A multi-parametric programming approach for the simultaneous process scheduling and control – Application to a domestic cogeneration unit

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Abstract

We present a strategy for the simultaneous solution of scheduling and advanced control based on our recently introduced PAROC framework and software platform. A rolling horizon optimization and scheduling model is proposed linked to a control-aware process model that is able to capture the system dynamics. A bridging model approach is applied to handle the mismatch between (i) the scheduling and the control level in terms of time step and (ii) the schedule and the "high fidelity" process model in terms of model-process mismatch. The proposed approach is demonstrated with its application to a domestic cogeneration unit.

Keywords

multiscale, simultaneous scheduling and control, multi-parametric programming, cogeneration

Introduction

Determining the operating conditions that result into profitable, stable and sustainable processes has been in the epicenter of process systems engineering. A general representation of such a problem, involving the optimization of design and operational characteristics of a process is presented in (1).

$$
\begin{aligned}
&\min J = \int_0^t P(x, y, u_c, SP, Y, d, D)dt \\
&\text{s.t. } \frac{d}{dt}x = f(x, u_c, d, D) \\
&\quad y_{min} \leq y = g(x, u_c, d, D) \leq y_{max} \\
&\quad u_c^{min} \leq u_c = h(x, d, Y, D) \leq u_c^{max} \\
&\quad SP^{min} \leq SP = m(x, d, Y, D) \leq SP^{max} \\
&\quad Y \in \{0, 1\} \\
&\quad [x_{min}^T,\ d_{min}^T]^T \leq [x^T,\ d^T]^T \leq [x_{max}^T,\ d_{max}^T]^T \\
&\quad D_{min} \leq D \leq D_{max}
\end{aligned} \tag{1}
$$

(1) corresponds to a large scale mixed integer dynamic optimization (MIDO) problem, representing the interactions amongst process design, scheduling and control - as discussed in Pistikopoulos and Diangelakis (2016) and references therein. $x(t)$ are the states, $u_c(t)$ are the control derived variables, $y(t)$ are the outputs, $d(t)$ is the uncertainty, $SP(t)$ are the scheduling derived setpoints, $Y(t)$ are the binary decision variables, D are the design variables. Design and control interactions are discussed, amongst others, in Mohideen et al. (1996); Ross et al. (1999) and Sakizlis et al. (2003). The interactions of control and scheduling are discussed, amongst others, in Kopanos and Pistikopoulos (2014); Subramanian et al. (2013); Zhuge and Ierapetritou (2014) and Du et al. (2015).

Here, based on (1), we focus on the simultaneous solution of advanced control and scheduling, following the steps of the PAROC framework (Pistikopoulos et al., 2015), applied to a domestic CHP (Diangelakis et al., 2014).

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Simultaneous scheduling and control via the PAROC framework

The proposed methodology for the integration of scheduling and control consists of two main steps. Firstly, a control scheme is deployed for the process at hand. Secondly, the process with its developed control scheme is then used to derive an approximate model, based on which a scheduling formulation is derived.

Process Control Strategy (PCS)

Figure 1 presents the PAROC framework (Pistikopoulos et al., 2015) featuring the following.

PCS 1: "High Fidelity" Dynamic Modeling – The development of the "high fidelity model", its quality and robustness determine the validity of the framework. The modeling of the system takes place in gPROMS® Process Systems Enterprise (2016).

PCS 2: Model Approximation – The resulting dynamic models of the first step (most commonly DAE or PDAE programs), although sufficiently accurate compared to the real process, are far from ideal in terms of multi-parametric programming utilisation. Therefore, reduction techniques and identification methods are employed for to (i) reduce the model complexity while (ii) preserving the model accuracy.

PCS 3: Design of the Multi-Parametric Model Predictive Controllers – The design of the controllers is based on the validated procedure described in Pistikopoulos et al. (2012). The resulting multi-parametric program is solved via the POP® (Oberdieck et al., 2016) toolbox in MATLAB®, thus acquiring the map of optimal control actions.

PCS 4: Closed-Loop Validation – The procedure is validated through a closed loop procedure, where the controllers are tested against the original model of Step 1. This can happen either via the interoperability between software tools such as gPROMS® and MATLAB® via gO:MATLAB or via the straight implementation of the controllers in the gPROMS® simulation via the use of C++ programming and the creation of Dynamic Link Libraries.

This "high fidelity" model with the control scheme is then utilized for the derivation of the approximate model upon which the scheduling strategy is based.

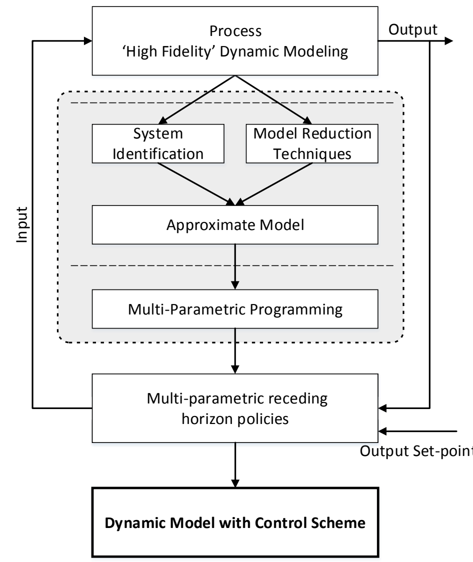

Figure 1. The PAROC framework.

Process Scheduling Strategy (PSS)

The solution of multi-parametric scheduling problems has been previously discussed in Ryu and Pistikopoulos (2007) and Wittmann-Hohlbein and Pistikopoulos (2013). In Kopanos and Pistikopoulos (2014) the procedure for deriving and solving scheduling problems via multi-parametric programming, using a state-space model representation (Subramanian et al., 2013) and a mp-MILP reformulation is presented. The model used in that work is based on a stochastic model approach as it does not consider information regarding the process itself or its dynamics. Utilizing such an approach without including process information, for an integrated scheduling and control application, will result into a schedule that is not consistent with the process and will therefore create a mismatch between the high-fidelity mathematical model and the state-space model representation used in the schedule. In this work we propose a scheduling formulation via a state-space model that is based on the closed-loop control behavior of the high-fidelity model. Furthermore, since the scheduling formulation is (i) typically a mixed integer linear programming problem that does not account for any mismatch between the schedule level and the process level and (ii) it focuses on the mid-term economic optimization of the process, the proposed scheme needs to account for those aspects. Taking into account that the control is of several orders of magnitude more frequent in terms of calculations, the use of a rolling horizon optimization formulation that will (i) bridge the time-scale gap between schedule and control and (ii) take into account the process model mismatch needs to be derived. The proposed methodology is presented in Figure 2 and described below.

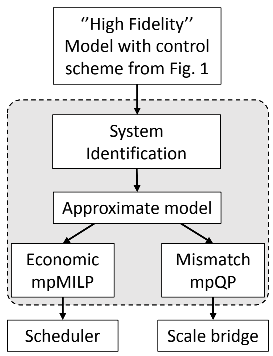

Figure 2. Derivation of scheduling and surrogate model.

PSS 1: "High Fidelity" Model with control scheme – The "high-fidelity" model with the control scheme is the basis upon the scheduling strategy is designed. It is the result of the application of the PAROC framework as described in the previous subsection.

PSS 2: Approximate model – A state-space model similar to the one in Kopanos and Pistikopoulos (2014) is then derived as follows. The fig:Approx model with the control scheme of the first step is utilized to create sets of data that correlate the desired set-points of the system operation with the real outputs. An identified approximate state-space model is created with (i) linear dynamics in discrete time, (ii) discretization step of several orders of magnitude larger than the time step of the control and (iii) awareness of the process dynamics.

PSS 3: mpMILP and mpQP reformulation and solution – The approximate state-space model is used to formulate a MILP problem that corresponds to the economic scheduling of the process at hand. The multi-parametric version of this procedure is described in detail in Kopanos and Pistikopoulos (2014). The resulting approximate model involves a mismatch against the online process model. This is handled by a scale bridging model procedure. Note that, as described in Equation 1, the unification of design, scheduling and control can be expressed by a large scale MIDO problem, both the mismatch and the different time scales considered is a mathematical artifact inherent to the procedures used to make the problem approachable via the available optimization techniques. In order to handle the aforementioned mismatch, a QP formulation is derived that minimizes the mismatch between the schedule predicted output and the "high fidelity" model with the control scheme. The scale bridging model recalculates the optimal scheduling action and the control set-points on the control time interval level. Note that in the case of multiple modes of operation as a result of an optimal schedule (e.g. in process with multiple products) multiple scale bridging models need to be derived. Both formulations are solved explicitly.

PSS 4: Closed-Loop Validation – The closed loop validation of the scheme involves all three stages and formulations. In a similar manner as in the PAROC framework the receding horizon optimization policies are tested against the original high fidelity model using the same computational tools. An overall depiction of the closed loop formulation is shown in Figure 3. In the figure, SP stands for set point, u stands for optimization variable and y for system output (and it denotes the feedback). OP is the operating policy that the solution at each stage dictates for the lower stage. Furthermore, MM is the process mismatch and the subscripts Ts and Tc correspond to the scheduling and control time intervals and denote the frequency of the information exchange between the different stages.

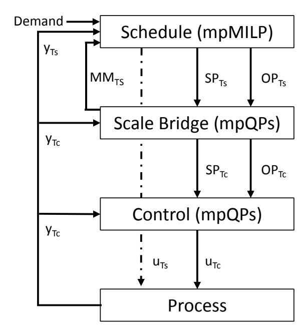

Figure 3. Closed loop application of the receding horizon optimization policies against the process "high-fidelity" model.

Combined Heat and Power (CHP) system

The approach described in the previous section is applied on the CHP model of our previous works (Diangelakis et al., 2014). For the sake of brevity the reader is referred to Diangelakis et al. (2016) for the full control scheme of the domestic CHP unit via decentralization and multi-parametric programming. Here we summarize the control scheme as follows:

1. Two decentralized - coordinated controllers handle

the power generation subsystem and the heat production subsystem, respectively.

2. The scheme features a dual operating mode as the CHP process can be regarded as a multi-product process. Power generation focused or heat production focused operation is subjected to economic criteria.

3. The controllers deployed are explicit Model Predictive Controllers.

Figures 4 and 5 summarizes the control schemes operation.

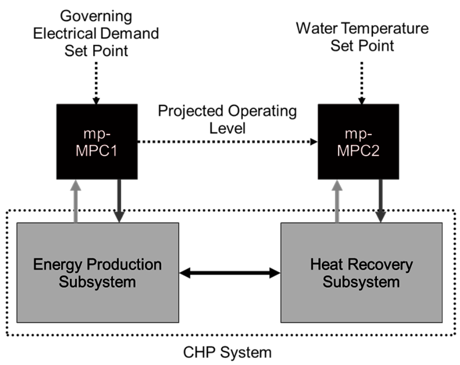

Figure 4. Power generation focused operation.

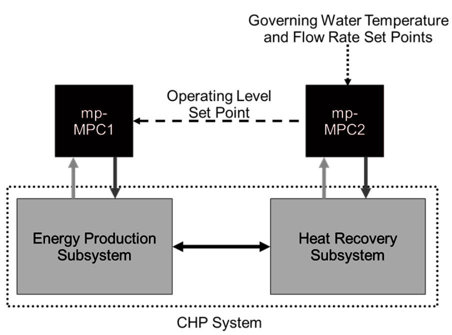

Figure 5. Heat production driven operation

As described in the previous section, the controlled "high fidelity" model is approximated using the System Identification Toolbox© in MATLAB©. Note that the controller scheme handles the power production level and the hot water temperature and flow. On the contrary, the scheduling approach described in Kopanos and Pistikopoulos (2014) focuses on electrical power and heat generation as a whole, i.e. it does not differentiate between water temperature and flow rate setpoints. Furthermore, the schedule takes into account the presence of a heat storage tank. The model used for scheduling purposes is presented in Equation 2.

$$\begin{aligned} E_{T+Ts} &= 0.9989E_T + 99.4867R_T \\ B_{T+Ts} &= 97.977B_T + 0.9079E_T \\ &\quad + 11.9421Q_T - 11.9421D_T - 11.9421Z_T^T \\ P_T &= -E_T + W_T + Z_T^E \end{aligned} \tag{2}$$

Where E_T and B_T are the power generation and storage tank heat, R_T is the change in the power generation level, Q_T is the amount of heat acquired from external sources, D_T is the amount of heat disposed to external sinks at a cost, W_T is the amount of electrical power acquired from external sources and P_T is the amount of power disposed to an external sink at a profit. Finally, Z_T^T and Z_T^E are the electrical and thermal demands treated as uncertain but bounded parameters. Note that Ts stands for the scheduling time intervals which, in this case, is equal to 100 control intervals (Tc). The formulation of the mpMILP follows the formulation of Kopanos and Pistikopoulos (2014) with the exception that in this work the CHP system is considered to be always on operation. Binary decision variables model the selection between buying or selling power/heat as the simultaneously doing both is not permitted.

The time variant part of the model in Equation 2 is discretized to the control time step and utilized for the formulation of the scale bridge with focus on the process-model mismatch, i.e. the formulation of the scale bridge handles the mismatch between the system states of Equation 2 by providing a "more discrete" optimization variable profile. Given the selectivity of the schedule in terms of buying or selling heat two scale bridging optimization formulations need to be designed. Each formulation is a quadratic problem with box constraints on the optimization variables, states and mismatch that is reformulated to an mpQP and solved explicitly using Oberdieck et al. (2016). The critical regions for the schedule mpMILP as well as the scale bridge for acquiring heat through an external source are presented in Figures 6 and 7. The solution of the scale bridge of disposing heat to an external heat is omitted for the sake of brevity.

Note that the B_0 axis in Figure 7 extends from 0 to 150 while the one on Figure 6 from 0 to 10000. The reason the remaining parameter values were omitted from depiction is that the parametric actions for the given set of fixed parameters and for values of $150 \leq B_0 \leq 10000$, for this scale bridging solution is the same, i.e. there aren't any more critical regions but rather a simple ex-

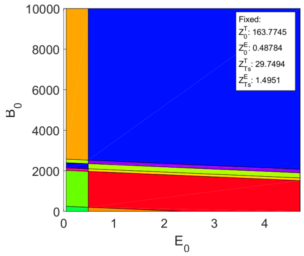

Figure 6. Scheduler regions with fixed demand.

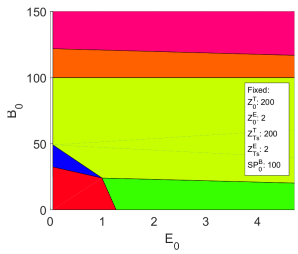

Figure 7. Scale bridge with fixed demand.

tension of the depicted region for $\approx 120 \leq B_0 \leq 10000$.

The closed loop validation of the system takes place as described in Figure 3. Note that the system operates in the power generation focused approach when the amount of power produced matches the demand (that is without selling or buying power) or when the ratio of the power demand covered by the production is larger than the ratio of the heat demand covered by the production. The exact opposite holds true for the heat production focused operation.

Results

The closed loop validation of the operational scheme that includes the schedule, 2 scale bridges and the control scheme is presented in Figure 8. Note that the setpoints for the control are not strict step change but rather the result of the scheduling and scale bridging optimization problems. Furthermore, Figure 9 shows how one of the optimization variables of the scheduling formulation problem, namely Q, the amount of heat acquired from external sources, provides the setpoint for the scale bridge models that calculate the optimal action in the control level time step while taking into account the process-model mismatch.

The controllers meet the setpoints as shown in Figure 8 with a mismatch of less than 2% for a steady setpoint. In case where a sudden setpoint change occurs, especially in the case of the temperature profile, the setpoint is met within the 10s windows of the scheduling problem time step. The momentary temperature violation of the water temperature at approximately 110s is the result of the challenging aspect of the schedule not being aware of the temperature but only of the heat production. The scale bridging models behavior as shown in Figure 9 manages to take the process-model mismatch into account. The difference in the Q profile is attributed to (i) the rapid change of the amount of power required from the grid, (ii) the change of policy from a power production focused operation to a heat production focused operation and (iii) the dynamics of the water flow system that is unable to reduce the flow rate so drastically thus causing the system to buy less heat from external sources than predicted. In this case the cost of operation predicted by the schedule is less than the actual cost because of the system dynamics. In an opposite case where less heat acquisition would be predicted but more would be acquired (for the aforementioned reasons) the cost would be affected in the exact opposite way. Although the mismatch is treated via this approach, the error of discretization is still present and this is a characteristic case of it.

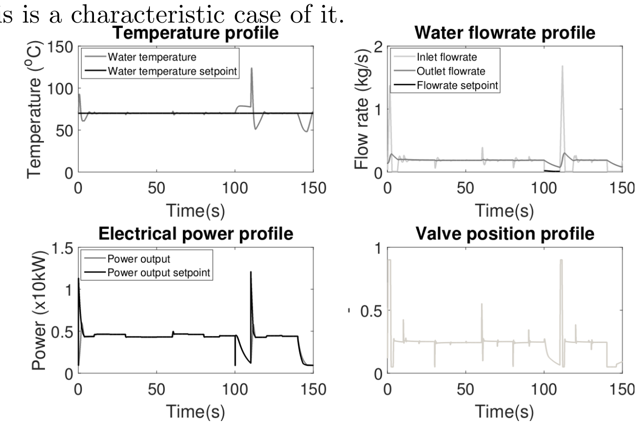

Figure 8. Closed loop validation results for simultaneous scheduling and control optimization.

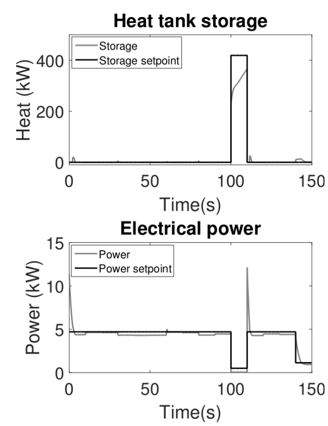

Figure 9. Closed loop validation results for simultaneous scheduling and control optimization.

Conclusions and future work

In this work we presented a framework for the simultaneous process scheduling and control via multiparametric programming and the PAROC framework. The framework was applied on a domestic cogeneration system. After the development of an advanced control scheme, we developed a control aware model that was utilized to derive and solve offline (i) the economic scheduling problem and (ii) the scale bridging model. The three different optimization levels were cross-validated against the original model of the process and their performance was assessed.

The advantages of the proposed approach are; (i) the state-space representation of the scheduling formulation is *a priori* control aware as it describes the controlled model response; (ii) the mismatch between the "high-fidelity" – controller model and the scheduling formulation is handled by a surrogate quadratic programming problem; (iii) the ability to obtain explicit expressions enables the closed loop validation and optimization within the advanced modeling environment. The proposed strategy provides one further step towards the unification of design and operational optimization (Pistikopoulos and Diangelakis, 2016).

The inclusion of more than one CHP units in the scheduling formulation and the ability for the unit to switch off is the subject of our ongoing work. Furthermore, different approaches in bridging the gap between the different rolling horizon optimization problems are investigated.

Acknowledgments

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