Petroleum production optimization – a static or dynamic problem?

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Abstract: This paper considers the upstream oil and gas domain, or more precisely the daily production optimization problem in which production engineers aim to utilize the production systems as efficiently as possible by for instance maximizing the revenue stream. This is done by adjusting control inputs like choke valves, artificial lift parameters and routing of well streams. It is well known that the daily production optimization problem is well suited for mathematical optimization. The contribution of this paper is a discussion on appropriate formulations, in particular the use of static models vs. dynamic models. We argue that many important problems can indeed be solved by repetitive use of static models while some problems, in particular related to shale gas systems, require dynamic models to capture key process characteristics. The reason for this is how reservoir dynamics interacts with the dynamics of the production system.

Keywords: production optimization; oil and gas; shale gas; formulations; dynamic models.

1. INTRODUCTION

Petroleum assets imply sizable amounts of hydrocarbons which are trapped in appropriate underground geological structures. After discovering and deciding on developing an asset wells are drilled and production starts. Typically the production lifetime of an asset contains three stages; the ramp-up, plateau and decline phases. During the ramp-up phase new wells are drilled and completed while the production rate steadily increases. During the plateau phase production stays fairly constant. New wells may, however, still be drilled while others are abandoned. Typically reservoir fluid composition changes with more and more less valuable products like water entering the production stream. Further, fluids may be injected to maintain reservoir pressure. During the decline phase production can no longer be maintained, thus production rates drop.

Asset production planning is performed on different horizons from a life-cycle perspective to daily production planning where decisions may be described through a control hierarchy as shown in Fig. 1. The uppermost level includes life-cycle related decisions such as selecting an investment strategy, appropriate technologies and an operations model.

The second highest level, level 2 in Fig. 1 typically refers to decision horizons of one to five years, even though this may vary significantly, and includes choices on production strategies. This includes drilling schedules, the location and completion design of new wells, injection rates and fluids, and target production rates to mention a few. Level 2 decisions are usually supported by simulator studies using high fidelity reservoir models.

Level 3 takes us to the operations domain since the planning horizon ranges from a few hours to a week, thus

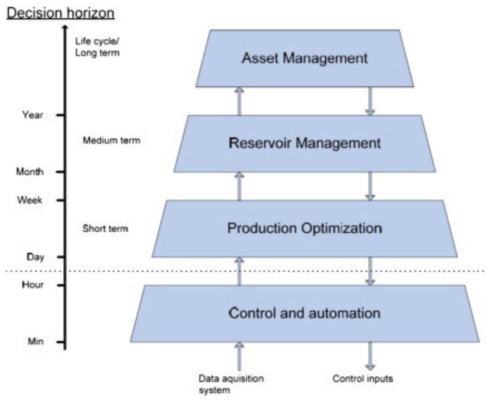

Fig. 1. A multilevel control hierarchy

from a process systems perspective this is equivalent to real-time optimization (RTO). We denote this by Daily Production Optimization (DPO). There are two important contrasts between level 2 and level 3 formulations. First, the shorter time horizon most often allows for the use simple reservoir models on level 3, and, second it is critical to include the production network in DPO formulations. The reason is that DPO production bottlenecks normally are found both in the reservoir as well as in the network.

The lowest level in Fig. 1 includes an automatic control systems which normally is implemented in proprietary control systems. This includes control loops for flow, pressure and level control functions to mention some of its functionality. It may also be noted that there will always exist a separate safety system, which is automatically activated in emergency situations.

There is a clear business case for mathematical optimization in DPO in the sense that decision support systems based on this methodology reports production increases in the range of 1-4% (Stenhouse et al., 2010; Teixeira et al., 2013). These improvements are more pronounced for fields in the late plateau and decline phases than earlier since the DPO bottleneck structure tends to become more complicated with time, e.g. due to increased water and gas production, and reduced reservoir pressure.

This paper contributes to *DPO formulations* by discussing static and dynamic formulations. The paper continues with an efficient formulation of the DPO problem, which is new and captures a broad class of problems since it includes both static as well as dynamic reservoir models. Examples of application classes in which a static or a dynamic formulation is appropriate are given. This section includes a new case study in addition to references to earlier cases, and forms the basis for a discussion on benefits and limitations of the static and dynamic formulations. Towards the end some conclusions are presented.

2. A GENERAL DPO FORMULATION

Consider the petroleum production system illustrated in Fig. 2, in which the produced fluid flows through well bores, manifolds, and flowlines, to finally enter the separators. At the separators the fluid phases, typically including oil, gas and water, are separated. It may be noted that the process diagram for the processing section in practice is far more complicated than shown in Fig. 2. Oil and/or gas are exported separately through transmission pipelines. One important feature of the production system is that each well flow can be routed to one of the flowlines by configuring the on/off valves in the manifold.

Upstream of the wells lies the reservoir. Thus, the reservoir defines the inflow boundary conditions on the production system. The downstream boundary conditions are given by the (nearly) constant pressure in the separators, which is maintained by regulatory control. In this paper we will mainly be concerned with production systems where the upstream and downstream boundaries are placed in the reservoir and the inlet separator, respectively.

Table 1. Utility sets

Set	Description
$\mathbf{N}$	Set of nodes $i \in \mathbf{N}$.
$\mathbf{E}$	Set of edges $e = (i,j) \in \mathbf{E}$, with $i, j \in \mathbf{N}$.
$\mathbf{K}$	Timesteps $\mathbf{K} = \{0, 1, \dots, N\}$. $\mathbf{K}^- = \mathbf{K} \setminus N$.
$\mathbf{E}_i^{\text{in}}$	Edges entering node i, i.e. $\mathbf{E}_i^{\text{in}} = \{e : e = (j,i) \in \mathbf{E}\}$.
$\mathbf{E}_i^{\text{out}}$	Edges leaving node i, i.e. $\mathbf{E}_i^{\text{out}} = \{e : e = (i,j) \in \mathbf{E}\}$.
$\mathbf{E}^{\text{snk}}$	Edges entering a sink node, i.e. $\mathbf{E}^{\text{snk}} = \{e : e = (i,j),\ \mathbf{E}_j^{\text{out}} = \emptyset\}$.
$\mathbf{E}^{\text{d}}$	Set of discrete edges, i.e. $\mathbf{E}^{\text{d}} \subset \mathbf{E}$.
$\mathbf{N}^{\text{d}}$	Nodes with discrete leaving edges, i.e. $\mathbf{N}^{\text{d}} = \{i : i \in \mathbf{N}, \mathbf{E}_i^{\text{out}} \subset \mathbf{E}^{\text{d}}\} \subset \mathbf{N}$.
$\mathbf{R}$	Set of phases - oil, gas and water $\{\text{oil}, \text{gas}, \text{wat}\}$.

We can now formulate a fairly general network optimization problem. The topology of the network is represented by a directed graph $G = (\mathbf{N}, \mathbf{E})$, with nodes $\mathbf{N}$ and edges $\mathbf{E}$ (Ahuja et al., 1993). In the sequel we adopt the notation in Grimstad et al. (2015). There are three mutually exclusive sets of nodes, $\mathbf{N}$, which all represent a junction: source nodes ($\mathbf{N}^{\text{src}}$), sink nodes ($\mathbf{N}^{\text{snk}}$) and intermediate nodes ($\mathbf{N}^{\text{int}}$), the latter representing junctions in the graph. An edge $\mathbf{E}$ connects two nodes and represents a pipe segment such as a well bore or a flowline, a valve, or an active element like a pump. A subset of edges, $\mathbf{E}^{\text{d}}$, represents chokes and on/off valves. These edges have two states: either open or closed. Thus, discrete edges are used to route the flow through the network by restricting the flow through the valve. It is advantageous to define certain utility sets, and certain requirements need to be placed on the graph structure, cf. Grimstad et al. (2015). Some utility sets are defined in Table 1 in order to compactify notation.

Table 2. Variables

Variable	Description
p_{ik}	Pressure in node $i \in \mathbf{N}$ at time $k \in \mathbf{K}$.
q_{rek}	Flow rate of phase $r \in \mathbf{R}$ on edge $e \in \mathbf{E}$ at time $k \in \mathbf{K}$.
Y_{ek}	Boolean variable associated with edge $e \in \mathbf{E}^{\text{d}}$ at time $k \in \mathbf{K}$. The edge may be open (Y_{ek} = True) or closed (Y_{ek} = False).

Alternative formulations like compositional models instead of three phases is sometimes necessary, especially for condensate reservoirs. The variables of the problem formulation are listed in Table 2. Note that the flow rates $\{q_{rek}\}$ are given as mass flow rates or as volumetric flow rates at standard conditions. For brevity, the phase flow rates on an edge $e \in \mathbf{E}$ at time $k \in \mathbf{K}$ are collectively denoted $\mathbf{q}_{ek}$, that is, with an oil, gas, and water phase, $\mathbf{q}_{ek} = [q_{\text{oil},ek}, q_{\text{gas},ek}, q_{\text{wat},ek}]^{\mathsf{T}}$.

The DPO problem is posed as the following general disjunctive programming (GDP) problem, denoted $\mathbf{P}$:

$$\text{maximize} \quad z = \sum_{r \in \mathbf{R}} \sum_{e \in \mathbf{E}^{\text{snk}}} \sum_{k \in \mathbf{K}} g_{rk}(q_{rek}) \tag{1}$$

subject to

$$\sum_{e \in \mathbf{E}_i^{\text{in}}} q_{rek} = \sum_{e \in \mathbf{E}_i^{\text{out}}} q_{rek}, \qquad \forall r \in \mathbf{R}, i \in \mathbf{N}^{\text{int}}, k \in \mathbf{K} \tag{2}$$

$$\zeta_{rik}(\mathbf{q}_{e,k+1}, \mathbf{q}_{ek}, p_{ik}) = 0, \quad \forall r \in \mathbf{R}, i \in \mathbf{N}^{\text{src}}, k \in \mathbf{K}^- \tag{3}$$

$$\mathbf{q}_{e,0} = \text{given}, \qquad \forall i \in \mathbf{N}^{\text{src}} \tag{4}$$

$$p_{ik} = \text{const.}, \qquad \forall i \in \mathbf{N}^{\text{snk}}, k \in \mathbf{K} \tag{5}$$

$$p_{ik} - p_{jk} = f_e(\mathbf{q}_{ek}, p_{ik}), \qquad \forall e \in \mathbf{E} \setminus \mathbf{E}^{\text{d}}, k \in \mathbf{K} \tag{6}$$

$$Y_{ek} \implies p_{ik} - p_{jk} = f_e(\mathbf{q}_{ek}, p_{ik}), \qquad \forall e \in \mathbf{E}^{\text{d}}, k \in \mathbf{K} \tag{7}$$

$$\neg Y_{ek} \implies \mathbf{q}_{ek} = 0, \qquad \forall e \in \mathbf{E}^{\text{d}}, k \in \mathbf{K} \tag{8}$$

$$\left(\underset{e \in \mathbf{E}_i^{\text{out}}}{\veebar} Y_{ek} \right) \vee \left(\bigwedge_{e \in \mathbf{E}_i^{\text{out}}} \neg Y_{ek} \right), \qquad \forall i \in \mathbf{N}^{\text{d}}, k \in \mathbf{K} \tag{9}$$

$$Y_{ek} \in \{\text{True}, \text{False}\}, \qquad \forall e \in \mathbf{E}^{\text{d}}, k \in \mathbf{K} \tag{10}$$

$$\sum_{e \in \mathbf{E}^{\text{snk}}} q_{rek} \le C_{rk}, \qquad \forall r \in \mathbf{R}, k \in \mathbf{K} \tag{11}$$

The objective function (1) is a sum of univariate, possibly nonlinear functions of flow rate. This general form allows for the inclusion of various cost and penalty terms. A particularly common objective in operational settings is

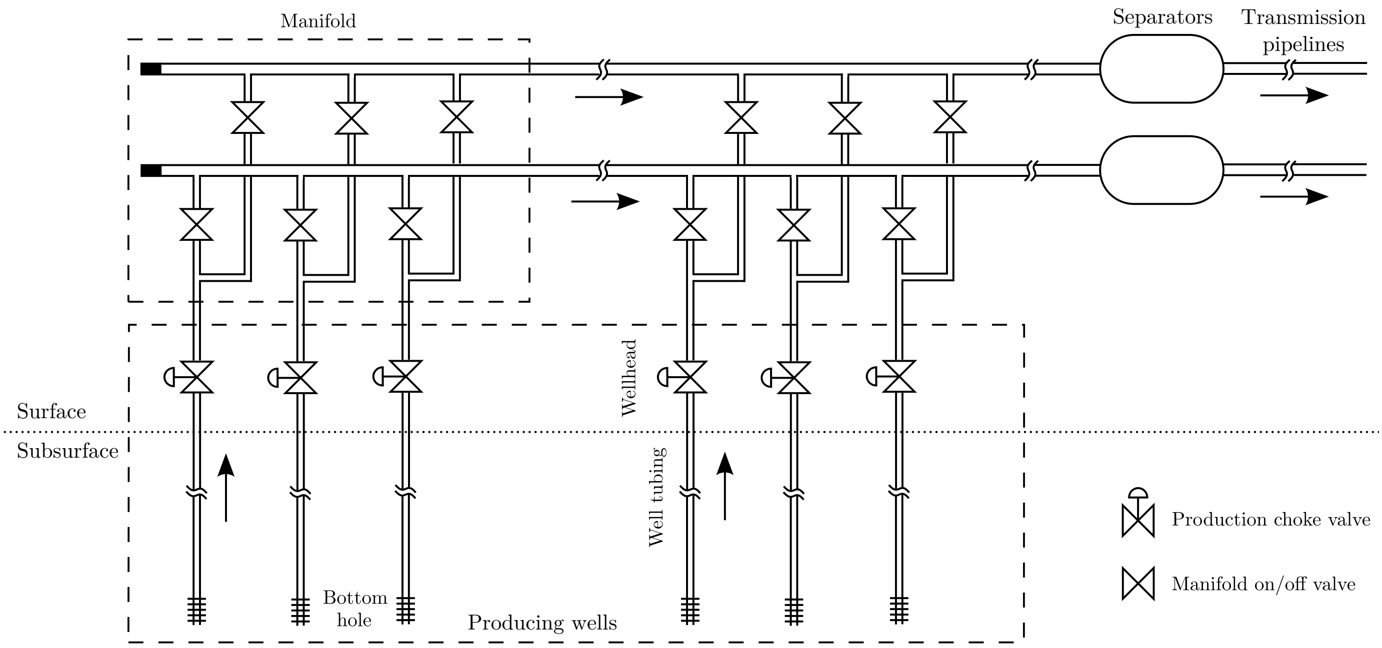

Fig. 2. A petroleum production system

the total oil production, which may be represented by $g_{rk}(q_{rek}) = q_{rek}$, for $r =$ oil.

The constraints in (2) are the mass balances for each phase, for all the internal nodes in the system. The upstream boundary conditions are covered by constraints (3), with initial conditions in (4), while the downstream boundary conditions are given by the (piecewise) constant separator pressure in (5). Downstream conditions may vary. However, a constant separator pressure is quite common and is adopted here.

Equations (6) and (7) define the momentum balance, or pressure drop, across pipe segments. These are modelled as a function of inlet flow rates and inlet pressure. For a discrete edge, the momentum balance is only enforced when the edge is open; otherwise, when the edge is closed, the flow rate is required to be zero according to (8). Note that P can easily be brought to the normal form of GDP by combining (7) and (8) to a set of disjunctions, and transforming (9) to a conjunctive normal form.

A node with discrete leaving edges may route the flow to *one or zero* of these edges. This logic is captured by the routing constraints in (9). These constraints do not allow for flow splitting—this way of operating the production system is rarely used since it is difficult to model. The final constraints in (11) are processing capacity constraints, typically included for the gas and water phase. Note that (10) just defines the Boolean variable Y_{ek}.

2.1 Discussion

Upstream conditions are of particular interest to this study and will thus be elaborated upon. Problem $\mathbf{P}$ defines a dynamic optimization problem on a prediction horizon $\mathbf{K}$ where the dynamics enter through the reservoir model (3), where a natural approach is to embed this into a receding horizon optimization strategy. This implies that $\mathbf{P}$ is solved repetitively at each timestep, and that only the first control move is actually implemented. In situations where the reservoir dynamics can be neglected a quasi dynamic approach may be used in which $\mathbf{P}$ is simplified by limiting the prediction horizon to one time step and substituting (3) with a static model, i.e.,

$$\zeta_{rik}(\mathbf{q}_{ek}, p_{ik}) = 0, \qquad \forall r \in \mathbf{R}, i \in \mathbf{N}^{\text{src}}, k = 0 \tag{12}$$

Further, (4) needs to be omitted in this case.

The static upstream inflow condition, which describes the mass flowrate from the reservoir into the well, may be linear or nonlinear depending on the actual well and reservoir characteristics near the well. Common models are linear productivity index models and nonlinear Vogel curves. There is a vast literature on dynamic reservoir models, also related to optimization, see e.g. Jansen et al. (2008) and references therein. Reservoir model (3) describes the well known class of black oil models, which include oil, water and gas phases. Such models are typically based on a spatial discretization of a PDE model (Aziz and Settari, 1979), and may for instance be solved using a Newton iteration scheme. It may be noted that there exisits more complex models, like compositional models as well as simpler, surrogate models. Streamline models is an example of the latter.

The static version of $\mathbf{P}$ is not new. Kosmidis et al. (2004) was the first to formulate a MINLP for the well oil rate allocation problem.

There are alternative ways to solve the DPO problem in $\mathbf{P}$. The problem is formulated as a GDP, which requires a special solver or a reformulation to a mixed-integer nonlinear programming (MINLP) formulation before it can be solved by conventional MINLP solvers. Then $\mathbf{P}$ may be solved directly using a derivative-free approach. Since equations (3), (6) and (7) normally are embedded in one simulator or several independent simulators for each well and pipeline, it is usually necessary to apply a derivative-free method since gradient information is seldom available. However, by acknowledging the fact that each nonlinear model, (3), (6) and (7), only has a few (local) inputs and that all integer variables appear linearly, it makes sense to replace each well and pipeline simulator with a piecewise continuous surrogate model. This transforms the MINLP problem into a MILP problem with all the benefits that come from such a formulation. One example of the latter is given in Codas and Camponogara (2012). Another option is to use piecewise polynomial proxy models instead of piecewise linear models. An interesting approach is the use of polynomials with compact support in which cubic B-splines seems to be particularly attractive. In the latter case this translates into a MINLP with only B-spline based nonlinear constraints, which may be solved with a dedicated solver (Grimstad and Sandnes, 2014).

3. CASE STUDIES

We now present three classes of DPO problems and start off with a problem from the shale-gas domain before discussing two other domains, namely oil rims and conventional oil reservoirs. Page constraints limits the depth of the last two applications while the first is presented in some detail.

3.1 Scheduling shale-gas wells with demand-side response

The case study, adopted from Knudsen et al. (2014), consists of a small field with dry, mature shale-gas wells. These wells have an unique ability to quickly recover from loss of production due to a well shut-in. Moreover, the land-based nature of shale-gas exploitation leaves many fields in immediate proximity to gas-intensive industries and power plants. Consequently, by systematically optimizing shut-ins with respect to current and predicted local demand, enables utilization of such wells as a proxy or substitute for conventional, third-party underground gas storage (Knudsen et al., 2014).

In the case study, we consider at set of 10 wells, all mature, mid-life wells producing at low rates, assumed to be located in proximity to a natural gas power plant (NGPP). An electric utility company (EUC) operates the NGPP, using it as a ramp-up source together with generation from intermittent renewables. We assume that wells produce gas onto a transmission pipeline with sufficient capacity, and we assume that the demand from the NGPP is given sufficiently early to compensate for a short transmission time on the pipeline. Hence, we omit the delay caused by the pipeline transmission in the optimization model.

The shut-in ability of dry, mature shale-gas wells is enabled by the characteristic shale-fracture system. This system, generated through hydraulic fracturing stimulation, ensures fast formation pressure build-up during well shut-ins, and a subsequent peak in production recovering the loss of production during shut-in. The dynamics of the pressure build-up, however, ranges from the time-scale of hours in and close to the fracture network, and on the time-scale of years further into the low-permeable shale matrix blocks (Knudsen and Foss, 2015). This composite dynamics, with fast near-well dynamics and steady-state like dynamics elsewhere in the reservoir, necessitates the use of a dynamic upstream model. To this end, we apply a dynamic shale-well proxy model (Knudsen and Foss, 2015) for $\zeta_{rik}(\cdot)$ in (3), which we tune using prediction-error filtering so as to fit the dynamic proxy model in the frequency range necessary to capture dominating dynamics during recurrent shut-in operations.

To solve $\mathbf{P}$, we apply a combined big-M and convex hull reformulation of the disjunction, see e.g. Grossmann and Trespalacios (2013). Further, we approximate the nonlinearities with piecewise linearization and implement a Lagrangian relaxation scheme to alleviate the computational burden of the large-scale MILP, see Knudsen et al. (2014) for details.

We assume that the EUC operates a set of intermittent renewable generation sources with hourly varying generation. The EUC uses the NGPP as ramping source in order to compensate for the variability in renewable generation and meet the power demand. We assume that the EUC by using weather information is able to predict some hours ahead its gas demand, which it requests from the operator of the shale-gas field. Consequently, the main objective for the DPO in this case is to meet the varying gas demands by scheduling well shut-ins. Every third hour, the shale-gas operator receives an updated gas demand for the next 24 hours from the EUC.

In order to meet the varying gas demands, the shale-gas operator must reoptimize its well schedule each time it receives an updated demand curve. To this end, we implement $\mathbf{P}$ on a receding horizon, reoptimizing the well schedule and operating pressures each time a new demand curve is received. We implent this with a prediction horizon of three days. Beyond the first 24 hours demand curve, we require that the set of shale wells must be able to meet the *average* gas demand, as a simple means of taking into account the uncertainty of the future demand. If the operator fails to meet the gas demand, a high penalty incur. This penalty enters into the objective (1). On the other hand, if the wells have excess capacity, the operator may choose to sell surplus gas on the spot market, though at a lower price compared to gas supplied to the NGPP.

In Fig. 3, we show the gas rates from the open-loop response computed at the first sampling instant. The set of wells is seen to exactly meet the initially provided gas demand and, due surplus capacity, some gas is additionally sold on the spot market. At the next sampling time, three hours later, the gas demand from the EUC changes due to variability in renewable generation. To prevent penalties, the operator updates the well schedule by incorporating the updated demand curve. As shown in Fig. 4, by optimizing over a receding horizon, the operator is able to exactly meet the changing gas demand by utilizing the gas-storage properties of the shale-gas wells. The resulting well schedule is highly dynamic, as shown by the shut-in pattern in Fig. 5. Some wells are frequently shut in and reopened in order to build-up pressure and save gas for the predicted future demands, while two of the wells produce continuously due to higher formation pressure support, seen by the solid lines.

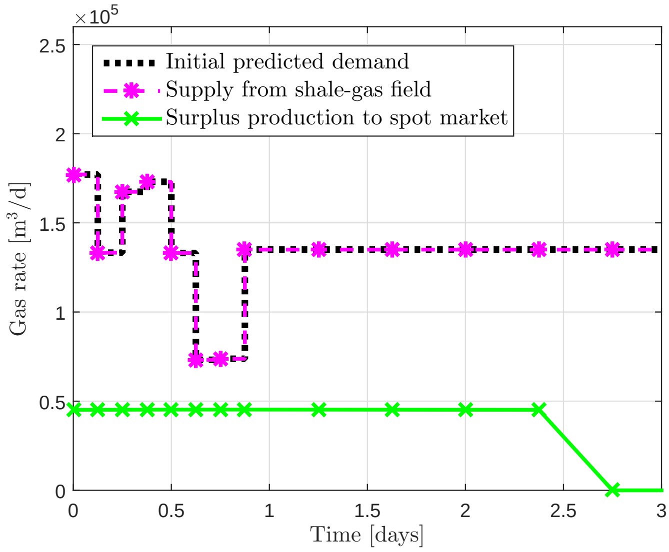

Fig. 3. Open-loop gas rates given initial demand from NGPP.

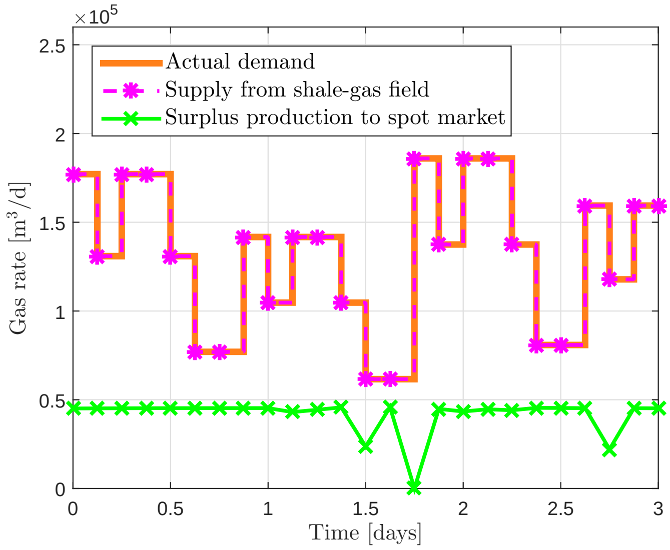

Fig. 4. Gas rates using receding horizon optimization.

3.2 Optimization oil rim performance

Oil rims are reservoirs where a thin oil rim, typically 10 – 25 metres, is sandwiched between a large gas cap above and water beneath. One well known case is the Troll oil field operated by Statoil. Oil rims can only be exploited economically through horizontal wells which are placed just above the water-oil-contact. The reservoir dynamics are fast. In particular the gas-oil-contact (GOC) may move downwards towards the well, in a matter of hours, in the event of increased choke opening. This can expose the well to free gas, a situation which limits oil production severely due to gas processing limitations, as modelled in (11). Nennie et al. (2009) compares alternative dynamic control strategies for production optimization of a thin oil rim and concludes that dynamic strategies enables increased production. Similar results are obtained in Sagatun (2010) and Hasan et al. (2013).

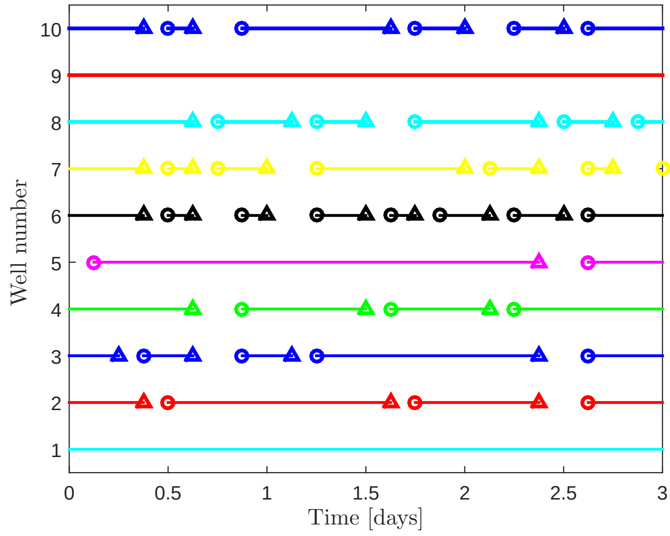

Fig. 5. Shut-ins and start-ups for the well scheduled optimized on a receding horizon, with total rates shown Fig. 4. The circle ◯ symbolizes a start-up of the well, while a △ symbolizes a shut-in.

3.3 DPO in conventional wells

The dynamics of conventional reservoirs is usually slow. In the DPO context this means that reservoir conditions around wells change slowly. In practice pressure and fluid changes only marginally from one day to another. In this case a static formulation, as discussed in conjuntion with (12) works well. There are numerous references on variations of this static strategy, which have appeared after the initial paper (Kosmidis et al., 2004). Gunnerud and Foss (2010) analyses a large problem inspired by a Statoil field in which it is necessary to exploit structure in order to solve **P**. In particular Lagrange relaxation and Dantzig-Wolfe decomposition is applied with encouraging results. Grimstad et al. (2015) extends the static optimization problem to include energy balances to account for flow velocity constraints in gas dominated flow for an application in cooperation with BP. The two latter papers relates to offshore applications while as Codas and Camponogara (2012) is linked to the complex onshore Urucu field operated by Petrobras. The use of a dynamic formulation becomes more relevant if the DPO problem is extended to longer time horizons, say one month ore more, since the reservoir dynamics then increases in importance. This latter case may be interpreted as an application in between level 2 and 3 in Fig. 1.

4. DISCUSSION

The cases referred to above makes a case for the use of both static and dynamic formulations. A large class of DPO problems are treated through a static formulation by acknowledging the time scale separation between reservoir dynamics and the prediction horizon for DPO, thus the use repetitive optimization on a static model suffices. On the contrary, however, there are cases where reservoir dynamics are important on a daily production horizon such as in shale-gas cases and oil rims.

The clear majority of cases uses a static DPO formulation for reasons discussed already. However, there is another reason for choosing the simpler option, static models, namely model accuracy. Model accuracy poses a particular challenge for the well models. Flow rate estimates from individual wells are inaccurate since multiphase flow meters are few and far between. Further, model calibration is performed quite infrequently since they rely on data from well tests.

A dynamic formulation may also be inferred because of demand side dynamics, a situation which is encountered in the shale-gas example discussed earlier. This case includes dynamics both in the upstream reservoir and downstream demand side. Another situation is if fast dynamics appear only on the demand side. This situation, with only demand side dynamics, is covered by $\mathbf{P}$ by optimizing on a suitable horizon $\mathbf{K}$ with a static reservoir model (3) while discarding (4).

The focus up until now has been on the need for dynamics related to reservoir behavior and the demand side, while the wells and network uses static models since $\mathbf{P}$ does not allow dynamic models for these parts. This is a valid assumption except for certain extended networks that are closely integrated with downstream processing. LNG plants may exemplify such systems. In Foss and Halvorsen (2009), where the Statoil Snøhvit offshore field was considered, the pipeline between 9 wells and the LNG plant is 140 km. The network dynamics are then in the range of $8 - 10$ hours and thus in this case the use of a dynamic formulation was considered and actually applied in the study to schedule well production. To limit complexity a dynamic surrogate model for the pipeline was used.

The focus of this paper has been DPO in upstream production. In cases with short DPO horizons such as a few hours a dynamic formulation may be in order, in particular in situations where the link to lower level control is particularly important. Slug flow is an issue which may link DPO closely to low level control. A recent paper (Codas et al., 2016) explores integration of low level control with nonlinear model predictive control for DPO. The analysis is performed for a system prone to slug flow. Improved results are reported, however, at the expense on a significant complexity increase.

It may be noted that uncertainty, which is an important topic, has intentionally been omitted in this paper due to page constraints.

5. CONCLUSIONS

A static optimization formulation suffices in many relevant DPO cases. Important exceptions are shale gas wells and thin oil rims.

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