17th European Symposium on Computer Aided Process Engineering – ESCAPE17
V. Plesu and P.S. Agachi (Editors)

An efficient model implementation to solve a real-world cutting stock problem for a corrugated board boxes mill

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Abstract

In this paper the cutting stock problem for the corrugated board boxes industry is presented. The problem is solved by means of a two step strategy. First, patterns pre-generation model is formulated which are then used as input in a mathematical MILP model that optimizes the cutting process minimizing the paper trim-loss costs. Several parameters have been added to the system such that the planner can manipulate its values to produce a solution according to their customer demands. The system has been linked to the company ERP and is now in production.

Keywords Cutting stock, corrugated board, production plan, MILP.

1. Introduction

As was pointed out by Grossmann and Westerberg [1], companies must design and operate chemical processes effectively and efficiently to survive in today's competitive world. Following their reasoning chemical engineering in the context of Process System Engineering (PSE) has evolved in the past decades from being rooted in the concept of unit operations to one based on engineering science and mathematics. They have proposed a new definition of PSE where the discipline is concerned with the improvement of decision-making processes for the creation and operation of the chemical supply chain. It deals with the discovery, design, manufacture, and distribution of chemical products in the context of many conflicting goals. Although in this work we do not deal with a chemical process, the theoretical issues and the goal pursued corresponds to a model generation for the prediction of performance, and decision making for an engineered system, which is also a main concern for PSE area [1].

In this work a real-world industrial problem of production planning and cutting optimization for corrugated board boxes is presented. The industry studied performs their activities in a very competitive market. To aggregate value, the company should improve both, customer satisfaction and production costs. An efficient production plan improves company competitiveness providing convenient product prices and just in time order deliveries. For modeling the cutting stock problem and the production plan of carton corrugated sheets, the objectives pursued are computational efficiency, planner intervention in the problem inputs and constraints, integration to the company information system.

2. Problem Statement, background

The production of corrugated board boxes sets up a complicated scenario. A variable number of papers must be combined in order to form the board. Paper reels of different width and lengths provided by different suppliers can be used to produce the corrugated board and cut the sheets. Diverse paper layers are used to form the board: liner paper and fluted paper. The most used boards in the industry are: the *single wall*, witch includes two external liner layers and a middle fluted one, and the *double wall board*, formed by three liner layers, two external and one central, and two fluting ones located between the liner central and the externals, respectively. Different possible flute types related to the structural properties required on the board are also involved.

After the corrugation step, the board goes to the cutting stage, where a variable number of corrugated board sheets are obtained from which a box is finally conformed. The cutting stage involves slitting and cross section knives that must be assembled according to the boxes size required in the purchase orders. The cutting machine has *Nlong* knives that cut the board lengthwise and *Ntrans* knives that make the transversal sections. The *Nlong* knives allow the cutting of at most *Nlong-1* sheets per board wide. Although using *Nlong* knives *Nlong+1* board parts could be obtained, the two external ones are discarded because the layers are not perfectly glued. This gives a minimum waste Per_{min}. The *Ntrans* knives limit to *Ntrans* the lengths to cut. The paper waste must be minimized because it has an important impact in the final product cost. The problem is NP-hard due to the huge number of product combinations and variables to manage.

3. Model formulation and implementation

Because of the problem complexity, the approach used to solve the corrugation and cutting problem has key influence on the solution quality and efficiency.

The strategy selected for the implementation must guarantee not only an optimal solution but also reasonable resource consumption. Two main strategies exists [2] to find the optimal solution of similar problem: a) two steps procedures that pre-generates feasible cutting patterns and then use mixed integer linear program (MILP) models to solve the cutting problem, or b) one step strategy where the non-convex formulation can be transformed to overcome bilinearities.
In this case, the first approach mentioned is used because reaching a solution in one step is a very difficult task, due to the high combinatory involved. The initial step generates feasible cutting patterns for a set of orders, and then a MILP optimization model is solved which selects a subset of the patterns and the length to cut to satisfy the demand and stock constraints. The MILP objective function is to minimize trim-loss cost.

3.1. Pattern pre-generation model.

The equations presented in this section consider that customer's pending orders, paper reels stock and its cost are known in order to define a set of feasible patterns. The information resulting from this set is then used as input in the trim-loss MILP optimization model.

$$Wf_P = \sum_{i} N_{iP} . W_i \qquad \forall P, \forall K_j, j = 1..l \quad (1)$$

$$Per_{max} \geq Per_{min} \quad (2)$$

$$WTP_{Kj} - Per_{max} \leq Wf_P \leq WTP_{Kj} - Per_{min} \qquad \forall P, \forall K_j, j = 1..l \quad (3)$$

$$Co_P = \sum_j Cp_{Kj} \cdot (WTP_{Kj} - Wf_P) \qquad \forall P, \forall K_j, j = 1..l \quad (4)$$

$$\sum_i N_{iP} \leq Nlong\text{-}1 \qquad \forall P, \forall I \quad (5)$$

$$\sum_i Yp_{iP} \leq Ntrans \qquad \forall P, \forall i \quad (6)$$

$$Nl_{iP} = Nl_{iiP} \qquad \forall P, \forall i \neq ii \quad (7)$$

$$TP_{Kj\,i\,P} = TP_{Kj\,ii\,P} \qquad \forall P, \forall K_j, j = 1..l, \forall i \neq ii \quad (8)$$

$$O_{iP} = O_{iiP} \qquad \forall P, \forall i \neq ii \quad (9)$$

Where Wf_P is the pattern wide, calculated in eq. (1) as the sum of the N_{iP} number of sheets of order i in the pattern P per W_i that represents the sheet wide of order i. WTP_{Kj} corresponds to the paper type wide used for layer K_j. Equation (3) assures that the width of each pattern P has at most a maximum waste of Per_{max} and at least a minimum Per_{min} in each layer K_j. In eq. (4) Co_p, corresponds to the cost of the trim-loss per meter of pattern P, calculated by multiplying the paper cost per meter (Cp_{Kj}), and the trim loss per meter in layer K_j, denoted by (WTP_{Kj} - Wf_P). In eq. (5) N_{iP} could be at most *Nlong-1*.
When combining different orders they must have the same board class, meaning that liner and flute layers K_j must be of the same type. The sum of Yp_{iP} determines the number of different orders assigned to pattern P, limited to *Ntrans* transversal knives (eq. 6). $Nl_{i\,P}$ and $Nl_{ii\,P}$ indicate the number of layers of orders i and ii assigned to pattern P. In eq. (7) $Nl_{i\,P}$ and $Nl_{ii\,P}$ of orders assigned to pattern P must be the same. Paper type for each layer K_j for orders i and ii, $TP_{Kj\,i\,P}$ and $TP_{Kj\,ii\,P}$ respectively, must be the same in order to be combined in a pattern (eq. 8). In eq. (9) orders assigned to P, must present the same flute $O_{i\,P}$ and $O_{ii\,P}$. If all those constraints are satisfied the pattern P is conformed and its characteristics are saved in order to feed the MILP optimization model.

3.2. MILP optimization model.

$$\text{Min } Z = \sum_P Co_P \cdot x_P \qquad (10)$$

subject to:

$$\sum_{P \in REL_{P\,Kj\,TP\,AP}} x_P \cdot \alpha_{Kj} \leq S_{TP\,AP} \quad \forall K_j,\ \forall TP,\ \forall AP \in REL_{P\,Kj\,TP\,AP} \quad (11)$$

$$\sum_{P \in PAT\_PED_{P\,i}} N_{i\,P} \cdot x_P / L_i \geq D_i \quad \forall i \qquad (12)$$

$$\sum_{P \in PAT\_PED_{P\,i}} N_{i\,P} \cdot x_P / L_i \leq D_i \cdot (1 + \beta_i) \quad \forall i \qquad (13)$$

$$x_P \geq CRmin_P \cdot y_P \qquad \forall P \qquad (14)$$

$$x_P \leq CRmax_P \cdot y_P \qquad \forall P \qquad (15)$$

In eq. (10) the objective function is defined, representing the paper trim-loss cost, where x_P is the pattern length and Co_P the cost of the trim-loss per meter of pattern P. Equation (11) determines that sum of the length of each paper layer K_j in all patterns P must not exceed the length of that paper in stock, $S_{TP\,AP}$. Parameter α_{Kj} is a coefficient for the paper consumption in layer K_j e.g. for liner

layers α_{Kj} is 1, for fluting layers it is greater depending on its profile. Equations (12) and (13) are the demands constraints. L_i is the length of the sheet of order i. In eq. (12) the number of sheets produced per order i in patterns P must be greater than the demand D_i, while eq. (13) allows over-production upper bound β_i, giving flexibility on the cutting plan. In eq. (14) if a pattern P is executed ($y_P=1$) it must be longer than or equal to a minimal run length $CRmin_P$, in eq. (15) $CRmax_P$ is big enough in order to activate eq. (14).
In most cases, some characteristics of the problem cannot be considered in the model formulation. Human expertise should not be disregarded because it can provide a competitive advantage over the system solution. Some parameters have been posed in the model such that the planner can handle them to analyze several scenarios. The planner can handle the following parameters: maximum waste allowed Per_{max}, maximum and minimum number of patterns per order, the number of longitudinal and transversal knives in the cutting machine, minimum run length $CRmin_P$ and can select between mandatory and optional orders to consider, while the plan for mandatory orders must be solved, optional are used to combine and produce a best set of patterns. An interface written in Java has been implemented such that the planning and cutting system can be linked to the company ERP (an Oracle E-Business Solution).

4. Results

In order to illustrate the results obtained with the model executing small example with ten orders are shown in Table 1. Twenty-two paper classes and 15 possible widths were used. The first model pre-generates 240 patterns; eleven of them were selected in the final solution. Table 1 shows the patterns and the main results obtained for this run. The value of the objective function is \$480.76 calculated as the sum of trim-loss cost for pattern P (P_P). The execution time was 0.312 sec.
Models have also been executed in real productive scenarios, for example with 25 orders, 1,577 patterns were generated and 27 of them have been chosen in the final solution. Objective function value was \$1,521.53 and the execution time was 0.453 sec. The trim loss was reduced by a 30% comparing to the plan obtained by the company expert with the old system, whose cost was of \$1,956.70. Another example including 32 mandatory orders and 60 optional used to combine generates 2,501 cutting patterns while the final solution uses 45 of them. The objective function value was \$12,811.25, saving abut 15% comparing to the solution obtained with the old system, the time obtained for this run was 0.941 seconds. One important feature of the mathematical model is the reduced time spent to reach the solution which allows the planner to evaluate in short time several scenarios by manipulating the parameter values, comparing with the old system where the planner spent around four hours to generate a valid solution.

Table 1. Results obtained with 10 orders example

P	i	$N_{i,p}$	ii	$N_{ii,p}$	K_1	K_2	K_3	K_4	K_5	W_p	X_p	$C_{OP}*X_P$
					TP/AP	TP/AP	TP/AP	TP/AP	TP/AP			
1	Ord1	1	--	--	K3/1050	O2/1050	T1/1050	- / -	- / -	1030	1895	$22.93
13	Ord2	1	--	--	B1/1280	O2/1300	T1/1280	- / -	- / -	1260	1125	$20.14
181	Ord8	2	--	--	K1/1280	O2/1300	O2/1300	- / -	- / -	1238	1837	$56.57
193	Ord9	2	--	--	O2/1500	O1/1500	O2/1500	- / -	- / -	1380	585	$34.92
194	Ord10	2	--	--	K5/1000	O4/1000	T2/1050	O3/1000	T3/1050	940	679	$67.83
206	Ord3	1	Ord10	1	K5/1000	O4/1000	T2/1050	O3/1000	T3/1050	950	663	$57.68
230	Ord4	2	Ord6	1	K3/1500	O4/1500	T2/1500	- / -	- / -	1475	505	$ 9.14
231	Ord5	1	Ord8	1	K1/1350	O2/1350	O2/1350	- / -	- / -	1330	3290	$36.54
234	Ord4	1	Ord6	2	K3/1500	O4/1450	T2/1500	- / -	- / -	1390	500	$32.15
236	Ord7	2	Ord9	1	O2/1500	O1/1500	O2/1500	- / -	- / -	1480	2054	$20.34
237	Ord3	1	Ord10	2	K5/1450	O4/1450	T2/1500	O3/1500	T3/1500	1420	1670	$122.52

5. Conclusions

A two step model for planning the cutting process in the corrugated board industry has been developed. First a cutting pattern generation algorithm is executed followed by the solution of a mixed integer linear program model. This strategy has been selected because the model is simpler avoiding the use additional techniques to convexify non-convex non-linear constraints. On the other hand the patterns generated by the first step of this model can be manipulated by the planner via several parameters, in this way he can evaluate several scenarios and select the best solution to satisfy their customer demands. It also provides a more robust and faster problem solution. Comparing to the actual cutting plans generated by the experts of the company, improved solutions are obtained reducing up to a 30% the trim-loss cost. No simplifications were done to represent the real productive context. The system has been tested for a couple of months giving very good results, now is in production. Having linked the system to the company ERP besides of the advantages of the integration, via the ERP the control of the production plan generated by the model is facilitated giving an extra feature to the whole system.

References

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