# ANALYSIS OF DETERMINISTIC ONLINE SCHEDULING

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*Abstract*

The relationship between the (open-loop) optimization problem that is repeatedly solved online and the quality of the (closed-loop) schedule that is implemented, is poorly understood, even in the deterministic case. We investigate various attributes of the open-loop problem and the rescheduling algorithm that affect the quality of closed-loop schedules, viz. rescheduling frequency, scheduling horizon length, and optimality gap. We find that it is beneficial to reschedule periodically even when there are no "trigger" events. Also, we show that solving the open-loop problem suboptimally does not necessarily lead to poor closed-loop solution due to the presence of feedback. Finally, we explore objective function modifications as well as addition of constraints to the open-loop problem as methods to improve closed-loop performance.

*Keywords*

Chemical production scheduling, Rescheduling, Mixed-integer programming, Closed-loop solution.

## Introduction

Chemical production scheduling is an important problem in the process industry (Harjunkoski et al., 2014). Although much work has been done on building accurate models and effective solution methods (Méndez et al., 2006; Maravelias, 2012; Velez and Maravelias, 2014), the aspect of rescheduling has received limited attention. Rescheduling has been emphasized in some works (Li and Ierapetritou, 2008), but in most cases scheduling is thought to be a static open-loop problem wherein if rescheduling is carried out, the emphasis is only on restoring feasibility or optimality to the unexecuted part of the current schedule. In reality, however, the actual implemented schedule (closed-loop schedule) results from successive solutions to multiple open-loop problems. In this work, we first show through motivating examples, how the relationship between open-loop and closed-loop scheduling is poorly understood, even in the deterministic case, when no uncertainty is present. Thereafter, we investigate how the design of the online scheduling problem, viz. the rescheduling frequency, scheduling horizon length, and optimality gap, affects the quality of the resulting implemented closed-loop schedule. In addition, we explore the effect of objective function modifications and constraint addition on closed-loop performance.

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## Model

The state-space model proposed by Subramanian et al. (2012) is used for all numerical studies in this paper (Eqs. 1–8).

$$\bar{W}^n_{ij(t+1)} = \bar{W}^{n-1}_{ijt} \quad \forall j, i \in I_j, t, n \in \{1, 2, ....\tau_i\} \tag{1}$$

$$\bar{B}^n_{ij(t+1)} = \bar{B}^{n-1}_{ijt} \quad \forall j, i \in I_j, t, n \in \{1, 2, ....\tau_i\} \tag{2}$$

$$\sum_{i \in \mathbf{I}_j} \bar{W}^0_{ijt} + \sum_{i \in \mathbf{I}_j} \sum_{n=1}^{\tau_i - 1} \bar{W}^n_{ijt} \leq 1 \quad \forall\ j, t \tag{3}$$

$$\beta^{MIN}_{ij} \bar{W}^0_{ijt} \leq \bar{B}^0_{ijt} \leq \beta^{MAX}_{ij} \bar{W}^0_{ijt} \quad \forall\ i, j, t \tag{4}$$

$$\begin{aligned} S_{k(t+1)} - BO_{k(t+1)} &= S_{kt} - BO_{kt} + \sum_j \sum_{i \in \mathbf{I_j} \cap \mathbf{I_k^+}} \rho_{ik} \bar{B}^{\tau_i}_{ijt} \\ &+ \sum_j \sum_{i \in \mathbf{I_j} \cap \mathbf{I_k^-}} \rho_{ik} \bar{B}^0_{ijt} - \xi_{kt} - V_{kt} \quad \forall\ k, t \end{aligned} \tag{5}$$

$$\bar{W}^n_{ijt} \in \{0, 1\};\ \bar{B}^n_{ijt}, V_{kt}, S_{kt}, BO_{kt} \geq 0 \tag{6}$$

$$z_{\text{cost}} = \min \quad \sum_k \sum_t \pi^{INV}_k S_{kt} + \sum_k \sum_t \pi^{BO}_k BO_{kt} \tag{7}$$

$$z_{\text{profit}} = \max \quad \sum_k \sum_t \pi_k V_{kt} \tag{8}$$

One of the objectives: cost minimization or profit maximization can be employed. Backlogs ($BO_{kt}$) are fixed to zero for profit maximization, while excess sales ($V_{kt}$) are fixed to zero for cost minimization. The task start variable ($\bar{W}^0_{ijt}$) is binary, while the batch-size ($\bar{B}^0_{ijt}$) and the inventory ($S_{kt}$) variables are non-negative. $\rho_{ik}$, $\tau_i$, $\beta^{MIN}_{ij}/\beta^{MAX}_{ij}$, $\xi_{kt}$, $\pi_k$, and $\pi^{INV}_k/\pi^{BO}_k$ denote mass conversion coefficients, task processing times, batch-size lower/upper bounds, net of material deliveries and orders due, material price, and inventory/backlog cost, respectively. A discrete time-grid with spacing of $\delta = 1$ hour is used.

The model is kept identical for each open-loop problem, except for the updated parameter values. The scheduling horizon (moving horizon) length is denoted by MH and the rescheduling frequency by RF. RF=$x$ implies that rescheduling is carried out every $x$ hours. The relative estimated optimality gap termination criterion for the optimization is denoted by OPTCR (Wolsey, 1998).

## Motivating Examples

In the subsections to follow, we present examples that showcase how for the deterministic (re)scheduling problem even without any stochastic features, surprising closed-loop solutions can be obtained.

*Empty closed-loop schedule*

We solve a profit maximization problem, for PN-1 (Fig. 1), wherein, sales are allowed at all times; with odd number of time-periods in the moving horizon, we can get an empty closed-loop schedule (Fig. 2A).

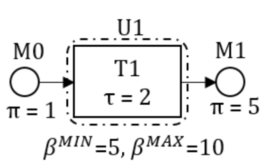


*Figure 1. Process network-1 (PN-1)*

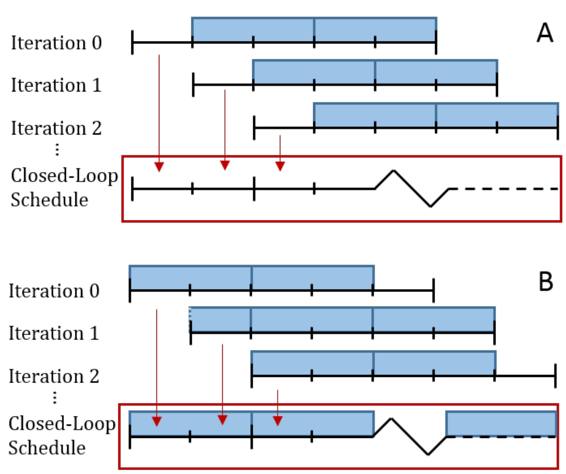


*Figure 2. (A) Empty closed-loop schedule for PN-1 (Fig. 1). (B) Left shifting of tasks by making open-loop objective greedy gives a non-empty closed-loop schedule. (Here MH=5, RF=1, and OPTCR=0%)*

With the task taking even number of time-periods for processing, but only odd number of time-periods being available in the scheduling horizon, multiple optimal solutions exist, and one of them is to have two tasks start after a single idle time-period. This open-loop solution results in an empty closed-loop schedule, since task start decisions are always postponed and never implemented. To circumvent this problem, an ad-hoc fix that can be applied to the open-loop problem is to left-shift open-loop schedules by favoring early sales in the objective (greedy objective). Thus we get a non-empty closed-loop schedule (Fig. 2B). Although, here by modifying the objective function we could achieve improvement, in the general case it is not known what modifications are needed to obtain a good closed-loop solution.

*Optimality of open-loop*

Let us consider PN-2 (Fig. 3) with orders of size 3 tons for each product material due every $3 \pm 1$ hours (integer uniform distribution). The objective is to maximize profit. Excess-sales are allowed only at those times when orders are due. A greedy open-loop objective is used in which early sales bring more profit than latter sales so that tasks are left-shifted, and excess production is shipped out as soon as possible.

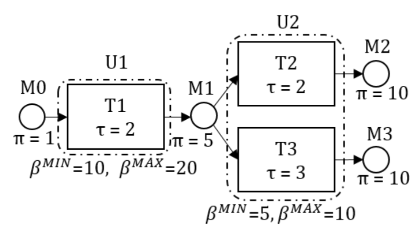


*Figure 3. Process network-2 (PN-2).*

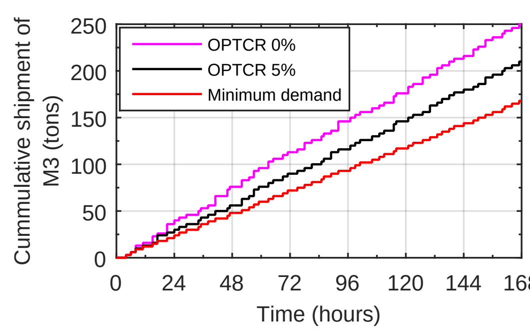


*Figure 4. Less favorable material M3 is produced more in closed-loop schedules when OPTCR=0% (magenta). Black represents OPTCR=5%. Production and shipment closer to minimum needed (red) is better.*

Since M2 and M3 sell at the same price, but the processing time for task T2 is less than that for T3, the most profitable schedule is where T2 dominates executions on unit U2, with minimum possible T3 executions sufficient to just meet the demand for M3.

A closed-loop schedule for 1 week obtained with MH=12, RF=1, and with each open-loop optimization solved to optimality (OPTCR=0%), has 25 executions of T3. Surprisingly, when each open-loop optimization is solved suboptimally (OPTCR=5%), the number of executions for T3 in the closed-loop schedule reduces to 21, which is a better closed-loop solution (see Figs. 4 and 9)! As we will discuss in the case study, this unexpected improvement in the closed-loop solution quality can be explained and addressed.

**Considerations in Online Scheduling**

Rescheduling is traditionally seen as an event-triggered activity wherein an event could be the arrival of an order, a unit breakdown, a processing delay, etc. However, we reason that rescheduling should also be performed periodically due to the consideration of additional time in the scheduling problem, which is also in essence a resource availability change. Rescheduling too often has an implicit cost associated with it due to the plant nervousness that it induces (Vieira et al., 2003; Kopanos et al., 2008). Hence, it is important to quantify how online scheduling affects the quality of the implemented closed-loop schedule, and then decide how often rescheduling should be performed.

In addition to rescheduling frequency, a key attribute of the open-loop problem is the scheduling horizon length because it dictates if orders farther into the future are accounted for while computing the current schedule. A myopic horizon can lead to bad early decisions, necessitating costly revision in the future. In general, it might be a good idea to employ as long a horizon as possible. In practice, however, longer horizons increase the problem size and might render computing the schedule challenging in real-time. Hence it is natural to ask what should be an appropriate length for the moving horizon.

The solution to large scale scheduling problems can be computationally very expensive. Due to the limited availability of online computation time, it is not always possible to solve the models to optimality. In general, it has been assumed that the closed-loop solution would deteriorate due to accumulation of these suboptimalities. However, no study has indeed confirmed this assumption. In fact, one can argue that suboptimal moves can be corrected due to feedback, consequently the closed-loop schedule remains minimally affected. Furthermore, due to the repetitive computations employing a finite moving horizon, it cannot be directly deduced that the open-loop optimization should even be attempted to be solved to optimality, especially since suboptimal solutions can be obtained faster allowing frequent rescheduling.

**Computational Results**

We investigate the effect of RF, MH and OPTCR on closed-loop performance for cost minimization and profit maximization. We present results for PN-2 with orders of M2 and M3 each due every $3\pm1$ hours (uniform integer distribution), and of size $3 \pm 0.3$ tons (uniform continuous distribution). We discern statistical significance by carrying out two-way ANOVA on the computational results (Wonnacott and Wonnacott, 1972). Similar trends were obtained from studies on other process networks.

Figures 5A(i-ii) show how closed-loop quality is affected by rescheduling frequency. The overall trend is that when rescheduling is carried out more frequently the closed-loop quality improves. This improvement with more frequent rescheduling frequency, however, is minimal with longer moving horizon lengths (MH=24, 30). Figures 5B(i-ii) show how the closed-loop quality for cost minimization is affected by moving horizon length. The overall trend is that as the moving horizon length increases the closed-loop quality improves. Infrequent rescheduling (RF=6) is affected more by shorter moving horizon length than frequent rescheduling (RF=1). Figures 5C(i-ii) show how the closed-loop quality changes with OPTCR. We present results with MH=24 hours, but MH=12, 18, and 30 hours also exhibit qualitatively identical trends. The overall trend is that as OPTCR increases, closed-loop quality deteriorates only slightly, and much less than the corresponding percentage of OPTCR. Frequent rescheduling is affected by non-zero OPTCR slightly less than infrequent rescheduling.

We also investigate the effect of frequency at which orders are due and the order-sizes. In Fig. 6A, we see that rescheduling often is better irrespective of the frequency at which orders are due. This is interesting, since rescheduling has been traditionally thought to be only needed on event-triggers, but the computational evidence here shows otherwise. In Fig. 6B, we see that employing a longer moving horizon becomes even more important as the load increases. This is expected, since to meet a big order, multiple batch executions are needed which need to be planned ahead. In addition, when these big orders are temporally spaced out (infrequent), a short moving horizon, owing to its limited ability to look ahead, is unable to start production in a timely manner to meet these orders. Finally in Fig. 6C, we see that the deterioration in closed-loop quality due to non-zero OPTCR is not affected in any particular way by the frequency or size of orders.

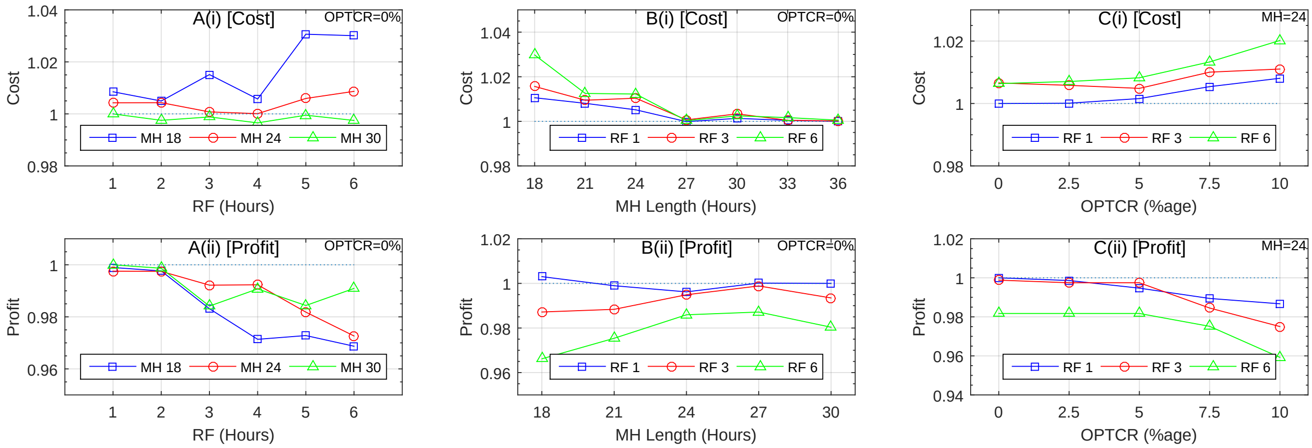


*Figure 5. Effect of RF, MH, and OPTCR on closed-loop quality. Each data point is an average cost of closed-loop schedules for 10 demand samples, and is scaled by mean closed-loop cost corresponding to (A) MH=30, RF=1; (B) MH=36, RF=1; (C) OPTCR=0%, RF=1, within each subplot, respectively. Each open-loop problem is solved to optimality. Top row (A(i)-C(i)) shows results for cost minimization, bottom row (A(ii)-C(ii)) shows results for profit maximization.*

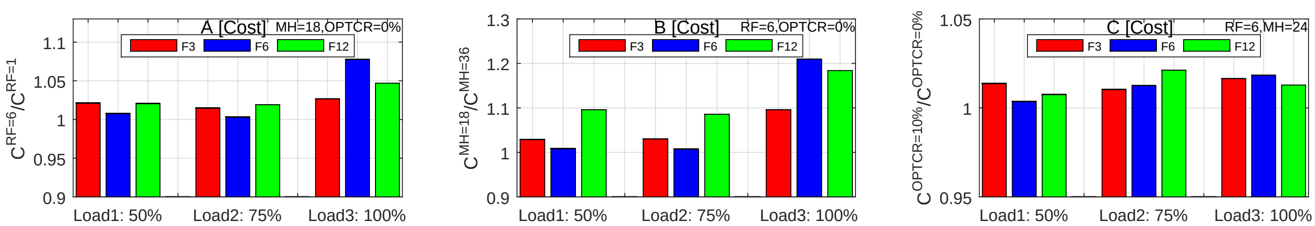


*Figure 6. Effect of frequency of orders and mean order sizes. $C^{*=\#}$ represents average cost of closed-loop schedules for 10 demand samples, for * design attribute set at # value. Fx denotes orders due on an average every x hours. Load, which is a result of both frequency and size of orders, is an approximate utilization of production capacity of the network for a given product split.*

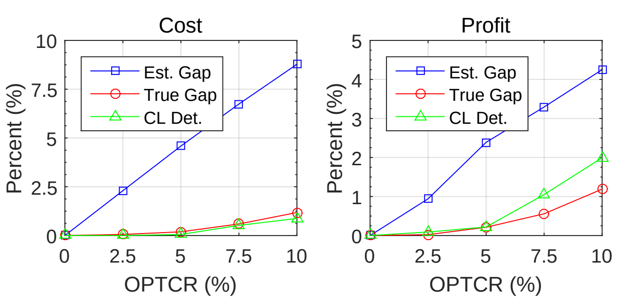


*Figure 7. Mean estimated relative gap (Est. Gap) and mean true relative gap of open-loop optimizations (True Gap), and the resulting mean closed-loop deterioration (CL Det.), as a function of OPTCR. Mean over 30 closed-loop schedules for 10 demand samples, computed using MH=24 with RF=1,3,6.*

## Discussion

In general, higher rescheduling frequency leads to better closed-loop quality, however, beyond a threshold, closed-loop performance does not improve. Short moving horizons can be myopic, thus unable to take good current decisions, but lengths beyond a certain threshold do not result in significant improvement, because in this case, orders located in the latter part of the long horizondo not affect decisions that will be immediately implemented. An OPTCR lesser than a certain threshold (in our results 5%), causes insignificant deterioration in the closed-loop quality. This can be attributed to the fact that the true gaps are significantly smaller than the estimated gaps, and also due to the feedback. For example, for cost minimization for OPTCR=5%, true gaps and estimated gaps are on an average 0.24% and 4.28% respectively, which translate to just 0.22% average deterioration in closed-loop quality (Fig. 7). Hence, in general, it can be an acceptable trade-off to not solve each open-loop problem to optimality or close to optimality.

Having investigated how rescheduling frequency, moving horizon length and suboptimality of solution to open-loop problem affect the closed-loop quality, individually, it is natural to ask if these also have any simultaneous effect. When we analyze this cross relation (Fig. 5), we observe that a longer moving horizon, completely compensates for slower rescheduling frequency. This is true, however, only for the deterministic case, where once an order enters the moving horizon, there is no uncertainty in its size or due-time. In the stochastic case, if the order changes while it is within the moving horizon, more frequent rescheduling would be required.

In addition, being able to reschedule frequently, even if obtaining suboptimal solutions, may have economic benefits. It is not very clear, however, if the resulting closed-loop schedule from a longer moving horizon is less prone to deterioration in quality than a shorter moving horizon, due to suboptimal computations.

## Case Study

We now again consider the motivating example in which the open-loop objective favors early sales (greedy objective). For suboptimal computations with greedy objective, we observe consistently that the closed-loop profit improves with increasing OPTCR (Fig. 8)! The cumulative demand for material M3 over 1 week is 168 tons, hence, the minimum number of executions required for task T3, with $\beta_{T3,U2}^{MAX} = 10$, to meet this demand is 17. An increase in the number of executions of T3 in comparison to the minimum needed, is expected due to the finiteness of the moving horizon. A finite (myopic) moving horizon (MH=12) does not account for demands far ahead in future and ships out any excess inventory of M3 as early as possible (due to the greedy objective); when a new demand of M3 enters the horizon, execution of a new T3 is required to meet this demand. The greedy objective aggravates this effect by shipping early and not maintaining inventory of M3 to meet future demand. In a suboptimal solution, early sales are reduced and excess inventory of M3 is held which is later used to meet new demand. Thus, a move that appears to be suboptimal in the current iteration (favoring inventory over sales) turns out to be a good move in the long run, leading to a better closed-loop schedule. This improvement with OPTCR though holds only till a certain threshold value (here 5%); beyond this value the open-loop solutions cannot be repaired. For example, for OPTCR=10% both T2 and T3 have a small number of executions resulting in the introduction of idle time and thereby less sales and profit.

This myopic behavior can be corrected by, for example, adding a constraint disallowing excess-sales of M3 in the first half of the moving horizon. Since sales are allowed in the second half, excess M3 can still be produced in the first half. As shown in Fig. 9, this added constraint leads to the best possible closed-loop schedule (for both OPTCR=0% and OPTCR=5%) with the minimum required 17 executions of T3. In the same figure we also show the closed-loop schedule obtained using a non-greedy open-loop objective (no favoring of early sales). For this objective, although the number of executions of T3 are 18 (close to the best minimum 17), there are missed opportunities (idle times) in the schedule. These idle times can be explained by the right-shifting of tasks due to multiplicity of solutions.

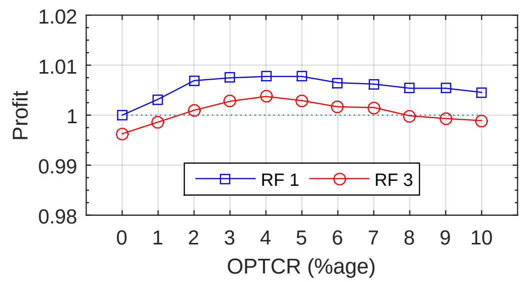


*Figure 8. Effect of OPTCR on closed-loop quality for profit maximization greedy objective. MH=24 is used. Each data point is an average profit of closed-loop schedules for 100 demand samples, scaled by mean closed-loop profit corresponding to OPTCR=0%, RF=1.*

In Fig. 10, we show the effect of using different rescheduling algorithms (using RF=3 and MH=12). We see that addition of constraint results in the best possible closed-loop performance for OPTCR=0% and 5%, but not so for OPTCR=10%. This is because the constraint was tailored to address the problem of executions of T3 over T2, but not to address idle times that are present when OPTCR=10% is used.

This example thus shows how adding constraints to open-loop problem is a powerful and effective way to improve closed-loop performance; however, finding the appropriate constraints for each situation is a challenge and requires further investigation.

## Conclusions

We presented a framework for the analysis of (online) closed-loop schedules. We first showed that even in the deterministic case, the relationship between the open-loop problem solved online and the closed-loop schedule is poorly understood. Applying methods to improve solution to the open-loop problem, does not necessarily translate to good solutions for the closed-loop problem. We studied how rescheduling frequency, moving horizon length and suboptimal solutions of open-loop problem affect the quality of closed-loop schedules. We found that it is important to reschedule periodically, even when there are no “trigger” events, something that is in contrast with the current rescheduling approaches. Also, we showed that suboptimal open-loop solutions do not “accumulate”, but instead, are corrected through revisions due to feedback. Lastly, we explored objective function modifications and addition of constraints to the

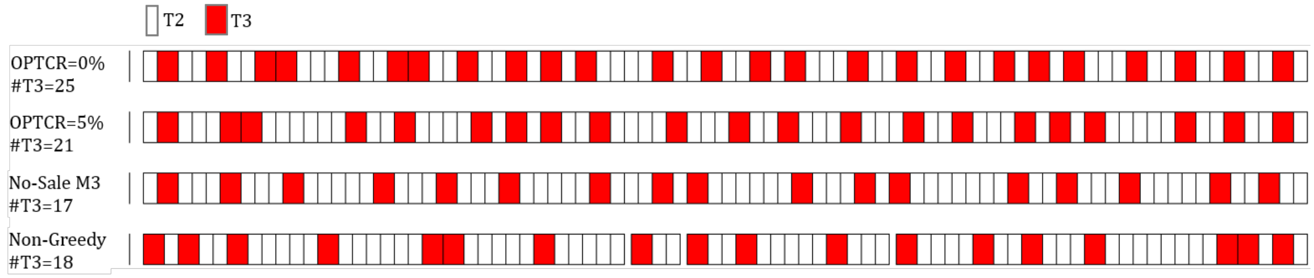


*Figure 9. Closed loop schedules for unit 2; MH=12 with OPTCR values 0% (first), 5% (second) and with added constraint (third); with OPTCR=0%, T3 is executed 25 times, while, with OPTCR=5% T3 is executed 21 times. The added constraint (for both OPTCR=0% and 5%) brings down the executions of T3 to the best minimum of 17. Non-Greedy (fourth) corresponds to MH=12, RF=1, OPTCR=0% for open-loop objective that does not favor early sales; it has 18 executions of T3 but has idle times (missed opportunities).*

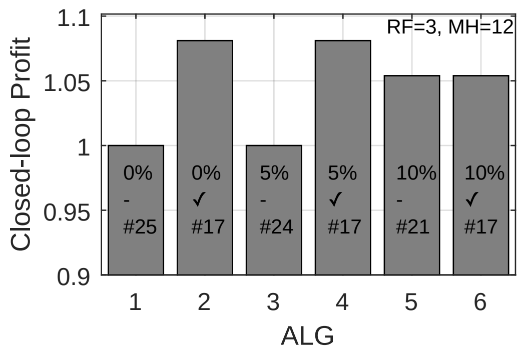


*Figure 10. Several rescheduling algorithms (ALG). In each bar, the %age denotes OPTCR, checkmark denotes addition of no excess-sale of M3 in first half of horizon constraint, and # denotes the number of executions of T3.*

open-loop problem as methods to improve closed-loop performance. Although adding constraints can possibly lead to lower quality open-loop solutions, they can ultimately result in higher quality closed-loop solutions. Further work is needed to find appropriate constraints that could improve closed-loop solution in the general case. The scope of this work was limited to deterministic online scheduling, since as shown through the motivating examples, the deterministic case itself is laden with several “paradoxes”. However, uncertainty is also a major concern, thus studying online scheduling under uncertainty is an interesting future direction.

## Acknowledgments

The authors would like to acknowledge support from the National Science Foundation under grants CMMI-1334933 and CBET-1264096, as well as the Petroleum Research Fund under grant 53313-ND9.

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