17th European Symposium on Computer Aided Process Engineering – ESCAPE17
V. Plesu and P.S. Agachi (Editors)

An Efficient Global Event-Based Continuous-Time Formulation for the Short-Term Scheduling of Multipurpose Batch Plants

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Abstract

In the last decade, the PSE community has achieved remarkable results in relation to the problem of short-term scheduling of batch processes. Numerous optimization models, able to solve different problem variants, have been reported. Currently, the great majority of the formulations adopt a continuous-time approach given its advantages over other alternative representations. This contribution presents a simple continuous-time formulation based on the global event concept that relies on general sequencing and precedence constraints. Computational results show that the proposed formulation outperforms other continuous-time approaches without affecting the solution quality.

Keywords: short-term scheduling; multipurpose batch plants; continuous-time formulation; global events; general sequence and precedence constraints.

1. Introduction

Many models able to tackle the problem of short-term scheduling of multipurpose batch plants have been developed in the past few years. Recently, Méndez et al. [1] presented an exhaustive review of the state-of-the-art in this challenging area. Likewise, Shaik et al. [2] compared different ways of dealing with time in a continuous manner; by means of time slots, unit-specific and global event points. In this contribution, a global event-based continuous-time formulation is introduced. Due to the fact that time slot and event point-based methods require solving a given problem many times (by gradually increasing the number of events until the solution does not present an improvement), it is essential to reduce both the number of iterations and the computational load required for reaching the optimal solution at each iteration. Consequently, the aim of the proposed approach is to provide a simple and efficient formulation that is easily understandable and applicable, as well as provides good quality solutions in reasonable CPU times.

2. Fundamental Concepts of the Proposed Approach

This new formulation, like [3], is based on global event points that represent the ending of tasks. It can be applied to either sequential or network processes and is able to take into account variable batch sizes and processing times, sequence-dependent changeovers and various storage policies, such as unlimited, finite (both in dedicated and shared units) and non-intermediate storage.
The key issues of the proposed model are the following: (i) a continuous-time representation is adopted; (ii) a predefined set of event points, common to all the units across the process, captures the endings of the task executions; (iii) variables that model task start and finish times are eliminated, only global event times are included; (iv) in order to consider changeover times efficiently slack variables are added to the model. These slack variables artificially force the end of processing tasks to not coincide with the model event points. This relaxation results in a decrease on the number of global events and allows an enhanced accommodation of the event grid; (v) general sequencing constraints, which apply on tasks being executed in the same unit, add flexibility to the resulting MILP models. These constraints are imposed during an extended period that may include various global event points; (vi) big-M constraints are not required; (vii) general precedence constraints ensure the fulfillment of material balances by forcing the precedence of tasks that produce a given material over those that consume it; (viii) renewable resources (utilities, manpower, etc.) can be taken into account since the model is based on global event points; (ix) a special state-task network structure is adopted. It is based on a pre-ordering of activities that results from an analysis of the STN, the existence of initial inventory, etc. This representation, which is referred as Ordered STN (OSTN), permits estimating a number of events' lower bound. Moreover, it allows a meaningful reduction of the number of binary variables and constraints that are posed at each iteration.

3. Mathematical Formulation

3.1. Equipment Allocation Constraints

$$\sum_{i \in I_j, i \notin I_k} Y_{i,j,k} \le 1, \quad \forall\ j \in J; k \in K \tag{1}$$

3.2. Relation Among Binary Variables

$$\sum_{ii \in I_j,\, ii \in I_i,\, ii \notin I_k} Y_{ii,j,k} \le |J| X_{ik}, \quad \forall\, i \in I;\, k \in K;\, k < |K| \tag{2}$$

3.3. Global Event Points Sequencing Constraints

$$T_k \ge T_{k-1}, \quad \forall\, k \in K;\, k > 1 \tag{3}$$

3.4. Global Event Times' Lower Bounds

$$T_k \ge \sum_{i \in I_j,\, i \notin I_k} \left(a_{i,j} Y_{i,j,k} + b_{i,j} B_{i,j,k} \right) + S_{j,k}, \quad \forall\, j \in J;\, k \in K \tag{4}$$

$$S_{j,k} \le H \left(\sum_{i \in I_j,\, i \notin I_k} Y_{i,j,k} \right), \quad \forall\, j \in J;\, k \in K \tag{5}$$

3.5. General Sequencing Constraints

$$T_k \ge T_{kk} + \sum_{i \in I_j,\, i \notin I_k} \left[a_{i,j} \left(Y_{i,j,k} + \sum_{ii \in I_j,\, ii \notin I_{kk}} Y_{ii,j,kk} - 1 \right) + b_{i,j} \left(B_{i,j,k} - B^{\max} \left(1 - \sum_{ii \in I_j,\, ii \notin I_{kk}} Y_{ii,j,kk} \right) \right) \right] + S_{j,k} \tag{6}$$
$$\forall\, j \in J;\, k, kk \in K;\, k > kk$$

3.6. General Sequencing Constraints for Sequence-Dependent Changeover Times

$$\begin{aligned} T_k \ge\; & T_{kk} + a_{i,j} \left(Y_{i,j,k} + \sum_{ii \in I_j,\, ii \notin I_{kk}} Y_{ii,j,kk} - 1 \right) + b_{i,j} \left(B_{i,j,k} - B^{\max} \left(1 - \sum_{ii \in I_j,\, ii \notin I_{kk}} Y_{ii,j,kk} \right) \right) + S_{j,k} - S_{j,kk} \\ & + \sum_{ii \in I_j,\, ii \notin I_{kk}} \sigma_{ii,i,j} Y_{ii,j,kk} - \sigma^{\max} \left(1 - Y_{i,j,k} \right), \quad \forall\, k, kk \in K;\, k > kk;\, j \in J;\, i \in I_j;\, i \notin I_k \end{aligned} \tag{7}$$

3.7. Batch Size Constraints

$$B_{i,j,k} \le B_{i,j}^{\max} Y_{i,j,k}, \quad \forall\, k \in K;\, i \notin I_k;\, j \in J_i \tag{8}$$

$$B_{i,j,k} \ge B_{i,j}^{\min} Y_{i,j,k}, \quad \forall\, k \in K;\, i \notin I_k;\, j \in J_i^{B\min} \tag{9}$$

3.8. General Precedence Constraints

$$T_k - T_{kk} \ge a_{i,j} \left(Y_{i,j,k} + X_{i,kk} - 1 \right) + b_{i,j} \left(B_{i,j,k} - B^{\max} \left(1 - X_{i,kk} \right) \right) + S_{j,k}, \tag{10}$$
$$\forall\, k, kk \in K;\, k > kk;\, i \notin I_k;\, j \in J_i$$

3.9. Material Balance Constraints

$$Inv_{m,k} = Inv_m^0 - \sum_{i \in I_m^c,\, i \notin I_k} \sum_{j \in J_i} Cs_{i,m} B_{i,j,k} + \sum_{i \in I_m^p,\, i \notin I_k} \sum_{j \in J_i} Pd_{i,m} B_{i,j,k} - \sum_{i \in I_m^c,\, i \notin I_k} \sum_{j \in J_i} Cs_{i,m} B_{i,j,k+1},$$
$$\forall\, m \notin M^a;\, k \in K;\, k = 1 \tag{11}$$

$$Inv_{m,k} = Inv_{m,k-1} + \sum_{i \in I_m^p,\, i \notin I_k} \sum_{j \in J_i} Pd_{i,m} B_{i,j,k} - \sum_{i \in I_m^c,\, i \notin I_k} \sum_{j \in J_i} Cs_{i,m} B_{i,j,k+1},$$
$$\forall\, m \notin M^a;\, k \in K;\, 1 < k < |K| \tag{12}$$

$$Inv_{m,k} = Inv_{m,k-1} + \sum_{i \in I_m^p,\, i \notin I_k} \sum_{j \in J_i} Pd_{i,m} B_{i,j,k}, \quad \forall\, m \notin M^a;\, k \in K;\, k = |K| \tag{13}$$

3.10. Initial Consumption Constraints

$$\sum_{i \in I_m^c,\, i \notin I_k} \sum_{j \in J_i} Cs_{i,m} B_{i,j,k} \le Inv_m^0, \quad \forall\, m \notin M^a;\, k \in K;\, k = 1 \tag{14}$$

3.11. Storage Constraints

$$Inv_{m,k} \le SC_m, \quad \forall\, m \in M^{md};\, k \in K \tag{15}$$

3.12. Storage Constraints for Shared Tanks

$$Inv_{m,k} \le SC_m W_{m,k}, \quad \forall\, m \in M^{ms};\, k \in K \tag{16}$$

$$\sum_{m \in M_t} W_{m,k} \le 1, \quad \forall\, t \in T;\, k \in K \tag{17}$$

3.13. Demand Satisfaction Constraints

$$Inv_{m,k} \ge D_m, \quad \forall\, m \in M^s;\, k \in K;\, k = |K| \tag{18}$$

3.14. Total Operating Time Constraint

$$T_k \le H, \quad k \in K;\, k = |K| \tag{19}$$

$$T_k \le Mk, \quad k \in K;\, k = |K| \tag{20}$$

3.15. Total Profit Calculation

$$TP = \sum_{m \in M^S} MP_m Inv_{m,k} + \sum_{m \in M^V} MP_m \left(Inv_{m,k} - Inv_m^0\right) - \sum_{m \in M^P} MP_m \left(Inv_m^0 - Inv_{m,k}\right), \quad k \in K;\, k = |K| \tag{21}$$

4. Results and Discussion

A benchmark case study (Example 3 of Janak et al. [4]) is dealt with to illustrate the efficiency of the new model. This example involves sequence-dependent changeover times, shared storage tanks as well as variable batch sizes and processing times. Relevant data and comparative results with other formulations can be found in [3]. The associated OSTN representations are shown in Fig. 1.
The initial pre-ordering shows that tasks T12 and T22 are unable to end at the first time point since no preliminary inventory of intermediate materials is available. Similarly, tasks T13 and T23 cannot end before the finishing of tasks T12 and T22, respectively (i.e., before time point number 3). In consequence, at least three global time points are necessary. In turn, the final sequence is depicting the latest allowed task completions. For example, it is not convenient to finish tasks T11 and T12 after the $|K|$-2 time point, since the material produced by them would not be used before the end of the scheduling horizon.

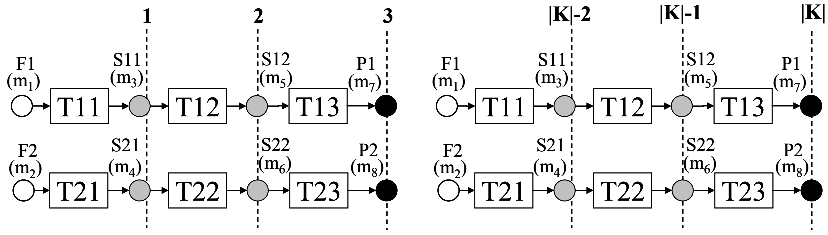

Figure 1. OSTN representation for the chosen example

Due to lack of space only one problem instance, that pursues a maximum-profit objective, is reported in this paper. The GAMS/CPLEX 10.0 solver was used to implement the proposed MILP model on a Pentium IV (3.0 GHz) PC with 2 GB of RAM. Computational results are presented in Table 1. As seen, they exhibit a very good performance.

Table 1. Model and solution statistics for the problem instance: $D_{P1}=D_{P2}=2$ *ton, H=12 h*

K	CPU time (s)	Nodes	RMILP (10^3 \$)	MILP (10^3 \$)	Binary variables	Continuous variables	Constraints	Nonzeros
5	0.07	21	9.000	8.000	62	64	248	968
6	0.20	114	12.000	9.000	78	79	343	1484
7	2.11	1711	14.636	9.000	94	94	452	2110

5. Conclusions

A new continuous-time formulation to tackle the short-term scheduling problem of multipurpose batch plants has been presented. Despite its simplicity, it can address various problem complexities quite efficiently. The proposal was tested by means of several examples. In all cases, a small number of variables and constraints were generated and problems were solved in a low CPU time.

Nomenclature

(a) Sets/Indices

K/k,kk = global event points
I/i,ii = tasks
J/j = units
$\boldsymbol{I_k}$ = tasks for which their ending at T_k is unfeasible or uneconomical
M/m = materials
T/t = shared tanks
$\boldsymbol{I_j}$ = tasks that unit j can perform
$\boldsymbol{J_i}$ = units that can perform task i
$\boldsymbol{J_i^{Bmin}}$ = units on which the minimum batch size condition applies for task i
$\boldsymbol{I_m^c}$ = tasks that consume material m
$\boldsymbol{I_m^p}$ = tasks that produce material m
$\boldsymbol{I_i}$ = tasks producing a material required by task i
$\boldsymbol{M^s}$ = sold materials
$\boldsymbol{M^v}$ = intermediate materials with economic value
$\boldsymbol{M^p}$ = purchased materials
$\boldsymbol{M^a}$ = materials with "as-required" availability
$\boldsymbol{M^{md}}$ = materials stored in dedicated units with maximum capacity
$\boldsymbol{M^{ms}}$ = materials stored in shared units with maximum capacity
$\boldsymbol{M_t}$ = materials that can be stored in shared tank t

(b) Parameters

H = time horizon
$\boldsymbol{a_{i,j}}$ = fixed duration of task i in unit j
$\boldsymbol{b_{i,j}}$ = variable duration of task i in unit j
$\boldsymbol{\sigma_{ii,i,j}}$ = sequence dependent changeover time
$\boldsymbol{\sigma^{max}}$ = maximum changeover time
$\boldsymbol{B_{i,j}^{max}}$ = maximum batch size of i in unit j
$\boldsymbol{B_{i,j}^{min}}$ = minimum batch size of i in unit j
$\boldsymbol{B^{max}}$ = maximum batch size
$\boldsymbol{Inv_m^0}$ = initial amount of material m
$\boldsymbol{Cs_{i,m}}$ = mass balance coefficient for the consumption of material m by task i
$\boldsymbol{Pd_{i,m}}$ = mass balance coefficient for the production of material m by task i
$\boldsymbol{D_m}$ = fixed demand of material m
$\boldsymbol{MP_m}$ = price (value) of material m
$\boldsymbol{SC_m}$ = maximum storage capacity for material m

(c) Variables

$\boldsymbol{Y_{i,j,k}}$ = 1 if task i finishes in unit j at T_k
$\boldsymbol{X_{i,k}}$ = 1 if a predecessor of task i is finished at T_k
$\boldsymbol{W_{m,k}}$ = 1 if material m is stored at T_k
$\boldsymbol{T_k}$ = time corresponding to event point k
$\boldsymbol{S_{j,k}}$ = slack time for unit j at T_k
$\boldsymbol{B_{i,j,k}}$ = batch size of task i finishing at T_k in unit j
$\boldsymbol{Inv_{m,k}}$ = amount of material m at T_k
TP = total profit
Mk = makespan

Acknowledgements. This work has been supported by CONICET, UNL and ANPCyT (PICTs 12628 & 14717).

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