

REDUCED ORDER MODELING FOR RESERVOIR INJECTION OPTIMIZATION AND FORECASTING

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Abstract

Reservoir reduced order modeling includes empirical, fundamental, and hybrid model forms. The objective is to capture time-varying relationships and facilitate decisions made by optimizers to maximize reservoir value in the near and long-term. The models used for simulation are often too complex and computationally demanding for direct application in optimization. A number of techniques are summarized that either reduce or simplify the model form to capture dynamic relationship between injectors and producers in complex fields. A case study is also included with Model 2 from the 10th Society of Petroleum Engineers comparative solutions project. Dynamic models of injector to producer relationships are created with and without fundamental reservoir information in the form of gain constraints. Higher order reduced models and gain constraints reduce the data requirement to obtain a satisfactory fit. Future needs are discussed such as quantification of model parameter uncertainty, optimal design of experiments for closed-loop identification, and improved modeling to capture nonlinear effects.

Keywords

Reduced order models, Dynamic data reconciliation, Enhanced oil recovery, Injection optimization.

Introduction

Reservoir model reduction techniques generally fall into two categories: (1) methods that are data driven and do not require fundamental knowledge of the reservoir and (2) physics-based reduced order models that exploit production data or reservoir knowledge. Physics-based models are further classified into two classes of reduced order models including those that are derived from finite element, first principles simulators and those that begin as lumped parameter models. The advantage of using physics-based models is that predictions are better extended outside of the training data but that additional geologic information can be difficult to acquire and assimilate into the predictive models. Commercial reservoir simulators use high-fidelity, physics-based models discretized onto finite-volume grids to provide predictions of reservoir performance. While these sim-

ulators attempt to provide an accurate description of reservoir geology and flow properties, the large size of these models make them computationally expensive to solve, especially in optimization (Awasthi et al., 2007) where many simulations of the model may be required to reach convergence. Often, such models have millions of grid blocks with associated state dimensions that are often multiples of the number of grid blocks (Foss, 2012). This makes injection optimization difficult and time-consuming without the aid of model reduction techniques or new reduced order models. This paper examines existing types of reduced order models and model reduction techniques that are applied to optimization of reservoirs, as shown in Figure 1. These methods enable history matching and optimization with limited computational resources. The reduced order models can then be used to guide decisions about reservoir injection.

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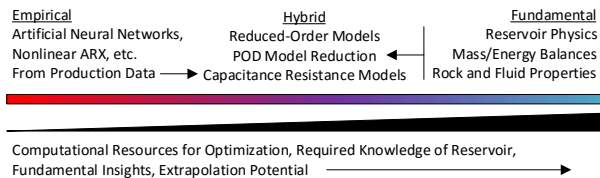


Figure 1. Empirical (based solely on data), fundamental, and hybrid models are all used for optimization with a range of trade-offs centered on computational expense and availability of data and reservoir information

Model Reduction Techniques

Model reduction is implemented to reduce simulation run time or investigate dominant reservoir dynamic factors. The process takes large models and creates smaller ones by reducing the number of variables, referred to as upscaling the model. Reservoir models can be reduced into both linear and non-linear models. Many linear models are not of practical use because they require frequent reconstruction that outweighs computational speed benefits (Heijn et al., 2004). This section focuses on non-linear model reduction techniques. Although non-linear models take more time to construct, the range of operation is greater for improved simulation performance.

One of the most common nonlinear model reduction methods is Proper Orthogonal Decomposition (POD) (van Doren et al., 2006; Fragoso et al., 2014; He et al., 2013; Cardoso and Durlofsky, 2009; He et al., 2014; Klie, 2013; Heijn et al., 2004; Alghareeb and Williams, 2013; Suwartadi et al., 2015; Cardoso et al., 2009). The POD method reduces the model size of high-fidelity simulators with a similarity transform (T) between the original states (x) and a reduced set of states ($\bar{x} = T x$) that capture the dynamics of the system. This reduces the number of parameters and state variables in simulations through either truncation where non-essential transformed states are set to zero or residualization where these same states are fixed at nominal values (Hedengren and Edgar, 2005). Residualization preserves steady state relationships to preserve important relationships such as mass or energy conservation and is preferred over the truncation approach. Cardoso et al. (2009) develops and applies POD for reservoir systems. Various model reduction techniques are applied to two-phase (oil and water) reservoir flow with POD having potential as a reduced order representation for reservoir simulation

due to the ability to capture non-linear dynamics (Heijn et al., 2004). POD is used to reduce model size while incorporating an Ensemble Kalman Filter (EnKF) for history matching of reservoirs (He et al., 2014). POD is also used to create reduced order models and apply these models in adjoint optimization methods. The reduced order model is nested within the high-fidelity model to improve simulation runtime by up to 35% with nearly identically-optimized solutions when compared with the high-fidelity model (van Doren et al., 2006; He et al., 2014; Jansen and Durlofsky, 2016).

Although speedup can be achieved with the POD method, these improvements are limited because the Jacobian must be determined and projected into the reduced order space at each time step during the simulation (Cardoso, 2009). To overcome these challenges, Trajectory Piecewise Linearization (TPWL) can be used in conjunction with POD (Jansen and Durlofsky, 2016). TPWL is implemented in order to avoid construction of the full Jacobian and residual matrices at each time step. Using a limited number of training runs, TPWL creates reduced order state and Jacobian matrices. The procedure is then represented in a reduced order space using POD (Cardoso and Durlofsky, 2009). A more detailed description of TPWL can be found in Cardoso and Durlofsky (2010) and Cardoso (2009). TPWL and POD have been used together to speed up reservoir optimization times by an order of 500 (Fragoso et al., 2014). Improvements to TPWL are made that provide improved resolution at grid blocks near wells and other important areas in He et al. (2011). TPWL is used to reduce model size in order to improve a history matching algorithm involving an EnKF (He et al., 2014). These different reduction techniques provide a method to run simulations more quickly by allowing the reduced model to run a majority of the iterations, while the full-fidelity model is only used to update the reduced model.

Other model reduction techniques are less common in literature. Various model reduction techniques are compared for use in low-order controllers, and Krylov based methods are determined to be efficient for large reservoirs (Gilden et al., 2007). Krylov methods and other projection techniques are compared in Gilden et al. (2006). The Discrete Empirical Interpolation Method (DEIM) has become an increasing area of interest for model reduction (Klie, 2013; Alghareeb and Williams, 2013; Ghommem et al., 2013). DEIM is coupled with POD or other model reduction methods to improve the modeling of system nonlinearities. An in-depth de-

description of DEIM can be found in Chaturantabut and Sorensen (2010). Future work involves improving these model reduction techniques for multiphase flow as well as accounting for the changing bottom-hole pressure associated with well shut-ins. Another area of future research is quantifying the relationship between computation speedup and accuracy. Scaling these procedures for use on large fields with tens or hundreds of wells is of future interest as well.

Reduced Order Models (Proxy Models)

Instead of reducing high-fidelity models, another method of improving simulation time is to use stand-alone reduced order models. These models can be based on the physical properties of the reservoir, or they can be completely empirical with no physical basis. A common objective of many model types is to determine the dynamic relationship between injector and producer wells. With this type of model, engineers can determine which wells to shut in and which wells are contributing most to production gains and long-term reservoir extraction targets. This type of model can also be used to determine various geological features in the reservoir, such as faults and heterogeneities (Panda and Chopra, 1998; Liu et al., 2007). The downside of this class of model is that it requires production and injection data, and so it is not useful before the field has been developed. Preliminary artificial data can, however, be obtained by simulating injection and production data from high fidelity simulators. Developing a reduced order model from simulators has an advantage over POD based methods because only input-output data is collected instead of modifying the source code of the simulator. A design of experiments (Darby and Nikolaou, 2014; G. D. Barros et al., 2015) to best optimize well performance and reduce model parameter uncertainty is a future opportunity that has not been adequately addressed in the literature.

Data Driven Models

Data driven models allow for evaluation of reservoir dynamics using only production and injection data. One of the advantages of empirical models is that they often do not require geological data from the reservoir. These types of models are useful for legacy fields, in which the reservoir has been passed from various companies and geologic data is no longer known or was compiled long ago. Lee et al. (2008) used a finite impulse response

model (FIR) to determine flow units between injection and production wells. The FIR model requires a large number of parameters to achieve comparable accuracy with other empirical models, making it computationally inefficient. Lee et al. use a multivariate autoregressive model to quantify the relationship between injection and production wells. The model was found to be more robust to noise than the FIR model (Lee et al., 2010). An Auto-Regressive eXogenous inputs (ARX) model is a time series representation that includes past externally determined inputs u and the output variable of interest y in the form $y_k = \sum_{i=1}^{n_y} \alpha_i y_{k-i} + \sum_{i=1}^{n_u} \beta_i u_{k-i}$. The parameters α_i and β_i are determined through regression to measured dynamic data (ym and um) with a least squares objective $\sum_{i=1}^n (ym_i - y_i)^2$. In this case, n is the number of data points, n_u is the number of β coefficients, and n_y is the number of α coefficients. A nonlinear autoregressive exogenous (NARX) model of the form $y_k = \sum_{i=1}^{n_y} f(y_{k-i}) + \sum_{i=1}^{n_u} g(u_{k-i})$ is used to simulate a naturally fractured reservoir under gas-drive (Sheremetov et al., 2014). The NARX model is more easily trained and converges faster than feedforward ANNs and the ANN architecture considerably affects model output.

An autoregressive model with two parameters per injector employs an extended Kalman filter to continually update the model parameters (Liu et al., 2007). The filter is used to quickly infer relationships between wells and even determine faults and other geological heterogeneities. Daoyuan Zhai (2010) further validated this model and more easily determined relationships between injection and production wells. A constrained Kalman filter is used to ensure that the injector-producer relationships are physically plausible. These data driven models allow engineers without prior knowledge of reservoir geology to understand the dynamics and infer geologic structures between different wells within the reservoir. The lack of fundamental insight provided by data driven models, and the inability to extrapolate beyond the training data, are weaknesses of data driven models when compared to reduced order or physics based models. However, with constraints or other information to constrain the empirical models, extrapolation potential is improved.

Artificial Neural Networks (ANNs) are common data driven models used in the petroleum industry. ANNs have been an area of intense research in the petroleum industry in recent years. ANNs are empirical models that are used as proxies to improve reservoir simula-

tion time in optimization and history matching problems. Each input neuron is given a weight, and these impulses move through the network until the output neurons are reached where a solution can then be read (see Figure 2). Because ANNs are data driven models, the accuracy of these models are solely dependent on the quality of the data used for training. The greatest advantage of ANNs is the ability to model nonlinearities with little computational effort or physical understanding of the process. However these models are difficult to train and have pitfalls such as overtraining, extrapolation, or lack of validation. For these reasons ANNs require user knowledge and understanding in order to select the proper architecture of the network and train routine. Sampaio (2009) gives an overview of methods for training and developing ANNs for use as proxy modeling in reservoir simulation and history matching.

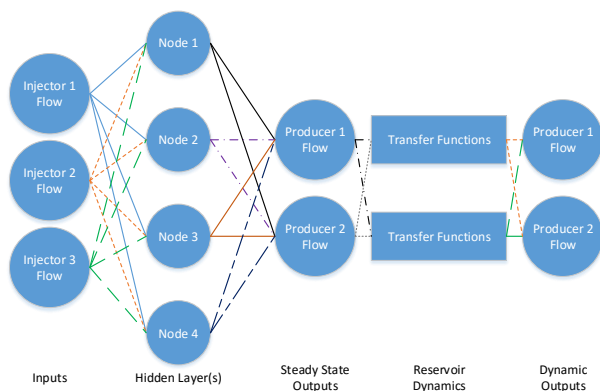


Figure 2. Schematic of a sample ANN architecture with output dynamics

ANNs are applied as proxy models to improve reservoir simulation time, history matching, optimize reservoir production, and to discover inter-well connectivities. This type of model also allows for quicker evaluation of reservoir heterogeneities such as faults and pinchouts (Panda and Chopra, 1998). ANNs are also used for history matching injection and oil production to maximize future production while minimizing injection (Nikravesh et al., 1996). These models provide information to reservoir engineers can use to avoid reservoir damage caused by over-injection while maximizing production. As mentioned earlier, Sheremetov et al. (2014) used a hybrid ANN model to predict reservoir performance under gas drive. This work was successfully applied to a simple reservoir and future studies are anticipated to apply this methodology to larger, more

complex reservoirs. ANNs are found to provide a reasonable substitute for reservoir simulators as a history matching tool, providing improved simulation time, although with lower confidence in the results. High fidelity simulators are suggested to validate the results of history matches obtained via ANN proxy models (Costa et al., 2014). ANNs are applied to proxy models for history matching on the Brugge field and an Iranian reservoir with fewer simulation runs than the methods compared in Foroud et al. (2014). ANNs are also used with Markov Chain Monte Carlo techniques to perform history matching on synthetic reservoirs (Maschio and Schiozer, 2014). ANNs have also been applied to steam flooding in heavy oil fields (Amirian et al., 2015; Panjalizadeh et al., 2014). Many of the studies mentioned in this section are done with simple or synthetic reservoirs. Application of these techniques on larger and more realistic fields is needed to confirm the robustness of these modeling methods. Another area of future research is in the optimal design of ANNs. Little has been done in this area, specifically for ANNs as proxy models for reservoir injection optimization.

Physics based models

Reduced order models with reservoir physics have also been developed. The models mentioned above are mostly data driven, and are not particular to modeling petroleum reservoirs. One example of the physics based approach is a streamline model, that predicts flow on streamlines rather than through grids in finite volume modeling. Streamline models are used to determine how injected fluids flow and affect production. These models provide significant speed-ups in computation time by assuming incompressibility of the reservoir fluids. This allows the pressure and transport equations to be decoupled and solved more easily. Streamline models have been applied extensively to waterfloods (Safarzadeh et al., 2014; Park and Datta-Gupta, 2013; Bostan et al., 2013; Ghori et al., 2007; Thiele and Batycky, 2003; Bostan et al., 2011). These models are used to determine the efficiency of individual injection wells (Thiele and Batycky, 2003; Bostan et al., 2011, 2013; Safarzadeh et al., 2014). This allows engineers to understand which injectors to shut in, as well as understand subsurface flow in heterogeneous reservoirs.

Similar to streamline models, the Capacitance Resistance Model (CRM) is a reduced order model that allows for the evaluation and optimization of waterflood injec-

tion schemes over the time scale of months (Sayarpour et al., 2009). Only the injection and production data are required, although bottom hole data can be used to obtain a more accurate model (Weber, 2009; Sayarpour et al., 2009). Mamghaderi and Pourafshary (2013) developed a CRM model that accounts for the cross flow of reservoir fluids between reservoir layers. This increases the computation time and number of parameters of the model, but allows for more accurate production predictions to be made in layered reservoirs. Like the data driven models, the CRM model is best suited to legacy assets, allowing engineers to quickly and easily estimate the connectivity and time constants between wells. However the CRM parameters are time invariant, therefore the model may not predict well over the whole life of the well without refitting the parameters. Refitting the parameters can happen either as a batch process as additional production is available or as a Bayesian estimation approach such as an Ensemble Kalman Filter (Jafroodi and Zhang, 2011).

To achieve a good fit with any of the data driven models discussed above, it is necessary to train the model. Without the right data all of the dynamics of the model may not be excited and can lead to poor predictions. Thus, it is important that data driven models are trained on data with sufficient variation in injector flows. Perturbation of injection rates is required for training linear reduced order models (Rezapour et al., 2013) although this is typically not an issue in practice due to maintenance and other activities that require injectors to be shut off periodically. Good training data contains excitation of the dynamic modes of the reservoir while also remaining in the linear regime for which the model is still valid. It is important also to note that because reservoirs are time varying systems, any data that is used for training is only valid over a certain time and therefore models must be periodically retrained to retain accuracy. This poses a practical problem because it may not be economically favorable to perturb injection rates for model training purposes. Injection is scheduled to meet these two constraints (Rezapour et al., 2013). In the later production stages reservoir dynamics become less nonlinear, favoring the use of simpler linear models.

Table 1 provides a comparison of the different modeling methods reviewed. Each method has advantages (+) and disadvantages (-) that make particular strategies desirable for situations depending on geologic data, production data, computational speed limitations, prediction horizon requirements, presence of a gas cap ver-

sus incompressibility, and need for extrapolation outside of the training data.

Table 1. Summary of model advantages and disadvantages

Proper Orthogonal Decomposition
(+) Faster convergence than stochastic methods
(+) Does not require production data
(-) Requires geologic data
(-, without TPWL) Limited speedup because of Jacobian reconstruction at each time step
(-, with TPWL) Requires preprocessing for TPWL construction
Capacitance Resistance Model
(+) Ideal for legacy fields with production data
(+) Requires limited geologic reservoir information
(-) Requires extensive production data that must include changes in injection rates
(-) Poor prediction over reservoir production life-span due to time-invariant parameters
Streamline Method
(+) Improved computation speed over rigorous simulation
(+) Easily determine efficiency of injector wells through streamlines
(-) Assumes incompressibility
(-) Only suitable for two phase (water and oil) systems
Linear Model Identification (ARX, FIR, etc.)
(+) Requires limited geologic reservoir information
(+) Linear time series model with reliable convergence
(-) Requires production data with input perturbations
(-) Higher order models may be over-parameterized
Artificial Neural Network
(+) Model nonlinearities with no reservoir knowledge
(+) Requires no geologic data
(-) Poor predictions outside of training data range
(-) Quality depends heavily on production/injection data

Case Study: SPE 10 Benchmark Model Reduction

An industry-standard waterflood injector case study is used to illustrate model reduction methods with linear model identification. Model 2 from the 10th SPE (Society of Petroleum Engineers) comparative solutions project provides a large complex reservoir suitable for this study (Christie et al., 2001). An ARX model of varying orders and with or without constraints is used in this study. Several different linear dynamic model

forms can be used to fit data generated by a process or dynamic simulation. For this work, the ARX model form is selected. The ARX relates previous values of inputs and outputs to the current model output estimate (see Equation 1).

$$y_k = \sum_{i=1}^{n_y} \alpha_i y_{k-i} + \sum_{i=1}^{n_u} \beta_i u_{k-i} \quad (1)$$

where α and β are model parameters and n_u and n_y denote model order in the input and output parts of the model. An advantage of the ARX model is it is linear in the model parameters. A disadvantage is that, in the presence of noise, the model is biased if the selected model order is not sufficiently high. For the example considered here, this will not be an issue since the data is generated from a fundamental model without noise.

A constraint is imposed in this case study to bound the model steady-state gains between every injector-producer pair. From material balance considerations, each gain must be less than one. The model gain, expressed as a function of the ARX parameters α_i and β_i , is given by Equation 2.

$$K = \frac{\sum_{i=1}^{n_u} \beta_i}{1 - \sum_{i=1}^{n_y} \alpha_i} \quad (2)$$

To satisfy the material balance in the reservoir constraints are placed on the model gains as shown in Equation 3.

$$0 \leq K \leq 1 \quad (3)$$

The objective of the estimator is to align the model and measured values by adjusting the parameters α_i and β_i . A squared error objective function of the following form shown in Equation 4.

$$\min_{\beta_i, \alpha_i} \Theta = \sum_{i=1}^n (y_i - y_{m_i})^2 \quad (4)$$

where n is the number of data points. The minimization is performed subject to the above gain constraint as shown in Equation 3.

Figure 3 is a graphical representation of the reservoir and Table 2 show the specific well locations for each injector and producer. The simulation is performed using the CMG IMEX simulator.

Figure 4 shows the four injector inputs over the simulation time span. Output data is recorded every 25 days

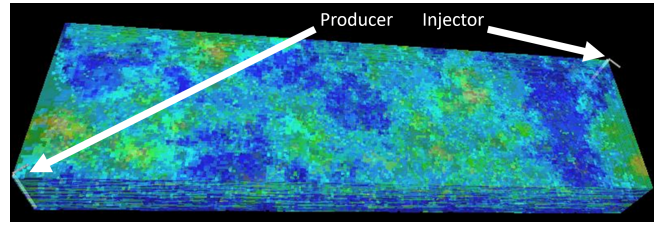


Figure 3. Example of injector and producer placement in the SPE 10 benchmark.

Well Name	X Location	Y Location
Injector 1	9	20
Injector 2	21	20
Injector 3	33	20
Injector 4	45	20
Producer 1	9	200
Producer 2	21	200
Producer 3	33	200
Producer 4	45	200

Table 2. Well Locations on the SPE 10 reservoir model

for the first 650 days and then collected less frequently as time approaches 2000 days.

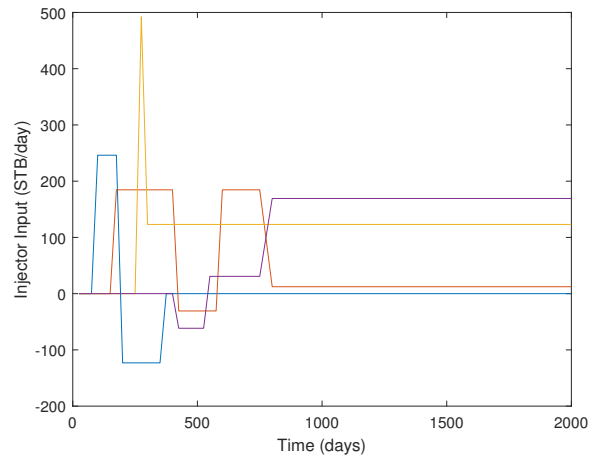


Figure 4. Injectors flow rate profiles over 2000 days of simulation.

Results and Discussion

An unconstrained second order ARX model is initially generated using data for all 2000 days. A more rigorous approach is to optimally design the input signals (Panjwani and Nikolaou, 2016; Darby and Nikolaou, 2009). Figure 5 shows the resulting model with the arbitrary input signals.

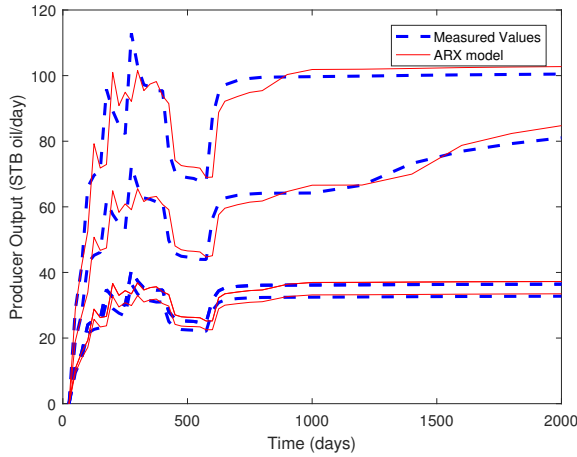


Figure 5. Second order ARX model without constraints.

Additional ARX models are generated using successively limited amounts of data to test the influence of limited data availability on the regression method. Each time the data is incrementally reduced and a model is generated using the remaining data. Figure 6 shows an example of one of these resulting models where only 450 days of data are used to build the ARX models.

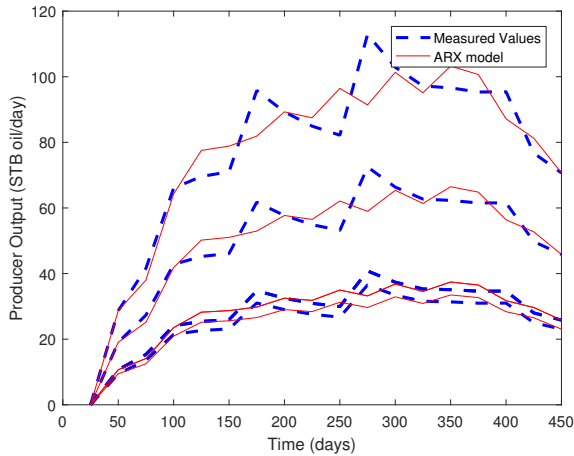


Figure 6. Limited data with second order ARX model without constraints.

Figure 6 shows that the model quality declines significantly without the remaining data. Further analysis shows that four of the 16 gain constants from this model result in negative values, which would physically represent that the injector wells produce oil. Because this is not plausible to have reverse flow, one way to improve this model is to clip all negative gains at zero. With an already poor model, such an adjustment exacerbates the inaccuracy of this model and renders it of little value for

optimization. As an alternative to clipping, lower gain constraints of zero and a sum of gains less than one are added to the ARX models for the identification. Figure 7 shows the 2000 day constrained ARX model.

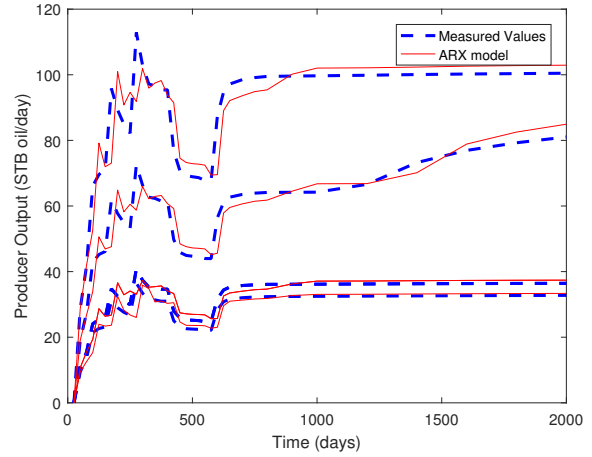


Figure 7. Second order ARX model with constraints.

Notably, Figures 7 and 5 both fit the data very well and would be expected to produce reliable control results. However, it requires 2000 days of data in order to achieve this level of accuracy. By adding constraints to the ARX model, the model quality improves significantly. Figure 8 shows an example of a data-limited ARX model with 450 days of data.

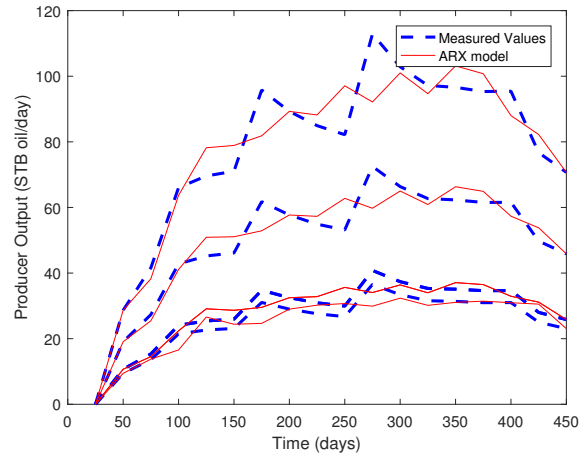


Figure 8. Limited data, second order ARX model with constraints. While the fit is excellent, an analysis of the gains is necessary to demonstrate the superior model quality with constraints.

Figure 9 shows a comparison of constrained versus unconstrained ARX models with limited data. For each ARX model, the sum of the absolute value of the 16

injector to producer gains are taken as a concise measure of the model accuracy. While this is not a perfect measure, it is a consolidated metric that can be used to evaluate the steady-state performance of the multivariate system. Dashed lines are plotted to show the model gains regressed from 2000 days of data. As data is reduced, both the constrained and the unconstrained ARX models deviate from initial gain values. The constrained models perform significantly better than the unconstrained models, especially with limited data, because the constraints include physical information that would otherwise not be included. Because constrained models can implement balance equation constraints, the models can be expected to maintain better accuracy when only limited data is available. Without constraints, the models may fit the data better over the training data but unrealistic parameters give undesirable control models.

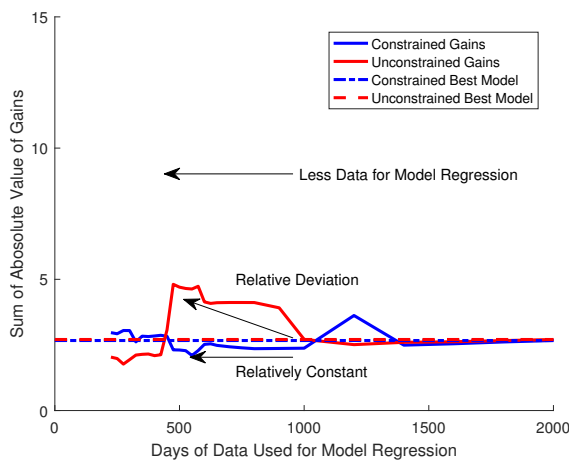


Figure 9. Constrained second order ARX models compared to unconstrained models.

Benefits of Higher Order Models

Similar constrained and unconstrained analysis is completed for third and fourth order models, as shown in Figures 10 and 11. Higher order models can capture more of the process dynamics but don't have a significant advantage like adding constraints.

Figure 12 shows the results of unconstrained second, third, and fourth order models as data is limited. The fit with only eight data points is not included in the fourth order plot due to failure to converge, possibly due to overparameterization with the limited data.

As was the case in the previous section, the metric for goodness of fit is consistency in the gains as data is limited. For the unconstrained case, the second and third

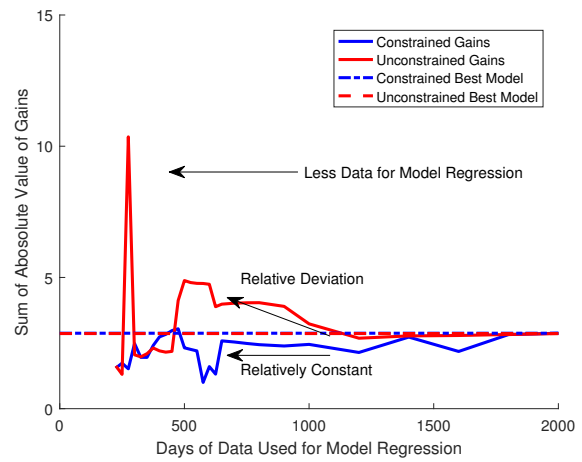


Figure 10. Constrained third order ARX models compared to unconstrained models.

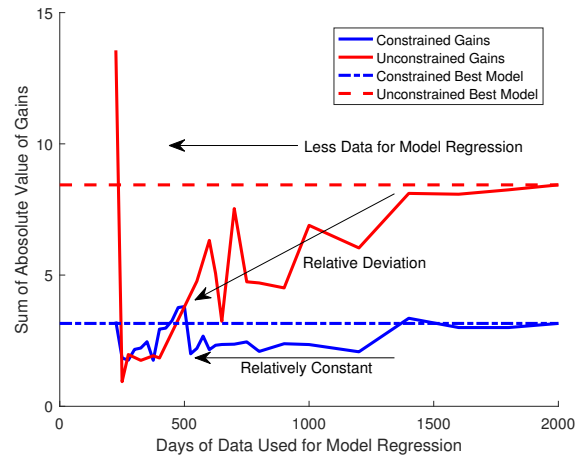


Figure 11. Constrained fourth order ARX models compared to unconstrained models.

order models maintain approximately the same gains until about half of the data is removed. The fourth order models maintain consistency until slightly more than a quarter of the data is removed. On the other hand, Figure 13 shows the constrained case, where there is noticeably less deviation seen by all three ARX orders from original gain values.

The advantage of both constraints and higher orders is that model quality is higher with limited data. One characteristic of high order models is that numerical solvers have difficulty converging when there is not enough data because there are simply too many possible solutions (overparameterized). This can be seen by eight of the models with fourth order unconstrained ARX analysis did not converge, but all of these converged when constrained. By adding constraints, the

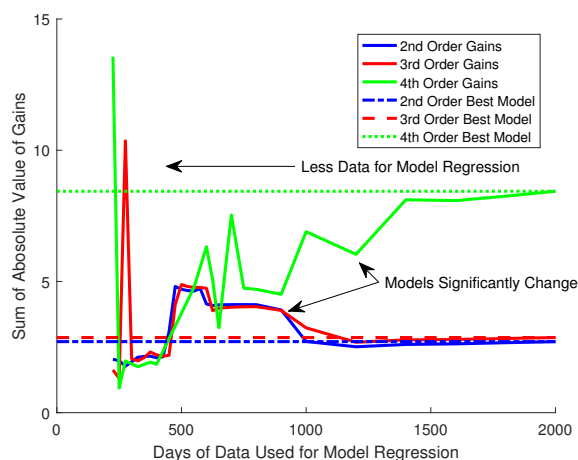


Figure 12. Unconstrained ARX models of various orders (number of α and β parameters).

solution space is significantly reduced and reliable models are produced with limited data.

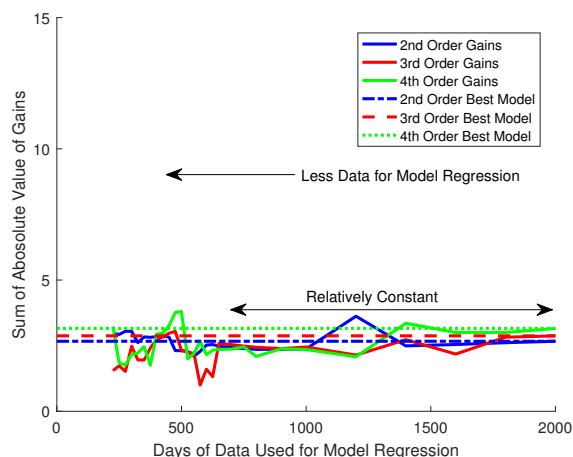


Figure 13. Constrained ARX models of various orders.

The 4 injector and 4 producer reservoir history matching problem provides a compelling example of the value of adding constraints for model identification. Convergence of higher order models is improved significantly as well as consistency between the models of the full data set and those of the limited data sets. Future work should include extension of this trend to more complex systems, higher order models, extension to an ℓ_1 -norm objective function, and model identification with other methods such as Output Error (OE) to reduce the potential for biasing.

Conclusion and Future Work

This brief review article discusses reduced order modeling techniques that enable optimization algorithms for injection planning and forecasting. Model reduction can be performed by either reducing high fidelity simulator models or by creating new reduced order data driven models. Both methods have been shown to significantly improve computation time allowing for optimization algorithms to be used. While many reduced models have been applied to waterflooding, there are relatively few models for enhanced oil recovery techniques, such as steam and polymer flooding. There is also future work in quantifying the uncertainty of model parameters. There has been some progress regarding design of experiment to determine the best type of data to train reduced order models, however this is an area that has received little attention. Linear time variant systems are inherently difficult to model and continued improvements in modeling important reservoir non-linearities are necessary future research subjects. Constrained identification allows for improved estimation under limited data. Results show that enforcing parameter constraints in conjunction with estimation allows for use of less data during training and improves model fit. Higher order constrained models also show improved training with less data. The benefits of higher order models can still be achieved under limited data by adding constraints. More research is required in creating accurate models of these systems while remaining computationally efficient.

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