RESOLVING CONFLICTS AMONG STAKEHOLDERS IN REAL-TIME OPERATIONS

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Abstract

Planning, scheduling, and control decisions often involve conflicting priorities from multiple stakeholders (e.g., due to different perceptions of risk). We present a new framework for multi-stakeholder optimization to compute optimal compromise solutions among stakeholders. In this setting, stakeholder opinions are interpreted as random variables, establishing a parallel between stochastic and multiobjective optimization. Risk metrics are used to shape the distribution of stakeholder satisfactions. We demonstrate the approach by considering the operation of a combined heat and power utility system that monetizes excess capacity by participating in energy markets. Stakeholders express their perceived risk by discounting (weighting) potential revenues from different market products. We find that despite strongly conflicting priorities, the proposed framework is able to identify efficient compromise solutions where each stakeholder is at least 95% satisfied. We compare compromise solutions computed using the conditional value-at-risk and entropic value-at-risk metrics and discuss how the selection of risk metric impacts the distribution of stakeholder satisfactions.

Keywords

combined heat and power systems, electricity markets, flexibility, multiobjective optimization, stochastic programming

Introduction

Operational decision-making often must balance conflicting technical, economic, safety, environmental, and social objectives. Applications include design of sustainable and resilient supply chains (Gebreslassie et al., 2012), planning and schedule with economic and environmental metrics (Grossmann and Guillén-Gosálbez, 2010), operation of large multi-product facilities (Blömer and Günther, 1998), and control of multi-product reactors and separation systems (Logist et al., 2009). Such settings involve a high degree of ambiguity as objectives are difficult to monetize and decision-makers often disagree on how to establish priorities. This is especially true for hidden objectives such as safety, equipment wear-and-tear, value of preventive maintenance, soft operating limits, perceived risk, etc. Ambiguity is often addressed by calculating Pareto solu-

tions that capture trade-offs between objectives. These strategies, including the Normal Boundary Intersection method (Das and Dennis, 1998; Jia and Ierapetritou, 2007; Logist et al., 2009), become computationally intractable in settings with many objectives. Moreover, such settings often only consider a single decision-maker that must pick a "suitable" Pareto solution. This poses a limitation as many decisions involve multiple stakeholders with conflicting opinions on how to prioritize many metrics. For instance, the U.S. Environmental Protection Agency (EPA) recently identified over a hundred metrics for evaluating the sustainability of different system designs (Ruiz-Mercado et al., 2012). Recently, we have proposed a multi-stakeholder context with the goal of computing optimal compromise solutions (Dowling et al., 2016b). In this work, we observe that this perspective aligns with many decision-making settings arising in operations were engineers and operators may disagree on the relative importance of different objectives. As such, ambiguity is addressed by using sam-

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ples of stakeholder opinions and computing Pareto solutions that minimize a metric of collective dissatisfaction. We prove that minimizing entropic value-at-risk (EVaR) and conditional value-at-risk (CVaR) results in Pareto optimal compromise solutions. The size of the proposed formulation scales linearly with the number of objectives and stakeholders, making it suitable for settings with many objectives and stakeholders.

Definitions

We define the decision variable vector $x \in \Re^{n_x}$ and we assume this to lie in the compact and nonempty feasible set $\mathcal{X} \subseteq \Re^{n_x}$. We consider n objective functions $f_i: \mathcal{X} \to \Re$ for $i \in \mathcal{O} = \{0...n-1\}$ and the corresponding objective vector $\mathbf{f}(x) := (f_0(x), f_1(x)..., f_{n-1}(x))$. We assume that the objective functions remain bounded in \mathcal{X} . We define the multiobjective optimization (MOO) problem,

$$\min_{x \in \mathcal{X}} (f_0(x), f_1(x), \dots, f_{n-1}(x)). \tag{1}$$

Throughout the paper the minimization operand implies global minimization. We scale the objective functions to lie in the interval [0,1] by using the coordinates of the utopia and nadir points as follows. We define,

$$\underline{f}_i := \min_{x \in \mathcal{X}} f_i(x), \quad i \in \mathcal{O}$$
 (2a)

$$\underline{x}_i := \underset{x \in \mathcal{X}}{\operatorname{argmin}} f_i(x), \quad i \in \mathcal{O}.$$
 (2b)

Here, the coordinates of the utopia point are given by \underline{f}_i . Traditionally the coordinates of the nadir point are given by:

$$\overline{f}_i := \max\{f_i(\underline{x}_0), f_i(\underline{x}_1), ..., f_i(\underline{x}_{n-1})\}, \quad i \in \mathcal{O}.$$
 (3)

We previously observed this may lead to a *pessimistic* nadir point (Dowling et al., 2016b). Instead we consider an *alternate* nadir point definition:

$$\underline{x}_{i}^{*} := \underset{x \in \mathcal{X}}{\operatorname{argmin}} \ \mathbf{w}^{T} \mathbf{f}(x)
\text{s.t.} \ f_{i}(x) \leq \underline{f}_{i},$$
(4)

where **w** satisfies $w_i = 0$ and $w_{i'\neq i} > 0$. The alternate nadir point $\bar{\mathbf{f}}^*$ with elements \bar{f}_i^* is defined as:

$$\overline{f}_i^* := \max\{f_i(\underline{x}_0^*), f_i(\underline{x}_1^*), ..., f_i(\underline{x}_{n-1}^*)\}, \quad i \in \mathcal{O}.$$
 (5)

We scale the objectives as,

$$f_i(x) \leftarrow \frac{f_i(x) - \underline{f}_i}{\overline{f}_i^* - \underline{f}_i}, \ i \in \mathcal{O}. \tag{6}$$

We can prevent visiting regions beyond the nadir points (of no interest) by imposing the constraints:

$$0 \le f_i(x) \le 1, \ i \in \mathcal{O}. \tag{7}$$

Thus using a pessimistic nadir point impacts scaling and leads to a larger feasible space.

The multiobjective optimization setting is illustrated in Figure 1. We use the following standard definitions of (weak) Pareto optimality (Miettinen, 1999):

Definition 1 (Weak Pareto Optimality) A decision x^* with objectives $f_i(x^*)$, $i \in \mathcal{O}$ is a weak Pareto solution of MOO if there does not exist an alternate solution \bar{x} with objectives $f_i(\bar{x})$, $i \in \mathcal{O}$ satisfying $f_i(\bar{x}) < f_i(x^*)$ for all $i \in \mathcal{O}$.

Definition 2 (Pareto Optimality) A decision x^* with objectives $f_i(x^*)$, $i \in \mathcal{O}$ is a Pareto solution of MOO if there does not exist an alternate solution \bar{x} with objectives $f_i(\bar{x})$, $i \in \mathcal{O}$ satisfying $f_i(\bar{x}) \leq f_i(x^*)$ for all $i \in \mathcal{O}$ and at least one index i satisfying $f_i(\bar{x}) < f_i(x^*)$.

Any Pareto solution of MOO is a weak Pareto solution. There are different methods to compute elements of the Pareto set, such as the weighting method. Consider a weight vector $\mathbf{w} \in \mathbb{R}^n$. We define the elements of \mathbf{w} as w_i and we assume that these satisfy the condition:

$$w_i \ge 0, \quad i \in \mathcal{O}$$
 (8a)

$$\sum_{i \in \mathcal{O}} w_i = 1. \tag{8b}$$

In some cases we require a stronger condition:

$$w_i > 0, \quad i \in \mathcal{O}$$
 (9a)

$$\sum_{i \in \mathcal{O}} w_i = 1. \tag{9b}$$

Consider now the weighted problem:

$$\min_{x \in \mathcal{X}} \mathbf{w}^T \mathbf{f}(x). \tag{10}$$

A minimizer of (10) is Pareto optimal if (9) holds and weakly Pareto optimal if (8) holds. This implies that pessimistic nadir points, (2) - (3), are constructed from only weakly Pareto optimal solutions, whereas alternate nadir points, (4) - (5), are constructed from Pareto optimal solutions. See Dowling et al. (2016b) for proofs of these statements.

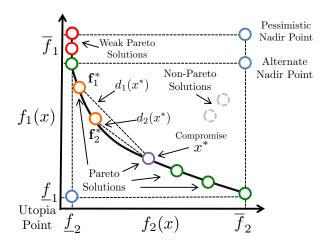


Figure 1. Multiobjective and multi-stakeholder settings.

Multi-Stakeholder Framework

Consider m stakeholders and assume that each stakeholder $j \in \mathcal{S} := \{0, ..., m-1\}$ prioritizes the multiple objectives of MOO according to the weight vector $\mathbf{w}_j \in \mathbb{R}^n$. We define the elements of \mathbf{w}_j as $w_{j,i}$ and we assume them to satisfy either (8) or (9). Each stakeholder $j \in \mathcal{S}$ seeks to solve its individual weighted optimization problem:

$$x_j^* := \underset{x \in \mathcal{X}}{\operatorname{argmin}} \ \mathbf{w}_j^T \mathbf{f}(x). \tag{11}$$

The solution of problem (11) yields the objective vector $\mathbf{f}_{j}^{*} := \mathbf{f}(x_{j}^{*})$ with elements $f_{j,i}^{*}$ and corresponding weighted cost $\mathbf{w}_{j}^{T}\mathbf{f}_{j}^{*}$. This weighted cost is ideal in the sense that it assumes that stakeholder j does not compromise with the rest of the stakeholders. We note that x_{j}^{*} is a weak Pareto solution of MOO if \mathbf{w}_{j} satisfies (8) and is a Pareto solution if \mathbf{w}_{j} satisfies (9).

To deal with conflicting priorities among multiple stakeholders we need to measure the satisfaction/dissatisfaction of stakeholders with a given decision. To do so, we define the dissatisfaction function of stakeholder j at decision x as:

$$d_{j}(x) := \mathbf{w}_{j}^{T}(\mathbf{f}(x) - \mathbf{f}_{j}^{*})$$

$$= \mathbf{w}_{j}^{T}\mathbf{f}(x) - \mathbf{w}_{j}^{T}\mathbf{f}_{j}^{*}$$
(12)

We define the vector of dissatisfactions $\mathbf{d}(x) := (d_0(x), d_2(x), ..., d_{m-1}(x))$. From optimality of x_j^* with respect to (11) we have that $\mathbf{w}_j^T \mathbf{f}(x) \geq \mathbf{w}_j^T \mathbf{f}_j^*$ and thus $d_j(x) \geq 0$ for all $x \in \mathcal{X}$ and $j \in \mathcal{S}$. Moreover, because the values of the objective functions $f_i(x)$ lie between zero and one and the weights satisfy either (8) or (9), we have that $d_j(x) \in [0,1]$ for all $x \in \mathcal{X}$.

To clarify these concepts consider two arbitrary decisions \bar{x}, x that yield $d_j(\bar{x}) < d_j(x)$ for a given stake-

holder j. This means that stakeholder j will be more satisfied under decision \bar{x} than under decision x. Because of a possible conflict in priorities, however, another stakeholder j' might be less satisfied under decision \bar{x} than under decision x (i.e., $d_{j'}(\bar{x}) > d_{j'}(x)$). To compute a stakeholder compromise we thus need to solve the multistakeholder optimization (MSO) problem:

$$\min_{x \in \mathcal{X}} (d_0(x), ..., d_{m-1}(x)). \tag{13}$$

Stakeholder priorities may be viewed as random variables. We propose computing compromise solutions by minimizing a risk metric of the distribution of stakeholder dissatisfactions:

$$\min_{x \in \mathcal{X}} \ R(\mathbf{d}(x)), \tag{14}$$

where $R: \mathbb{R}^n \to \mathbb{R}$ is a function that we call a risk metric. We will show that certain types of risk metrics yield Pareto efficient compromise solutions.

Definition 3 Consider two vectors $\mathbf{d}, \bar{\mathbf{d}} \in \mathbb{R}^m$. We say that the risk metric $R : \mathbb{R}^m \to \mathbb{R}$ is strongly monotone if $\bar{d}_j < d_j$, $\forall j \in \{0, ..., m-1\} \implies R(\bar{\mathbf{d}}) < R(\mathbf{d})$.

We note that strong monotoneity holds for certain norms of a vector space (such as the L_p norm) but a risk metric does not necessarily need to satisfy the properties of a norm. In this work, we only require risk metrics to satisfy strong monotoneity.

Theorem 1 A minimizer of (14) is Pareto optimal solution of MOO if the risk metric $R(\cdot)$ is strongly monotone and (9) holds.

Proof: Assume decision x^* is a minimizer of (14) but is not Pareto optimal. This implies an alternate decision \bar{x} exists where $f_i(\bar{x}) \leq f_i(x^*)$ for all $i \in \mathcal{O}$ and $f_i(\bar{x}) < f_i(x^*)$ for at least one i. This implies $d_j(\bar{x}) < d_j(x^*)$ for all $j \in \mathcal{S}$ because $w_{j,i} > 0$, which implies $R(\mathbf{d}(\bar{x})) < R(\mathbf{d}(x^*))$. Thus x^* is not a minimizer of (14) and we establish a contradiction. \square

We consider variants of two risk metrics in this paper: Conditional Value-at-Risk (CVaR) (Rockafellar and Uryasev, 2000) and Entropic Value-at-Risk (EVaR) (Ahmadi-Javid, 2012), both adapted to finite-dimensional vector spaces. Both of these metrics are parameterized by a probability level $\alpha \in [0,1]$. CVaR $_{\alpha}(\mathbf{d})$ is the average of the largest entries of the vector \mathbf{d} (in the $1-\alpha$ tail) and can be computed as:

$$\text{CVaR}_{\alpha}(\mathbf{d}) = \inf_{v \in \mathbb{R}} \left\{ v + \frac{1}{(1-\alpha)m} \sum_{j=0}^{m-1} [d_j - v]_+ \right\}, \quad (15)$$

where $[v]_+ := \max\{v, 0\}$ for $v \in \mathbb{R}$. In Dowling et al. (2016b), we prove that CVaR is strongly monotone and thus Theorem 1 holds.

Next, we consider the entropic value-at-risk metric,

$$EVaR_{\alpha}(\mathbf{d}) = \inf_{v>0} \left\{ \frac{1}{v} \log \left(\frac{\sum_{j=0}^{m-1} e^{v \cdot d_j}}{(1-\alpha)m} \right) \right\}.$$
 (16)

Both CVaR and EVaR converge to the average (L_1 -norm scaled by 1/m) of the entries vector \mathbf{d} for $\alpha = 0$ and the worst-case entry (L_{∞} -norm) for $\alpha = 1$. Moreover, EVaR upper bounds CVaR:

$$\frac{1}{m} \|\mathbf{d}\|_{1} \le \text{CVaR}_{\alpha}(\mathbf{d}) \le \text{EVaR}_{\alpha}(\mathbf{d}) \le \|\mathbf{d}\|_{\infty}$$
 (17)

Thus by considering CVaR or EVaR in (14), we generalize previous approaches that minimize average or worst-case dissatisfactions (Dyer and Forman, 1992; Hu and Mehrotra, 2012).

Theorem 2 $EVaR_{\alpha}(\cdot)$ is strongly monotone.

Proof: Summations, exponentials and logarithms all preserve strong monotoneity, thus from inspection of (16) it is clear that EVaR is strongly monotone. \Box

This implies that solving (14) using EVaR also produces Pareto efficient compromise solutions. We finally note that many other risk metrics satisfy Property 3, including weighted sums of strongly monotone risk metrics (e.g., $\frac{1}{m} \| \cdot \|_1 + \text{EVaR}_{\alpha}(\cdot)$). This flexibility is useful, as the choice of risk metric shapes the distribution of stakeholder satisfactions. (We will demonstrate this in the next case study.)

Case Study: Market Participation of CHP

Operating strategies for industrial processes and supply chains are undergoing a paradigm shift caused by fluctuating energy prices. Large energy consumers can realize substantial cost savings by adjusting electricity demands (and generation) to take advantage of variability in prices. Recently, Dowling et al. (2016a) analyzed cost savings opportunities for combined heat and power generators using historical data for California markets. We found that strategic operation and market participation can result in up to 37% energy cost reductions. For a $100~\mathrm{MW_e}$ facility, this would represent around $2.5~\mathrm{million}$ USD per year. These savings result from participation in both Day-Ahead (DAM) and Real-Time Markets (RTM).

Directly participating in markets involves risks from price volatility. The RTMs are more lucrative but also more volatile than the DAM. The CHP operators thus seek to determine the time-varying generation and market participation schedule while satisfying on-site steam and electricity demands from manufacturing facilities. In this study, we account for risk by prioritizing different revenue streams and fuel costs:

min (DAM revenue, RTM revenue, Fuel costs)

This is analogous to operating a multi-product supply chain by prioritizing products based on perceived risks. In the CHP example, the multiple products are electrical energy and ancillary services that may be sold in different markets. Detailed market and CHP models are described in Dowling et al. (2016a). Stakeholders express their intuitions about risk by weighting the objectives. Alternately, CHP operation may be formulated as a stochastic program. This requires a detailed risk model, which is unavailable in many cases. In this case study, we consider ten stakeholders, whose priorities are given in Table 1. For this problem, the objectives are already expressed in consistent units and stakeholder weights may be interpreted as perceived risk. For example, stakeholder 3 discounts DAM and RTM revenues by 20% and 50%, respectively, and believes fuel costs are slightly over-predicted. In contrast, stakeholder 10 perceives negligible risk and does not discount market revenues. The weights are rescaled to satisfy (9b).

Table 1. Stakeholder preferences for three objectives.

| ${\bf Stakeholder}$ | $w'_{ m DAM}$ | $w'_{ m RTM}$ | $w_{ m fuel}'$ | | | | | |
|-------------------------------------|---------------|---------------|----------------|--|--|--|--|--|
| Cautions about RTM markets: | | | | | | | | |
| 1 | 0.8 | 0.2 | 1.1 | | | | | |
| 2 | 0.9 | 0.3 | 1.0 | | | | | |
| Cautious about DAM and RTM markets: | | | | | | | | |
| 3 | 0.8 | 0.5 | 0.9 | | | | | |
| 4 | 0.6 | 0.8 | 1.0 | | | | | |
| 5 | 0.7 | 0.7 | 0.9 | | | | | |
| 6 | 0.6 | 0.7 | 1.0 | | | | | |
| 7 | 0.6 | 0.5 | 1.0 | | | | | |
| Supportive of DAM and RTM markets: | | | | | | | | |
| 8 | 0.8 | 0.7 | 0.9 | | | | | |
| 9 | 0.7 | 0.9 | 1.0 | | | | | |
| 10 | 1.0 | 1.0 | 1.0 | | | | | |
| | | | | | | | | |

Using these weights, we solve (18) by minimizing stakeholder dissatisfactions using CVaR and EVaR risk

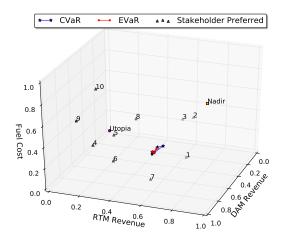


Figure 2. Stakeholder ideal solutions and compromise solutions shown in scaled objective space.

Figure 2 compares the utopia point, nadir point, stakeholder preferred solutions, and compromise solutions. The individual stakeholder preferred solutions are widely spread throughout the objective space while the compromise solutions are tightly clustered together. All of these solutions are guaranteed to be Pareto efficient and appear to lie on a plane. Figures 3 and 4 compare the distribution of stakeholder satisfactions for compromise solutions computed with CVaR and EVaR. With EVaR, the difference in satisfaction level between the most and least satisfied stakeholders shrinks as α increases from 0 to 1. This does not happen with CVaR. We recall that EVaR bounds CVaR, thus EVaR places more emphasis on extreme dissatisfactions of stakeholders. Although the individual stakeholder ideal solutions are spread out in the space, the compromise solutions present stakeholder satisfactions in the range of 96-99% for all stakeholders (stakeholders are highly satisfied with the compromise). In contrast, stakeholders 9 and 10 are only 77-78% satisfied when stakeholder 1 is the sole decision-maker (i.e., (11) is solved for j = 1). This illustrates that the proposed framework can identify Pareto solutions with meaningful interpretations in terms of stakeholder satisfaction (as opposed to standard multiobjective approaches). Moreover, under a systematic framework like the one proposed, stakeholders can compare the impact of their opinions on their individual satisfaction and on the satisfactions of the rest of the stakeholders. Such information can be used to facilitate negotiations.

Finally we compare the operating profiles for different solutions in Figure 5 and Table 2. Stakeholder 3 prefers participation in the DAM market, whereas stake-

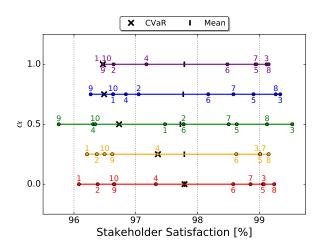


Figure 3. Stakeholder satisfactions for CVaR compromise solutions.

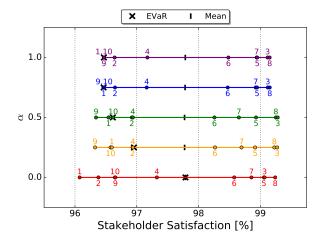


Figure 4. Stakeholder satisfactions for EVaR compromise solutions.

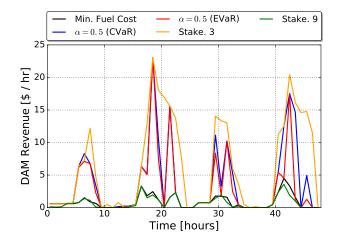


Figure 5. DAM revenue collection profile in time.

holder 9 prefers the RTM. The compromise solutions are between these two extremes, with CVaR slightly prefer-

ring DAM participation relative to EVaR. Both of these compromise solutions (CVaR and EVaR with $\alpha=0.5$) have nearly identical fuel costs (3.34 and 3.37 k\$, respectively) and total electrical energy sales (15.0 and 14.8 MWh_e). The downwards spike in DAM market revenue at time 20 hours corresponds with a massive upwards spike in RTM revenues, caused by RTM price volatility. DAM and RTM participation compete over the finite generation capacity.

Table 2. Comparison of compromise, single objective, and stakeholder preferred solutions.

| | Min. | CVaR | EVaR | Stakeholder | |
|--------------------------------------|------|----------------|----------------|-------------|------|
| | Fuel | $\alpha = 0.5$ | $\alpha = 0.5$ | 3 | 9 |
| Fuel Cost (k\$) | 2.98 | 3.34 | 3.37 | 3.57 | 3.77 |
| DAM Rev. (k\$) | 0.13 | 0.64 | 0.58 | 0.96 | 0.14 |
| RTM Rev. (k\$) | 0.50 | 0.96 | 1.02 | 0.83 | 1.99 |
| Fuel Used (MWh _t) | 218 | 247 | 247 | 261 | 276 |
| Energy Sold (MWh _e) | 0.0 | 15.0 | 14.8 | 20.8 | 26.7 |
| Avg. Price (\$/MWh _e) | - | 59.9 | 61.3 | 52.3 | 53.4 |

Conclusions

Operational decision-making settings often involve multiple decision-makers with conflicting priorities. We present a framework for computing compromising solutions and facilitating conflict resolution. Stakeholder opinions are cast as objective weights and are interpreted as random variables. We prove that any strongly monotone risk metric can be used to balance stakeholder dissatisfactions and calculate Pareto optimal compromise decisions. We demonstrate the benefits of the framework by considering the operation of a CHP system participating in multi-product electricity markets. Ambiguity from perceived risk in different products is captured via stakeholder opinions. The proposed framework is applicable to a broad class of operational settings including resilient supply chains, integrated design and operation planning, and process control. The method can also handle a large number of stakeholders and objectives and does not require computing Pareto sets.

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