17th European Symposium on Computer Aided Process Engineering – ESCAPE17
V. Plesu and P.S. Agachi (Editors)

A Bi-level Decomposition Scheme for the Integration of Planning and Scheduling in Parallel Multi-Product Batch Reactors

Muge Erdirik-Dogan[a], Ignacio E. Grossmann[a], John Wassick[b]

[a] *Department of Chemical Engineering, Carnegie Mellon University, Pittsburgh, PA 15213;E-mail: grossmann@cmu.edu*
[b] *Dow Chemical Company, Midland, MI*

Abstract

We address the simultaneous planning and scheduling of parallel multi-product batch reactors, a challenging problem that has been motivated by a real world application at the Dow Chemical Company. We propose a novel continuous time MILP model for the simultaneous planning and scheduling that is based on slot representation. While effective for short-term scheduling, the proposed model becomes computationally intractable for long planning horizons. Hence, we propose a rigorous bi-level decomposition algorithm that reduces the computational effort of the problem. We decompose the original problem into an upper and a lower level. We iteratively solve the upper and the lower level problems until the difference between the bounds is less than a specified tolerance.

Keywords planning, scheduling, batch plants

1. Introduction

The motivation behind the problem supplied by the Dow Chemical is that the current approaches for managing the planning and scheduling relies on the traditional two step process. The first step involves long range production planning while the second involves short term scheduling. The goal of production plan is to determine production targets for each asset. The goal of scheduling on the other hand is to determine the detailed timing of operations and sequencing so as to meet the targets set by the planning. However, due to the overestimation of the available production capacity at the planning level, production targets may not be realized at the scheduling. One of the major reasons of this overestimation is that the capacity losses due to change overs are not taken into account in this stage. Depending on the magnitude of these changeovers, they can significantly reduce the capacity available for production and could lead to inconsistencies between the planning and scheduling. The simplest alternative of solving this issue is to formulate a single model that spans the entire horizon. The limitation is that the model becomes intractable due to the exponential increase in computation. In order to overcome this problem, we propose a bi-level decomposition scheme that will generate solutions that are theoretically equivalent to performing simultaneous planning and scheduling over the entire horizon at a reasonable computational expense.

2. Problem Statement

Given is a plant that contains batch reactors that operate in parallel. The batch reactors are to be used to manufacture intermediates and final products. A subset of the final products is produced in a single reaction stage, while the remaining final products require intermediates, thus involving two reaction stages with intermediate storage. Each final product is fed to a dedicated storage tank. In order to formulate this problem we assume that we are given the products each reactor can produce, as well as the batch times and batch sizes for each product and the corresponding reactor. While the batch times and batch sizes are fixed, the number of batches of each product is a variable that is to be determined. Sequence dependent changeover times and the total time each reactor is available in each month are given. Given are also raw material costs, and storage tanks with associated capacities. Given is also a production horizon composed of a certain number of time periods given by due dates in which demands are specified as upper bounds. The problem is to determine the production plan and schedule in terms of production quantities for each reactor and the sequence of batches, so as to maximize the profit.

3. MILP Scheduling Model

In order to address the above problem, we first propose a novel continuous time MILP model for the simultaneous planning and scheduling that is based on slot time representation. Each slot represents one potential batch of the product that is assigned on that slot. Since the number of batches of each product is a variable to be determined by the model, the exact number of slots to be utilized is not known prior to solving the model. In order to avoid infeasible or suboptimal solutions, we postulate more than necessary number of slots for each unit and period. Hence, some slots may be left unoccupied. The assignments of products to these slots are to be determined to define the sequence of production on each unit, at each time period. The length of each slot is equal to the batch time of the product assigned on the slot plus the corresponding transition time. If none of the products is assigned to a particular slot, then the length of the slot is forced to zero. Slots do not have to be identical for each unit and each period. The number of postulated slots, start and end times vary for each unit and each time period.

The generic form of the proposed MILP model is as follows (see Erdirik and Grossmann (2007) for details):

Objective Function: The objective is to maximize the profit which is given by the sum of sales revenues, operating costs, inventory costs and total transition costs.

Assignments and Processing times: The key binary variable is $W_{i,m,l,t}$ which becomes 1 if product i is assigned to slot l of unit m during time t. Assignments of products to available slots define the sequence of production. In each slot at most one product can be produced, however the same product can be produced in more than one slot.

Detailed timing relations and sequence dependent transitions: Changeovers occur when the production in one unit is changed from one product to another. When the products assigned to two consecutive slots are different, the corresponding sequence dependent transition time is added to the batch time of the product. Hence, the length of the corresponding slot becomes the summation of the batch time of the assigned product and the corresponding transition time.

Mass and Inventory Balances: A subset of the end products is produced in a single stage whereas the rest of the products require intermediates. Due to the layout of the plant, once an end product is transferred to the dedicated storage tanks, it can not be retrieved back into the plant. Therefore once the production of the end product which is both an intermediate and an end product is completed, each batch is split and transferred to intermediate storage tanks and dedicated storage tanks. The products that are produced in a single stage and in two stages on the other hand, are directly transferred to the dedicated storage tanks and distributed to satisfy customer demands. This feature of the problem requires defining mass and inventory balances for the intermediates, products produced in 2 stages and products produced in a single stage separately. To guarantee feasible mass transfer, we keep track of the materials on a slot base.

4. Solution strategy/Decomposition Algorithm

To avoid the direct solution of the proposed MILP model, we propose a bi-level decomposition algorithm that is similar in spirit to the method by Erdirik-Dogan and Grossmann (2006). The problem is decomposed into an upper level planning and a lower level planning and scheduling problem. The upper level determines the products to be produced at each time period, assignments of products to available equipment as well as the number of batches of each product, production levels and product inventories. The upper level is based on a relaxation of the proposed MILP scheduling model where the detailed timing of production and changeovers are replaced by time balances yielding tight upper bounds on the profit. In the lower level, the MILP model presented in the previous section is solved by excluding products that were not selected and fixing the number of slots to the ones used in the upper level. A lower bound is obtained from the solution of the lower level since its solution corresponds to a feasible solution of the original problem. The lower level determines production and inventory levels as well as the detailed timing and the sequence of production. The procedure iterates until the difference between the upper and the lower bounds is less than a specified tolerance. In order to expedite the search we add integer and logic cuts to the upper level. For long time horizons, computational expense for solving (DP) can be high. For those instances, we circumvent this difficulty by applying a rolling horizon algorithm.

5. The Upper Level Model

In this section, we outline the aggregated MILP model that is based on a network representation, which will be used to predict an upper bound on the profit. The basic idea relies on using mass balances and replacing the detailed timing of production by time balances that anticipate as best as possible the effect of sequence dependent changeovers through sequencing constraints. As will be shown, this has the effect of yielding a tight upper bound on the profit. The decisions that we are concerned with are (i) the assignments of tasks to available equipment at each time period, $YP_{i,m,t}$, (ii) number of batches of each task in each time period, $NB_{i,m,t}$, (iii) amount of material processed by each task in each unit during each time period, $FP_{i,m,t}$.

The generic version of the MILP Planning model is as follows (see Erdirik and Grossmann (2007) for details) (i) objective function (ii) material handled and capacity requirements, (iii) number of batches, (iv) Mass Balances on state nodes, (v) changeover times and costs, (vi) time balance constraints on equipment.

We account for the sequence dependent changeover times and costs without determining the detailed timings of the operations but through sequencing constraints similar to the ones from the traveling salesman problem. In order to do this, we propose to find the minimum transition time sequence within the assigned products within each period while maximizing the profit and satisfying the demands at the due dates. In this way the determination and allocation of number of batches of each task and their sequencing are determined simultaneously. The idea for the sequencing is to generate a cyclic schedule within each period that minimizes transition times amongst the assigned products, and then to determine the optimal sequence by breaking one of the links in the cycle as described in Birewar and Grossmann (1990).

To generate a cyclic schedule the decisions concern the sequence of production which is represented by the binary variable $ZP_{ii'mt}$, which becomes 1 if product i precedes product i' in unit m at time period t, and zero otherwise. The total number of links transitions, NL, within each cycle will be equal to the total number of products assigned to that period. According to the location of the link that is to be broken, a total of NL different schedules can be generated from each cycle. In order to determine the optimal sequence amongst the NL possible sequences, the cycle will be broken at the link with the highest transition time. The binary variable $ZZP_{ii'mt}$ represents location of the link to be broken to obtain the specific sequence. The total transition time within each period is then given by the summation of the transition times corresponding to each existing pair ($ZP_{i,i',m,t}$) minus the transition time corresponding to the link that is broken from the sequence ($ZZP_{i,i',m,t}$). In order to account for the transition times and costs across adjacent weeks, we need to determine the first and last element of each sequence obtained at each period. These elements correspond to the pair where the cycle is broken to form the sequence. According to their relative position in the cycle, the head of the cycle will correspond to the first element and the tail will correspond to the last element. And the transitions will be taken into account from the last product of period t to the first product of period t+1. Finally, the time balance on each equipment states that the total allocation of production times plus the total transition time within that period plus the transition time to the adjacent period cannot exceed the available time for each unit.

Examples

This example consists of five different products to be processed on two reactors R1, R2. Each reactor can process any of the products. Table 1 shows the problem sizes and solution times for the full space method and the proposed method for the case of one week schedule.

Table 1: Results for 5 Products, 2 Reactors, 1 Week

Method	Number of binary variables	Number of continuous variables	Number of Equations	Time (CPUs)	Solution ($)
Full Space	500	2,615	2,185	60.0	1,055,127
Proposed algorithm				1.2	1,055,127
Problem UB	140	207	335	0.6	1,055,127
Problem LB	500	2,615	2,185	0.6	1,055,127

Table 2: Results for 5 Products, 2 Reactors, 6 ,12 ,24 ,36,48 Weeks

Method	Number of binary variables	Number of continuous variables	Number of Equations	Time (CPUs)	Solution ($)
6 weeks					
Rolling Horizon	204	461	746	1.7	3,239,000
12 weeks					
Rolling Horizon	372	929	1496	31	5,575,000
24 weeks					
Rolling Horizon	708	1865	2996	34	10252000
36 weeks					
Rolling Horizon	1044	2801	4496	36	14,076,000
48 weeks					
Rolling Horizon	1380	3737	5996	767	20,342,000

The proposed algorithm yields the global solution of $1,055,127 in 1.2 CPUs whereas the full space method yields the same solution in 60 CPUs. GAMS/CPLEX 9.1 was used to solve these models with a 0.5 % optimality tolerance on a on an Intel 3.2 GHz workstation.. In Table 2, we present the results using a rolling horizon approach for the same example for 6, 12, 24, 36 and 48 weeks.

6. Conclusions

In this paper, an MILP model for the simultaneous planning and scheduling of a multiproduct batch plant has been presented where issues such as sequence dependent changeover times and two stage production with finite storage have been accounted for. While effective for short term scheduling, the resulting model becomes computationally intractable for long time horizons. Therefore, a bi-level decomposition algorithm was used that decompose the problem into an upper level and a lower problem. For the representation of the upper level, we have proposed an MILP planning model where we anticipate the effects of changeovers quite accurately without greatly increasing the computational effort. The results show that the proposed method is significantly faster than the full space solution. Moreover, the solutions obtained by the upper level planning model are very tight and for the cases where subcycles are not observed in the solution, the solutions obtained by the planning model are identical to the solutions of the original problem.

Acknowledgments. The authors would like to acknowledge financial support from the Pennsylvania Infrastructure Technology Alliance, Institute of Complex Engineered Systems, from NSF under Grant DMI-556090 and from The Dow Chemical Company.

References

1. Birewar, B. D.; Grossmann, I. E. Efficient Optimization Algorithms for Zero-Wait Scheduling of Multi-Product Batch Plants. *Ind. Eng Chem. Res.* 1990, 28, 1333-1345.
2. Erdirik-Dogan, M. and I. E. Grossmann, "Simultaneous Planning and Scheduling for Multiproduct Continuous Plants", *Ind. Eng. Chem. Res*, 45. 299-315 (2006).
3. Erdirik-Dogan, M. and I.E. Grossmann, "An Accurate Production Planning Model for the Scheduling of Parallel Batch Reactors with Changeovers", to be submitted for publication (2007).
4. Erdirik-Dogan, M. and I.E. Grossmann, "A Slot-based Scheduling Model for the Scheduling of Parallel Batch Reactors with Changeovers", to be submitted for publication (2007).
5. Mendez, C.A., J. Cerdá , I. E. Grossmann, I. Harjunkoski, and M. Fahl, "State-Of-The-Art Review of Optimization Methods for Short-Term Scheduling of Batch Processes," *Computers & Chemical Engineering* 30, 913-946 (2006).