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Parametric Programming & Control: From Theory to Practice

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Abstract

Parametric programming provides solution of an optimization problem as a function of the parameters in a computationally efficient manner, without exhaustively enumerating the entire space of parameters. In a model based predictive control framework, parametric programming can be used to obtain optimal control variables as an explicit function of the state variables. The main advantage of this approach is that it reduces on-line control and optimization to simple function evaluations which can be implemented on a simple computational hardware such as a microchip, opening avenues for many applications in chemical, energy, automotive and biomedical systems. This paper presents an overview of parametric programming and control, some recent achievements of its applications and its potential for fast, efficient and portable implementation of advanced model based control technology of the future.

Keywords: Model based control; on-line optimization; parametric control; biomedical control; automotive control; chemical process systems control.

1. Introduction

The last decades have seen the rapid increase in the use of optimization- based techniques for improved design, control and operation of various types of engineering systems. A prime difficulty in applying these types of techniques to real systems and processes arises from the unavoidable presence of variations in the problem parameters such as fluctuations in uncertain inputs and measurements, or variations in inherent system properties and characteristics. These variations readily translate to deviations from the prescribed optimal point, thus, either failing to exploit fully the benefits of the optimization based solution or requiring the repetitive solution of the problem for different values of the problem parameters. Parametric programming is a technique that determines computationally inexpensively the exact mapping of the optimal solution profile in the space of the system parameters. In this way the repetition of the problem solution is avoided, while the optimal solution can readily adapt to the system variability. In our group we have developed algorithms for multi-parametric (mixed integer) linear, quadratic, non-linear and dynamic optimization problems that are commonly encountered in (i) optimization under uncertainty, where the uncertainties are the problem parameters, (ii) multi-level and multi-objective optimization where the different objectives play the role of the parameters and (iii) model-based on-line control and optimization where the process states correspond to the parameters. In this paper, we will first give an overview of the mathematical foundations of multi-parametric programming for different classes of mathematical models. We will then discuss its application to model-based optimal control, with emphasis on how to design off-line affordable advanced parametric controllers for chemical, energy, automotive and biomedical systems.

2. Parametric Programming

Parametric programming is concerned with the solution of the following problem:

$$
\begin{aligned}
& J(x) = \min_{u} f(u,x) \\
& \text{s.t. } h(u,x) = 0 \\
& \quad g(u,x) \le 0 \\
& \quad x \in X
\end{aligned}
$$

where x is the vector of parameters, u is the vector of optimization variables, f is a scalar objective function, h is the vector of equations representing for example the model of a system, g is a vector of constraints such as a lower and upper bounds on x and u, and X is a compact, convex and real set. The solution of this problem is given by: $u(x) = u^i(x)$ if $x \in CR^i$ such that $CR^i \cap CR^j = \emptyset,\ i \neq j\ \forall i,j = 1, \ldots, N$ and $CR^i \subseteq X\ \forall i = 1, \ldots, N$. Note that a CR^i is a critical region where a particular solution $u^i(x)$ is valid. For example when f, g and h are linear and separable in u and x, a CR^i is a polyhedron and corresponds to a unique set of active constraints and $u^i(x)$ is an affine function of x; a graphical interpretation is given in Figure 1. The algorithms for obtaining $u^i(x)$ and CR^i depend upon whether f, g and h are linear, quadratic, nonlinear and convex or not and also on whether u and x are vectors of continuous or mixed continuous and integer variables. These algorithms with illustrative example are discussed in detail in [1]. In the next section we show how the solution obtained by using parametric programming can be used to significantly improve on-line implementation of model based controllers.

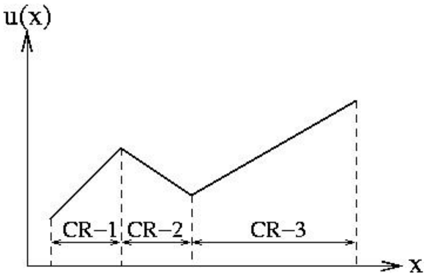

Figure 1. Parametric Programming

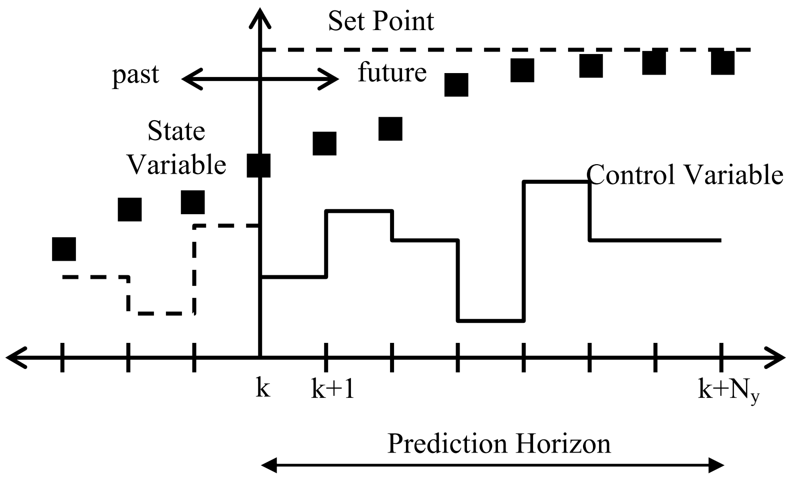

Figure 2. Model Predictive Control

3. Parametric and Model Based Predictive Control

Model Predictive Control (MPC) problems can be formulated as the following optimization problem:

$$\min_{u(k),\ldots,u(k+Nu)} J = \sum_{k=0}^{N_y} [x'(k)Qx(k)] + \sum_{k=0}^{N_u} [u(k)' Ru(k)]$$

$$\text{s.t.} \quad \begin{aligned} & x(k+1) = f(x(k), u(k)) \\ & x_{\min} \le x(k+1) \le x \le x_{\max}, k = 0,1,\ldots, N_c \\ & u_{\min} \le u(k) \le u_{\max}, k = 0,1,\ldots, N_c \end{aligned}$$

where x and u are the vectors of state and control deviation variables respectively, N_y, N_u and N_c are the prediction, control and constraint horizons respectively, Q and R are weights on deviations of the state and control variables, and k denotes a time interval. The basic idea of an MPC implementation is shown in Figure 2 where at the current time interval, k, the optimization problem is solved to minimize the state and control deviations from the set point by implementing the optimal vales of the control variables. Note that only the first control element is implemented and this sequence is repeated at the next time intervals, for the new state measurements or estimates, until the desired or set point values are obtained. The key advantage of MPC is that it can take into account the constraints on state and control variables but its major limitation is its on-line computational effort.

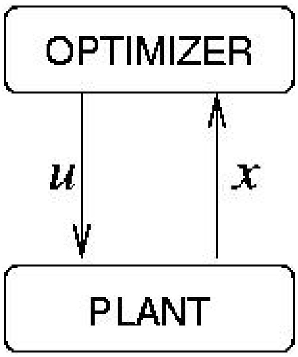

Figure 3. Traditional MPC

Parametric programming can be used to obtain u as an explicit function of x reducing the on-line model based control and optimization problem to a sequence of function evaluations. Figure 3 shows a traditional implementation of MPC whereas Figure 4 depicts implementation of the parametric control. The parametric profiles in the parametric control setting can be stored on a simple computational hardware such as a micro-chip. The key advantages of such an implementation are that it is computationally efficient since it requires simple function evaluations; it does not require any on-line optimization software; it is excellent news for safety critical applications, and advanced model based controllers can be implemented via portable control devices that are of great importance for drug delivery systems. Details of theory and applications of parametric control are presented in [2].

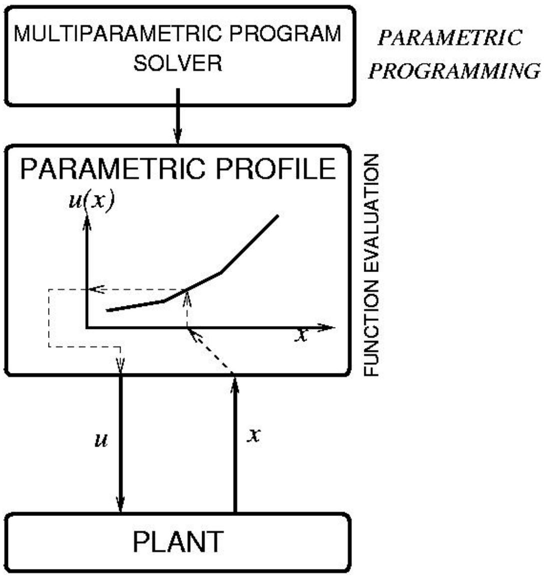

Figure 4. Parametric Control

The theoretical developments presented earlier have been tested on a number of applications including:

- Parametric Control of a Partially Simulated Exothermic Pilot Plant Reactor [3],
- Parametric Control of an Industrial Air Separation Unit [4],
- Parametric Control of an Active Valve Train Actuation System [5],
- Parametric Control of Blood Glucose for Type 1 Diabetes [6],
- Parametric Control of Surgery under Anesthesia [7].

4. Concluding Remarks

In this paper an overview of theory and applications of parametric programming and control with suitable references has been provided where further details can be found. The main advantageous feature of parametric programming is its ability to find the mapping of the optimal solution on the space of the parameters in a computationally efficient manner. This feature translates into a very elegant method for the implementation of advanced model based controllers with applications in chemical, automotive and biomedical systems as shown in Figure 5.

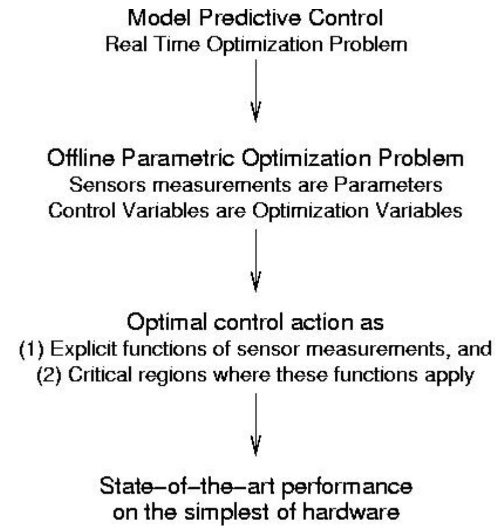

Figure 5. Parametric Control via Parametric Programming

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