Asymptotically Stable Economic Nonlinear Model Predictive Control without Pre-Calculated Steady-State Optimum

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Abstract

Over the past decade, economic Nonlinear Model Predictive Control (eNMPC) has gained attention along with setpoint tracking NMPC for industrial applications. In particular, eNMPC is able to make future predictions by directly maximizing economic performance of the system. However, eNMPC usually requires a prior calculation of the steady state for stabilization, which compromises the benefit of dynamic real-time optimization. The impact of this may cause control delays in applications, especially when model parameters or disturbances are updated. In this work, we derive an eNMPC formulation with a stabilizing constraint that is composed of optimality conditions of the economic steady-state problem, over each stage in the prediction horizon. The proposed formulation reduces control delays as it does not require steady states for stabilization. Instead, the optimality conditions lead the process to the steady state automatically after the estimated information is updated in the predictive model. The advantage of this eNMPC formulation is demonstrated on a continuous stirred tank reactor (CSTR) example with parameter perturbations. Additionally, we also investigate the impact of terminal conditions on the stability of the proposed dynamic system.

Introduction

Model Predictive Control (MPC) has applications in a wide range of areas such as robotics, aerospace engineering, and chemical engineering for trajectory or setpoint tracking. This is due to MPC's capability to handle complicated multi-input and multi-output (MIMO) dynamic systems with operating constraints for states, inputs, and outputs. In the past decade, Nonlinear Model Predictive Control (NMPC) grew as a popular choice as it captures the detailed dynamics of nonlinear models in the process, thus giving more accurate state and output predictions.

With traditional control procedures for chemical units, we usually calculate the setpoint or the reference in the Real-Time Optimization (RTO) layer based on an economic goal and steady-state plant model (Marlin et al., 1997). Then the setpoint is passed to an advanced controller such as MPC or NMPC to calculate the optimal control action based on the current plant state and the predictive model. This is done by solving an optimization problem that minimizes the difference between predictive states and the calculated setpoint at each sampling time (Biegler et al., 2015). Next, the control action obtained from the controller is implemented in the plant followed by the collection of the resultant plant mea-surements. Once the measurements are obtained, they are sent to a real-time estimator to determine the current plant state and infer the model parameters. Finally, the control loop is closed by updating parameters within the RTO and controller models, which re-initializes the controller with the new states.

Ideally, the solution of RTO problems and MPC problems should have the same frequency; this assures that the tracking direction always approaches an optimal goal as the parameters are updated. However, solving RTO problems at every sampling time may delay the on-line control. As a result, RTO problems are usually solved to update the setpoint on an hourly or even daily basis while control inputs are updated at the frequency of a few seconds or minutes. Thus, while this RTO approach prevents online computational delay, it may lead to sub-optimal control actions due to the out-of-date set-point, especially when there is mismatch in parameters between the plant and the RTO model.

To address this problem, the extension of MPC to optimize economic performance has been explored in Rawlings and Amrit (2009), Würth et al. (2009), and Rawlings et al. (2012). The concept of economic dynamic real-time optimization (DRTO) and NMPC with economic objectives

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merges the RTO and the advanced control layers together, and eliminates the inconsistencies between two layers. Instead of minimizing the trajectory difference between the reference setpoint, eNMPC directly considers an economic stage cost that represents meaningful benefits such as material costs, product quantities, or some other performance measure. This concept has been applied in Aske et al. (2008), Würth et al. (2009), and Lin et al. (2022).

Although eNMPC implementations can improve economic performance in practice, steady-state stabilization of eNMPC continues to be a challenge. Steady-state stabilization is an important characteristic for the control of chemical processes as such processes often need to avoid state divergence and persistent oscillations. Since the economic stage cost can be any cost of interest and not always a $\mathcal{K}_\infty$ function, needed for control theory, eNMPC cannot guarantee asymptotic stability. This therefore leads to a Lyapunov function without sufficient decrease to a stationary point. It is in contrast to setpoint tracking NMPC, which has a $\mathcal{K}_\infty$ function as a (usually quadratic) stage cost that provides asymptotic stability with appropriate terminal conditions and horizon length.

Recent studies have shown important stability results for eNMPC strategies. Diehl et al. (2011) demonstrated that if a strictly dissipative dynamic system has a storage function and satisfies strong duality at the equilibrium point, asymptotic stability can be ensured by defining a rotated stage cost which is a $\mathcal{K}_\infty$ function. However, not all systems have this dissipativity property and it is difficult to check and generalize for large-scale systems, such as distillation columns and complex polymer processes. To construct a stabilizing strategy for more general systems, Jäschke et al. (2014) modified the economic stage cost to create a $\mathcal{K}_\infty$ function, by adding a tracking term with large enough penalty parameter. However, the drawback of this strategy is that the extra tracking term may dominate the objective function and lead to reduced economic performance. To remove the tracking term in the objective, Zavala (2015) proposed to interpret eNMPC as a multi-objective optimization problem by including a stabilizing inequality as a constraint, which inherits stability properties of tracking NMPC while enhancing the economic performance at the same time. Griffith et al. (2017) further analyzed the robustness properties of this eNMPC formulation and applied it to a large-scale distillation column. Although this formulation demonstrates improved performance, it requires a pre-calculated steady-state optimum as information for the stabilizing constraint, which compromises the benefit of dynamic real-time optimization. Specifically, if there are any parameter updates from the estimator, the steady-state optimizer needs to determine the new setpoint on-line. In order to resolve this disadvantage, we reformulate the stage cost and the Lyapunov function included in the stabilizing constraint with the optimality conditions instead of the optimal steady state. This allows the optimal steady state to be adjusted or updated automatically as model parameters or disturbances are estimated.

The paper is organized as follows. In Section 2 we briefly introduce the eNMPC formulation. In Section 3 we present the Lyapunov Theory for asymptotic stability and two novel strategies for stability. In Section 4, we derive the proposed eNMPC formulation with the optimality conditions of the steady-state problem. Section 5 demonstrates how we stabilize a continuous stirred tank reactor (CSTR), which is not asymptotically stable with standard eNMPC. Finally, Section 6 gives some conclusions and future perspectives.

Economic NMPC Formulation

Consider the dynamic system of a plant described as the following discrete-time dynamic model with uncertain parameters,

$$x_{k+1} = f(x_k, u_k, d_k) \tag{1}$$

where $x_k \in \mathbb{R}^{n_x}$ is the process state vector, $u_k \in \mathbb{R}^{n_u}$ is the control vector, and $d_k \in \mathbb{R}^{n_d}$ denotes the parametric uncertainty defined at time step t_k with integer time index $k \geq 0$. The plant states and controls satisfy the constraints $x_k \in \mathbb{X}$, and $u_k \in \mathbb{U}$. The dynamic function $f : \mathbb{R}^{n_x + n_u + n_d} \rightarrow \mathbb{R}^{n_x}$ maps the information at time k to the state vector at time $k + 1$. Equation (1) can then be implemented within a NMPC problem that minimizes either the deviation from a setpoint or an economic cost, i.e.,

$$V(x_k) = \min_{v_l, z_l} F^\gamma(z_N) + \sum_{l=0}^{N-1} \psi^\gamma(z_l, v_l)$$

$$\text{s.t.} \ z_{l+1} = f(z_l, v_l, \hat{d}_k), \quad l = 0, ..., N-1 \tag{2a}$$

$$z_0 = x_k, \tag{2b}$$

$$z_l \in \mathbb{X}, \ v_l \in \mathbb{U}, \ z_N \in \mathbb{X}_f \tag{2c}$$

where v_l, z_l are the predicted control and state variables, respectively, at point l in the predictive horizon of length N, and $\gamma \in \{tr, ec\}$ denotes either the quadratic tracking stage cost or the economic stage cost considered in the objective of NMPC problem (2). F^γ is the terminal cost and $\hat{d}_k$ represents the estimate of uncertain parameter at time k.

Lyapunov Stability Theory and Previous Stabilizing Strategies

Definition 1 (Comparison Functions). *A function $\alpha : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is of class $\mathcal{K}$ if it is continuous, strictly increasing, and $\alpha(0) = 0$. A function $\alpha : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is of class $\mathcal{K}_\infty$ if it is a $\mathcal{K}$ function and $\lim_{s \rightarrow \infty} \alpha(s) = \infty$.*

Definition 2 (Lyapunov Function). *A function $V : \mathbb{X} \rightarrow \mathbb{R}_+$ that satisfies the following:*

$$\alpha_1(|x|) \leq V(x) \leq \alpha_2(|x|) \tag{3a}$$

$$V(x_k) - V(x_{k-1}) \leq -\alpha_3(|x_{k-1}|) \tag{3b}$$

where $\alpha_1, \alpha_2, \alpha_3 \in \mathcal{K}_\infty$ is said to be a Lyapunov Function for Eq. (1).

Theorem 1. *If system (1) admits a Lyapunov function for some $\alpha_1, \alpha_2, \alpha_3 \in \mathcal{K}_\infty$ and*

$$F^\gamma(z_{N|k}) - F^\gamma(z_{N|k-1}) \leq -\bar{\psi}^\gamma(z_{N|k-1}, v_{N|k-1}) \tag{4}$$

holds, then (1) is asymptotically stable on $\mathbb{X}$.

For setpoint tracking NMPC, Lyapunov function $V(x_k)$ is usually chosen as the optimal objective function with $\alpha_3(|x_{k-1}|) = \psi^{tr}(x_{k-1})$ in Eq. (3b), satisfying Definition 2 and thus resulting in asymptotic stability based on Theorem 1. On the other hand, since the economic stage cost ψ^{ec} of eNMPC can take any form of economic measures, Definition 2 does not necessarily hold if we continue to select the economic objective of (2) as the Lyapunov function. Therefore, additional work is required to stabilize eNMPC.

eNMPC stability can be established if the system satisfies the turnpike property proposed by Faulwasser et al. (2017) or if the system is dissipative as proved in Diehl et al. (2011). However, these properties are system-specific and can be difficult when applied in practice. Instead, Jäschke et al. (2014) derived a constructive way to calculate large enough tracking terms and modified the stage cost to ensure Definition 2 holds, as follows:

$$\psi^{mod}(z_l, v_l) = \psi^{ec}(z_l, v_l) + \rho \psi^{tr}(z_l, v_l) \quad (5)$$

where $\psi^{tr} = ||z_l - x_{ss}||^2 + ||v_l - u_{ss}||^2$ and $\rho > 0$ is a sufficiently large constant.

Although (5) helps to stabilize the closed-loop control of eNMPC, a large ρ limits economic performance and leads to conservative control. Instead, Zavala (2015) and Griffith et al. (2017) removed the tracking cost from the objective and included a stabilizing constraint accounting for Eq. (3b) in the eNMPC problem. We denote this eNMPC formulation as eNMPC-sc.

$$\min_{v_l, z_l} F^{tr}(z_N) + \sum_{l=0}^{N-1} \psi^{ec}(z_l, v_l)$$

$$\text{s.t. } z_{l+1} = f\left(z_l, v_l, \hat{d}_k\right), \quad l = 0, ..., N-1 \quad (6a)$$

$$z_0 = x_k, \quad (6b)$$

$$V(x_k) - V(x_{k-1}) \le -\delta \, \psi^{tr}(x_{k-1}, u_{k-1}), \, \delta \in (0,1] \quad (6c)$$

$$z_l \in \mathbb{X}, \, v_l \in \mathbb{U}, \, z_N \in \mathbb{X}_f \quad (6d)$$

where the Lyapunov function is defined with tracking stage cost $V(x_k) := F^{tr}(z_N) + \sum_{l=0}^{N-1} \psi^{tr}(z_l, v_l)$. $V(x_{k-1})$ and $\psi^{tr}(x_{k-1}, u_{k-1})$ are the Lyapunov function and the first tracking stage cost calculated at time $k-1$, respectively. This formulation preserves the economic cost in the objective function, and δ in Eq. (6c) can be adjusted to balance stability of the system with dynamic performance. However, to calculate ψ^{tr}, both strategies must calculate the optimal steady state (x_{ss}, u_{ss}). This additional computational burden undermines the purpose of eNMPC, which is to eliminate the need for steady-state optimization. Moreover, frequent updates to input disturbances and model parameters will require frequent recalculation of (x_{ss}, u_{ss}). In the next section we describe an eNMPC approach that does not need (x_{ss}, u_{ss}).

Proposed eNMPC Formulation without Pre-Calculated Steady States

In order to derive the formulation of eNMPC without the requirement of steady states for asymptotic stability, we first consider the following steady-state problem:

$$(x_{ss}, u_{ss}) := \operatorname*{argmin}_{x,u} \, \psi^{ec}(x,u) \quad (7a)$$

$$\text{s.t. } x = f(x,u) \quad (7b)$$

$$g(x,u) \le 0 \quad (7c)$$

Using barrier functions to take care of inequalities (7c), we reformulate the steady-state problem as,

$$\min_{x,u} \, \bar{\psi}^{ec}(x,u) = \psi^{ec}(x,u) - \mu \sum_{j=1}^{n_g} ln(-g_j(x,u)) \quad (8a)$$

$$\text{s.t. } x = f(x,u) \quad (8b)$$

where $\mu > 0$ is the barrier parameter.

The barrier function can approach the optimum without constraint violations by setting μ small in the objective. We then derive the Karush-Kuhn-Tucker (KKT) conditions of (8) as follows.

$$\nabla_u f(x,u)(I - \nabla_x f(x,u))^{-1} \nabla_x \bar{\psi}^{ec}(x,u) + \nabla_u \bar{\psi}^{ec}(x,u) = 0 \quad (9)$$

$$x - f(x,u) = 0 \quad (10)$$

It is important to note that the KKT conditions are sufficient if the Strong Second Order Sufficient Condition (SSOSC) and Linear Independent Constraint Qualification (LICQ) are satisfied. The latter is satisfied if $(I - \nabla_x f(x,u))^{-1}$ is always nonsingular. For convenience, we define,

$$Z(x,u)^T = [\nabla_u f(x,u)(I - \nabla_x f(x,u))^{-1} \, | \, I] \quad (11)$$

where $Z(x,u)$ is a null space representation for $\nabla(x - f(x,u))^T$. With Eqs. (9), (10), and (11), we can define a new stage cost $\bar{\psi}^{tr}(x_k, u_k)$ and Lyapunov function $\bar{V}(x_k)$ for stability as follows.

$$\bar{\psi}^{tr}(x_k, u_k) := ||f(x_k, u_k) - x_k||^2 + ||Z(x_k, u_k)^T \nabla \bar{\psi}^{ec}(x_k, u_k)||^2 \quad (12)$$

$$\bar{V}(x_k) := \sum_{l=0}^{N-1} \left(||z_{l+1} - z_l||^2 + ||Z(z_l, v_l)^T \nabla \bar{\psi}^{ec}(z_l, v_l)||^2 \right) + \bar{F}^{tr}(z_N) \quad (13)$$

Since $\bar{\psi}^{tr}(x_k, u_k)$ is now a $\mathcal{K}_\infty$ function, the descent property (3b) holds for Lyapunov Theory. These equations yield our new eNMPC formulation with stability property:

$$\min_{v_l, z_l} \bar{F}^{tr}(z_N) + \sum_{l=0}^{N-1} \psi^{ec}(z_l, v_l)$$

$$\text{s.t. } z_{l+1} = f\left(z_l, v_l, \hat{d}_k\right), \quad l = 0, ..., N-1 \quad (14a)$$

$$z_0 = x_k, \quad (14b)$$

$$\bar{V}(x_k) - \bar{V}(x_{k-1}) \le -\delta \, \bar{\psi}^{tr}(x_{k-1}, u_{k-1}), \, \delta \in (0,1] \quad (14c)$$

$$z_l \in \mathbb{X}, \, v_l \in \mathbb{U}, \, z_N \in \mathbb{X}_f \quad (14d)$$

where $\bar{V}(x_{k-1})$ and $\bar{\psi}^{tr}(x_{k-1}, u_{k-1})$ are the new Lyapunov function and the new first stage cost evaluated at $k-1$. In contrast to eNMPC-sc (6), the stabilizing constraint (14c) is composed of the optimality conditions of the steady-state problem (8). Therefore, it does not require the exact steady state for dynamic system stabilization.

For the finite horizon MPC, terminal conditions are necessary to account for the events beyond the horizon at each iteration. Either the terminal cost or terminal constraint or both are included in the MPC problem to ensure that the state at the end of the horizon is inside the terminal region and that recursive feasibility holds. However, most terminal conditions are constructed based on the stationary point, which we no longer need to include in the proposed eNMPC formulation. Therefore, we exploit the KKT conditions of Eqs. (9) and (10) again formulate a strict terminal region defined by the endpoint equalities and the terminal cost as follows.

$$x_N - x_{N-1} = 0 \tag{15a}$$

$$Z(x_N, u_{N-1})^T \nabla \bar{\psi}^{ec}(x_N, u_{N-1}) = 0 \tag{15b}$$

$$\bar{F}^{tr}(z_N) = \beta(\|x_N - x_{N-1}\|^2 + \|Z(x_N, u_{N-1})^T \nabla \bar{\psi}^{ec}(x_N, u_{N-1})\|^2) \tag{16}$$

where $\beta > 0$ is a sufficiently large weighing parameter. In our case study, we investigate the impact of terminal conditions by comparing the cases with and without terminal conditions.

Stability Property of Proposed eNMPC

Consider the stabilizing constraint (14c) with new defined Lyapunov function $\bar{V}$ and stage cost $\bar{\psi}^{tr}$. We rearrange the constraint and sum up both sides from $k = 1$ to $k = \infty$, which leads to,

$$\sum_{k=1}^{\infty} \delta \, \bar{\psi}^{tr}(x_k, u_k) \leq \sum_{k=1}^{\infty} (\bar{V}(x_{k-1}) - \bar{V}(x_k)) \tag{17}$$

After canceling the repeated terms on the right hand side, we can obtain the following inequality,

$$\sum_{k=1}^{\infty} \delta \, \bar{\psi}^{tr}(x_k, u_k) \leq \bar{V}(x_0) - \bar{V}(x_\infty) \leq \bar{V}(x_0). \tag{18}$$

Since this infinite sum is bounded above, we obtain,

$$\lim_{k \to \infty} \bar{\psi}^{tr}(x_k, u_k)$$
$$= \|f(x_k, u_k) - x_k\|^2 + \|Z(x_k, u_k)^T \nabla \bar{\psi}^{ec}(x_k, u_k)\|^2 = 0. \tag{19}$$

Thus, asymptotic stability holds and we converge to a KKT point of (8).

Simulation Example

We consider a nonlinear continuously stirred tank reactor (CSTR) from Diehl et al. (2011), where reaction $A \to B$ occurs.

$$\frac{dC_A}{dt} = \frac{q}{V}(C_{Af} - C_A) - kC_A \tag{20}$$

$$\frac{dC_B}{dt} = \frac{q}{V}(-C_B) + kC_A \tag{21}$$

C_A and C_B represent the concentrations of chemicals A and B, respectively, in mol/L. The control input is the flowrate of reactor q in L/min. The volume of the reactor V is $10\,L$, the rate constant of reaction k is $1.2\,L/(mol \cdot min)$, and the feed concentration of component A C_{Af} is $1\,mol/L$. The economic stage cost is given by the price of C_B and a separation cost, which are proportional to the flowrate q as defined in Diehl et al. (2011),

$$\psi^{ec}(C_A, C_B, q) = -q(2C_B - \frac{1}{2}) \tag{22}$$

with the steady-state values $(C_{Ass}, C_{Bss}, q_{ss}) = (0.5, 0.5, 12)$ and economic cost $\psi_{ss}^{ec} = -6$. Initial conditions of states are chosen as $C_{A0} = 0.1$ and $C_{B0} = 1.0$.

The continuous model consisting of Eqs. (20) and (21) is discretized by Lagrange-Radau collocation method on finite elements and included as the predictive model in eNMPC optimization problems (2) and (14), which are constructed in the optimization modeling platform Pyomo 6.0.2 (Bynum et al., 2021) and solved by the NLP solver IPOPT 3.12 (Wächter and Biegler, 2006). All simulations are run on an HPE-180t desktop operating on Ubuntu 18.04.3 with an Intel Core i7-930 CPU.

Regular eNMPC without Stabilization

In the first case, we present the result of the standard eNMPC of Eq. (2) with $\gamma = ec$ and the horizon length $N = 10$. In addition, we include neither terminal cost nor terminal constraint in the optimization problem. As shown in Figure 1, the dynamic system controlled is not asymptotically stable. The optimization-based controller obtains the maximum economic performance under a periodic cycle, which causes the states of C_A and C_B to oscillate during the entire process. Furthermore, compared to the setpoint tracking NMPC, the negative economic stage cost cannot provide a qualified Lyapunov function for Eq. (3a) to achieve asymptotic stability.
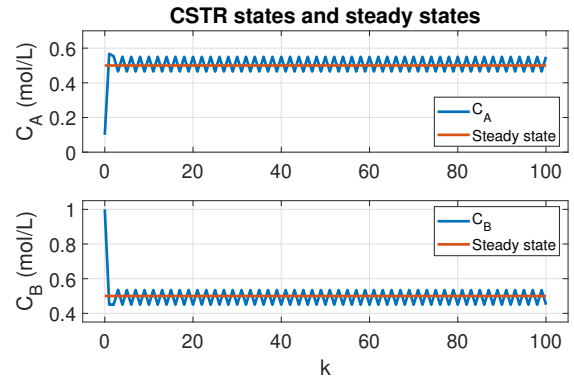

Figure 1: *Concentrations of A (top) and B (bottom) of CSTR under the closed-loop control by standard eNMPC.*

Stable eNMPC without Terminal Conditions

In the second case, the proposed eNMPC formulation of Eq. (14) with $\delta = 1$ is implemented for the CSTR, but the terminal conditions are not included in the optimization problem. We simulate the closed-loop results with different horizon length where $N = 10, 20, 30$. In the top of Figure 2, the state C_A appears to converge to its steady state. However, as we zoom in the state trajectory of C_A between $k = 60 - 100$ shown at the bottom of Figure 2, we note that C_A does not converge to the exact steady state, but to some "neighborhood" of the steady state. This is mainly because we do not

have terminal conditions in this case study. To be more specific, the important stability condition of terminal cost (4) is not considered when the terminal conditions are ignored.
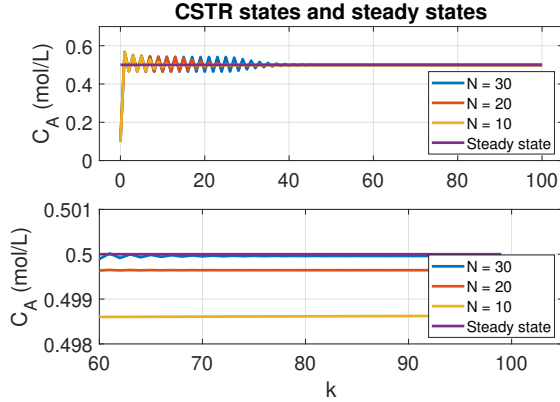

Figure 2: Concentration of A (top) and its enlargement (bottom) of CSTR under the closed-loop control by the proposed eNMPC without terminal conditions.

Stable eNMPC with Endpoint Constraints

In the third case, we include Eqs. (15a) and (15b) as endpoint constraints in the proposed stable eNMPC with $\delta = 1$ to satisfy Eq. (4). We run the simulations of closed-loop control with the horizon length $N = 10, 20, 30$. As presented in Figure 3, the concentration of A converges to the exact steady state optimum no matter how long the horizon is, in contrast to the previous results. The offset from the steady state is eliminated because the endpoint constraints ensure the states satisfy the KKT conditions at the end of the horizon, which makes Eq. (4) hold, with zero on both left and right hand sides.
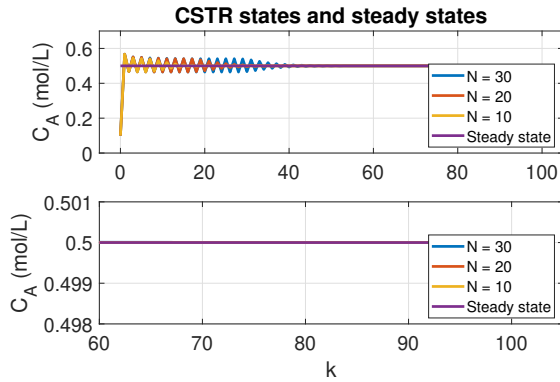

Figure 3: Concentration of A (top) and its enlargement (bottom) of CSTR under the closed-loop control by the proposed eNMPC with endpoint constraints.

Stable eNMPC with Quadratic Terminal Cost

In the fourth case, we include the terminal cost of Eq. (16) in the proposed eNMPC with $N = 10$. A large enough β is chosen to be 10^6. The states of CSTR, C_A and C_B, converge to their steady states as shown in Figure 4. Additionally, we investigate the effect of δ on the performance of the proposed eNMPC, which controls the rate of convergence in Eq. (14c). We run three simulations with different values of $\delta = 0.3, 0.6, 1.0$. In Table 1, we compare the economic

performance by summing up the scaled economic stage cost over 100 iterations of the closed-loop results. We notice that when δ is smaller, the oscillation lasts longer and the state converges to the steady state slower. However, we also observe that the sum of costs reveals a monotonic relationship with δ, where smaller δ results in better economic performance. This is due to the longer period of oscillations to the steady state. Generally, the proposed eNMPC allows the system to explore the periodic cycles to gain the best economic performance while the stabilizing constraint drives the state toward its optimal stationary point. The tradeoff between dynamic economic performance and stability is controlled by the parameter of convergence rate δ, which can be adjusted depending on the application in practice.
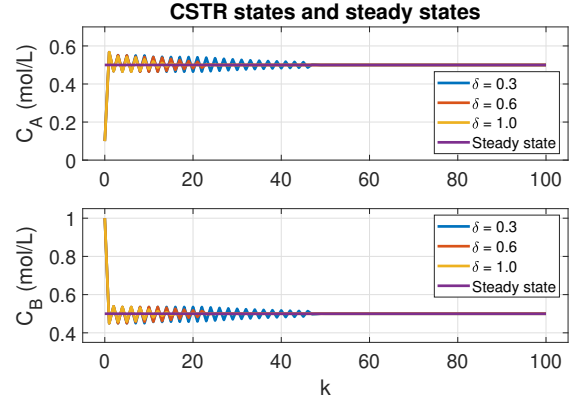

Figure 4: Concentrations of A (top) and B (bottom) of CSTR under the closed-loop control by the proposed eNMPC with terminal cost.

Table 1: Economic performance of proposed eNMPC with different δ

Proposed eNMPC	$\sum_{k=1}^{100} \psi^{ec}(x_k, u_k) - \psi_{ss}^{ec}$
$\delta = 0.3$	-27.0128
$\delta = 0.6$	-23.2065
$\delta = 1.0$	-21.3178

Stable eNMPC with Input Parameter Updates

In the last case, we simulate the closed-loop control of CSTR with the proposed eNMPC and quadratic terminal cost. The horizon length is $N = 10$ and $\delta = 1$. We demonstrate the main advantage of the proposed eNMPC by perturbing one of the parameters, C_{Af}, from 1.0 to 1.2 at $k = 50$. As shown in Figure 5, the system continues to be asymptotically stable even after the parameter perturbation. Both states converge to the first steady state after about 20 horizons. After the parameter C_{Af} is updated in the predictive model at $k = 50$, two states oscillate for a few iterations and then eventually converge to the second steady state. The stabilizing constraint of Eq. (14c) is temporarily deactivated at $k = 50$ in order to re-calculate the non-zero Lyapunov function due to the parameter update and then it is reactivated at $k = 51$. It is important to note that we do not need to solve for any steady-state optima explicitly during the whole process because once the parameter is updated in the model the optimality conditions considered in $\bar{V}$ and $\bar{\psi}^{tr}$ will reflect the new steady state automatically and the controller will stabilize the system by driving the state to it.
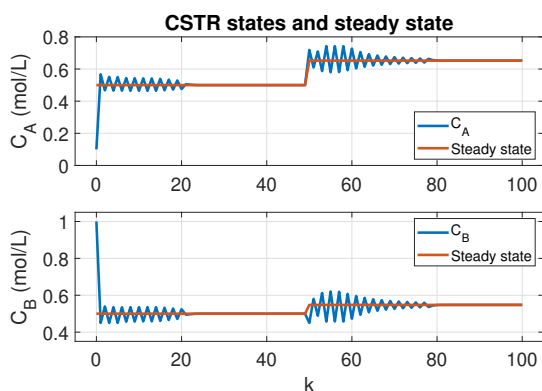
Figure 5: Concentrations of A (top) and B (bottom) of CSTR under the closed-loop control by the proposed eNMPC with parameter update at k = 50.

Conclusions and Future Work

In this work, we formulate a new economic NMPC without the requirement of pre-calculated steady states. This formulation ensures asymptotic stability of the closed-loop control by constructing a stabilizing constraint with optimality conditions of the steady-state problem. Therefore, once parameters or disturbances are estimated and updated in the dynamic model, the plant state is driven to the new steady state automatically. In addition, this formulation does not modify the economic stage cost, which allows the NMPC controller to give the best economic performance within the feasible region defined by the stabilizing constraint and the convergence rate parameter δ. We also investigate the effect of terminal conditions in the case study by comparing the results with either the terminal cost or endpoint constraints, as well as without any terminal conditions.

Our continuing work focuses on applying the proposed eNMPC formulation to large-scale dynamic models for distillation and membrane reactor systems. For these applications modern AD and sensitivity tools facilitate and simplify the tasks of processing gradients for large Differential Algebraic Equation (DAE) systems and assembling the KKT conditions in Eq. (13). These tools allow us to build efficient strategies to formulate and apply our proposed eNMPC approach to solve challenging dynamic applications in real-time.

In addition, we have observed satisfying results for stability with current terminal cost and endpoint constraints, and we are developing more general terminal conditions that are less strict. An example of this would be to define a terminal region that would enclose the end states for each horizon. Again, the anticipated advantage would be to avoid computing steady state optima for these terminal conditions.

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