 \\ x_{w,t}^{\text{GOR}_0} = x_{w,t-1}^{\text{GOR}_0} \\ a_{w,t} = a_{w,t-1} + 1 \end{array} \right] && \forall w \in \mathbb{W}, t \in \mathbb{T} \\
& \forall m \in \mathbb{M}_w \left[\begin{array}{l} y_{w,m,t} = 1 \\ x_{w,t}^{\text{GOR}} = r_{w,m}^{\text{GOR}}(a_{w,t}, x_{w,t}^{\text{GOR}_0}) \\ x_{w,t}^{\text{OIL}} = r_{w,m}^{\text{Oil}}(x_{w,t}^{\text{GOR}}) \\ x_{w,t}^{\text{GAS}} = r_{w,m}^{\text{Gas}}(x_{w,t}^{\text{GOR}}) \end{array} \right] && \forall w \in \mathbb{W}, t \in \mathbb{T} \\
& y_{w,m,0} = \bar{y}_{w,m} && \forall w \in \mathbb{W}, m \in \mathbb{M}_w \\
& a_{w,0} = \bar{a}_w && \forall w \in \mathbb{W} \\
& x_{w,0}^{\text{GOR}_0} = \bar{x}_w^{\text{GOR}_0} && \forall w \in \mathbb{W} \\
& y_{w,m,t}, z_{w,m,t} \in \{0, 1\} && \forall w \in \mathbb{W}, t \in \mathbb{T}, m \in \mathbb{M}_w \\
& z_{w,t} \in [0, 1] && \forall w \in \mathbb{W}, t \in \mathbb{T} \\
& x_{w,t}^{\text{GOR}_0}, x_{w,t}^{\text{GOR}}, x_{w,t}^{\text{OIL}}, x_{w,t}^{\text{GAS}}, a_{w,t} \in \mathbb{R}^+ && \forall w \in \mathbb{W}, t \in \mathbb{T}
\end{aligned} \tag{41}$$

B. MILP Formulation

$$\begin{aligned}
\min \quad & \sum_{t \in \mathbb{T}} \sum_{w \in \mathbb{W}} x_{w,t}^{\text{OIL}} \\
\text{s.t.} \quad & \sum_{m \in \mathbb{M}_w} y_{w,m,t} = 1 && \forall w \in \mathbb{W}, t \in \mathbb{T} \\
& \sum_{m \in \mathbb{M}_w} z_{w,m,t} = z_{w,t} && \forall w \in \mathbb{W}, t \in \mathbb{T} \\
& \sum_{m' \in \mathbb{A}_{w,m}} z_{w,m',t} + y_{w,m,t} \geq y_{w,m,t-1} && \forall w \in \mathbb{W}, t \in \mathbb{T}, m \in \mathbb{M}_w \\
& \sum_{m \in \mathbb{G}_w} z_{w,m,t} \leq \sum_{m \in \mathbb{G}_w} y_{w,m,t'} && \forall w \in \mathbb{W}, t \in \mathbb{T}, t' \geq t + T_w^{\text{Gmin}} \\
& \sum_{m \in \mathbb{H}_w} z_{w,m,t} \leq \sum_{m \in \mathbb{H}_w} y_{w,m,t'} && \forall w \in \mathbb{W}, t \in \mathbb{T}, t' \geq t + T_w^{\text{Hmin}} \\
& \sum_{h \in \mathbb{H}_w} \sum_{t' \leq t + T_w^{\text{Gmax}}} y_{w,h,t'} \geq 1 && \forall w \in \mathbb{W}, t \in \mathbb{T} \setminus \{0\} \\
& \sum_{g \in \mathbb{G}_w} \sum_{t' \leq t + T_w^{\text{Hmax}}} y_{w,g,t'} \geq 1 && \forall w \in \mathbb{W}, t \in \mathbb{T} \setminus \{0\} \\
& \sum_{w \in \mathbb{W}} x_{w,t}^{\text{GAS}} \leq \bar{F}_t && \forall t \in \mathbb{T} \\
& z_{w,m,t} \leq y_{w,m,t} && \forall w \in \mathbb{W}, t \in \mathbb{T}, m \in \mathbb{M}_w \\
& z_{w,m,t} \leq 2 - y_{w,m,t-1} - y_{w,m,t} && \forall w \in \mathbb{W}, t \in \mathbb{T}, m \in \mathbb{M}_w \\
& v_{w,m,t,0} = z_{w,m,t} && \forall w \in \mathbb{W}, m \in \mathbb{M}, t \in \mathbb{T} \setminus \{0\} \\
& v_{w,m,t,q} \leq v_{w,m,t-1,q-1} + (1 - y_{w,m,t}) && \forall w \in \mathbb{W}, t \in \mathbb{T} \setminus \{0\}, \mathbb{Q}_{w,m} \setminus \{0\} \\
& v_{w,m,t,q} \geq v_{w,m,t-1,q-1} - (1 - y_{w,m,t}) && \forall w \in \mathbb{W}, t \in \mathbb{T} \setminus \{0\}, \mathbb{Q}_{w,m} \setminus \{0\} \\
& \sum_{q \in \mathbb{Q}_{w,m}} v_{w,m,t,q} = y_{w,m,t} && \forall w \in \mathbb{W}, m \in \mathbb{M}, t \in \mathbb{T} \setminus \{0\} \\
& \sum_{p \in \mathbb{P}} u_{w,t,p} \hat{x}_{w,p}^{\text{GOR}} = x_{w,t}^{\text{GOR}} && \forall w \in \mathbb{W}, \forall t \in \mathbb{T} \\
& \sum_{p \in \mathbb{P}} u_{w,t,p} \hat{x}_{w,m,t}^{\text{OIL}} = \tilde{x}_{w,m,t}^{\text{OIL}} && \forall w \in \mathbb{W}, m \in \mathbb{M}, t \in \mathbb{T} \\
& \sum_{p \in \mathbb{P}} u_{w,t,p} \hat{x}_{w,m,t}^{\text{OIL}} \hat{x}_{w,p}^{\text{GOR}} = \tilde{x}_{w,m,t}^{\text{GAS}} && \forall w \in \mathbb{W}, m \in \mathbb{M}, t \in \mathbb{T} \\
& \sum_{p \in \mathbb{P}} u_{w,t,p} = 1 && \forall w \in \mathbb{W}, t \in \mathbb{T} \\
& \sum_{p \in \mathbb{P}} s_{w,t,p} = 1 && \forall w \in \mathbb{W}, t \in \mathbb{T} \\
& u_{w,t,p} \geq 0 && \forall w \in \mathbb{W}, t \in \mathbb{T}, p \in \mathbb{P} \\
& u_{w,t,0} \geq s_{w,t,1} && \forall w \in \mathbb{W}, t \in \mathbb{T} \\
& u_{w,t,p} \geq s_{w,t,p} + s_{w,t,p+1} && \forall w \in \mathbb{W}, t \in \mathbb{T}, p \in \mathbb{P} \setminus N \\
& u_{w,t,N} \geq s_{w,t,N} && \forall w \in \mathbb{W}, t \in \mathbb{T} \\
& \left[\begin{array}{c} z_{w,t} = 1 \\ x_{w,t}^{\text{GOR}_0} = x_{w,t-1}^{\text{GOR}} \end{array} \right] \vee \left[\begin{array}{c} z_{w,t} = 0 \\ x_{w,t}^{\text{GOR}_0} = x_{w,t-1}^{\text{GOR}_0} \end{array} \right] && \forall w \in \mathbb{W}, t \in \mathbb{T} \\
& \vee_{q \in \mathbb{Q}_{w,m}} \left[\begin{array}{c} v_{w,m,t,q} = 1 \\ x_{w,t}^{\text{GOR}} = r_{w,m}^{\text{GOR}}(q, x_{w,t}^{\text{GOR}_0}) \\ x_{w,t}^{\text{OIL}} = \tilde{x}_{w,m,t}^{\text{OIL}} \\ x_{w,t}^{\text{GAS}} = \tilde{x}_{w,m,t}^{\text{GAS}} \end{array} \right] && \forall w \in \mathbb{W}, t \in \mathbb{T}, m \in \mathbb{M} \\
& y_{w,m,0} = \bar{y}_{w,m} && \forall w \in \mathbb{W}, m \in \mathbb{M}_w \\
& x_{w,0}^{\text{GOR}_0} = \bar{x}_w^{\text{GOR}_0} && \forall w \in \mathbb{W} \\
& y_{w,m,t}, z_{w,m,t} \in \{0, 1\} && \forall w \in \mathbb{W}, t \in \mathbb{T}, m \in \mathbb{M}_w \\
& z_{w,t} \in [0, 1] && \forall w \in \mathbb{W}, t \in \mathbb{T} \\
& x_{w,t}^{\text{GOR}_0}, x_{w,t}^{\text{GOR}}, x_{w,t}^{\text{OIL}}, x_{w,t}^{\text{GAS}} \in \mathbb{R}^+ && \forall w \in \mathbb{W}, t \in \mathbb{T}
\end{aligned} \tag{42}$$