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A novel continuous-time MILP approach for short-term scheduling of multipurpose pipeless batch plants

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Abstract

This work presents an alternative MILP mathematical formulation for the short-term scheduling of pipeless batch plants based on a continuous-time representation that relies on the general precedence notion. Besides of the intrinsic characteristics of a pipeless plant, this model considers the moveable vessels as an additional resource that has to be allocated and sequenced. This fact allows proposing a strategy consisting of a sequential treatment of the resources to reduce the complexity of the problem.

Keywords: pipeless plants, batch scheduling, limiting resources, MILP model.

1. Introduction

Pipeless plants have been developed to increase the plant flexibility and minimize the waste material by reducing piping and avoiding complex cleaning operations [1]. In a pipeless batch plant, materials are transported from one processing stage to another in moveable vessels transported by AGV (automated guided vehicles) while processing is carried out at a number of fixed stations. This allows for multiple production tasks processed simultaneously increasing the efficiency and shareability of all the process equipment and peripheral facilities [2]. Considering the additional difficulties that naturally arise in the short-term scheduling problem of pipeless batch plants, several authors have proposed different approaches. One of the first attempts to address this complex problem was carried out by Pantelides et al. [3], who developed a mixed integer linear programming model relying on a uniformly discretized time horizon and the state-task-network (STN) representation. Although this method was able to deal with the major characteristics of this type of process, a huge number of binary variables were inevitably needed to represent the events at the boundary of predefined uniform time intervals. Afterwards, Realff et al. [4] combined the scheduling, design and layout of pipeless batch plants in a single optimization framework. Later on, Bok and Park [5] considered the scheduling problem as a matching problem and developed an alternative MILP formulation based on a two-coordinate representation in which the time slots, symbolizing the timetable of resources, are paired with the processing stages of products. However, in this formulation computational effort is highly dependent on the number of time slots, and especially on those not matching any stage of a product (superfluous).

In this work, the short-term scheduling of pipeless plant is addressed by a general precedence notion [6] model which handles allocation and sequencing decisions through a different set of binary variables. The mathematical formulation is described in next section. A motivating example taken from the literature is described in section 3. Finally, the problem size of this example is increased for discussing how to address this problem in order to obtain efficient solutions in reasonable time.

2. The mathematical model

Given are a set of products *p* and a set of batches *i* of each product which have to be manufactured in a series of consecutive processing stations *s*. A suitable moveable vessel *k* carries the material between every station where at least one processing unit *u* exists. The goal of this problem is the minimization of makespan. A brief summary of the nomenclature used in this model can be found in Table 1.

2.1. Processing units constraints

The following constraint enforces the allocation of a suitable processing unit ($u \in U_{ps}$) to every task (*p, i, s*). Within this constraint, binary variable Y_{pisu} is equal to 1 whether the task (*p, i, s*) is assigned to unit u and 0 otherwise.

$$\sum_{u \in U_{ps}} Y_{pisu} = 1 \qquad \forall p \in P, i \in I_p, s \in S_p \qquad (1)$$

Constraint 2 guarantees that if task (*p, i, s*) precedes task (*p', i', s'*) and both tasks are processed in the same unit u, task (*p', i', s'*) can not start until task (*p, i, s*) is finished, while constraint 3 states the opposite case of the aforementioned constraint, that is, task (*p', i', s'*) precedes task (*p, i, s*).

$$ST_{p'i's'} \geq FT_{pis} - M \cdot (1 - X_{pisp'i's'}) - M\left(2 - Y_{pisu} - Y_{p'i's'u}\right) \quad (2)$$

$$ST_{pis} \geq FT_{p'i's'} - M \cdot X_{pisp'i's'} - M\left(2 - Y_{pisu} - Y_{p'i's'u}\right) \quad (3)$$

$$\forall p, p' \in P, i, i' \in I_p, s, s' \in S_p, u \in (U_{ps} \cap U_{p's'}) : p < p' \; or \; (p = p', s < s')$$

Table 1. Nomenclature

Sets		*Continuous variables*	
p, p'	Products	ST_{pis} , FT_{pis}	Starting and completion times
i, i'	Batches	*Binary variables*	
u, u'	Processing units	Y_{pisu}	Allocation single task to a unit
k	Moveable vessels	Z_{pik}	Assignment batch to a moveable vessel
P	Set of products	$X_{pis,p'i's'}$	General precedence among a pair of tasks
I_p	Set of batches of a product	*Parameters*	
S_p	Set of stations	pt_{pu} , tt_{ps}	Processing and transfer times
U_{ps}	Set of available units	$\{s^f_{pi}\}$, $\{s^l_{pi}\}$	First and last station
K_p	Set of suitable moveable vessels	M	A very large number

Constraint 4 synchronizes a pair of tasks performed in two consecutive stages and constraint 5 sequences two batches of the same product executed in the same processing unit.

$$FT_{pis} \leq FT_{pis'} \qquad \forall p \in P, i \in I_p, s, s' \in S_p : s' = s + 1 \quad (4)$$

$$ST_{pi's} \geq FT_{pis} - M\left(2 - Y_{pisu} - Y_{pi'su}\right) \; \forall p \in P, i, i' \in I_p, s \in S_p, u \in U_{ps} : i' > i \quad (5)$$

2.2. Moveable vessels constraints

The set of available moveable vessel is an additional limiting resource that has to be considered in this model. Likewise, as it was done for the processing units, constraint 6 assigns a suitable moveable vessel ($k \in K_p$) by a decision variable (Z_{pik}) equals to 1 if that batch is assigned to that moveable vessel *k*.

$$\sum_{k \in K_p} Z_{pik} = 1 \qquad \forall p \in P, i \in I_p \tag{6}$$

Similarly as constraints 2 and 3, constraint 8 and 9 sequence two tasks of different products but this time assigned to the same moveable vessel.

$$ST_{p'i's'} \geq FT_{pis} - M(1 - X_{pis'p'i's'}) - M(2 - Z_{pik} - Z_{p'i'k}) \tag{7}$$

$$ST_{pis} \geq FT_{p'i's'} - M \cdot (1 - X_{pisp'i's'}) - M\left(2 - Z_{pik} - Z_{p'i'k}\right) \tag{8}$$

$$\forall p, p' \in P, i, i' \in I_p, s, s' \in S_p, k \in (K_p \cap K_{p'}) : p < p', s = \left\{s^f_{pi}\right\}, s' = \left\{s^l_{p'i'}\right\}$$

Constraint 9 sequences tasks sharing the same moveable vessel to make them not simultaneous in time.

$$\begin{aligned} &ST_{pis} \geq FT_{pi's'} - M\left(2 - Z_{pik} - Z_{pi'k}\right) \\ &\forall p \in P, i, i' \in I_p, s, s' \in S_p, k \in K_p : i' < i, s = \left\{s^f_{pi}\right\}, s' = \left\{s^l_{p'i'}\right\} \end{aligned} \tag{9}$$

2.3. Task duration and objective function

The following constraint establishes the duration of a task taking into account unit-dependent processing times, waiting times and the moveable vessel transfer times towards the corresponding station.

$$FT_{pis} \geq ST_{pis} + tt_{ps} + \sum_{u \in U_{ps}} pt_{pu} Y_{pisu} \qquad \forall p \in P, i \in I_p, s \in S_p \tag{10}$$

Alternative objective functions may be evaluated using this formulation, but in this case the minimization makespan (11) has been considered for simplicity.

$$\min \qquad MK \geq FT_{pis} \quad \forall p \in P, i \in I_p, s \in S_p : s = \left\{s^l_{pi}\right\} \tag{11}$$

3. Case study and results

The case study addressed was firstly introduced in Bok and Park [5]. This problem consists of a pipeless plant designed to manufacture three batches of different products following the same production sequence. Table 7 shows the seven stations that the products have to undergo and the available processing units at every station. Transfer times from/to the stations are also included in this table while setup times are included in the transfer times.

The formulation was implemented within the modeling language GAMS using CPLEX version 7.5. Table 4 summarizes the results obtained by the direct application of the proposed formulation compared with the results reported by [5].

Table 3. Processing times

Products	**Units**							
	U1	U2	U3	U4	U5	U6	U7	U8
P1	0.6	0.5	0.5	0.85	0.85	0.6	0.5	0.5
P2	0.5	0.5	0.7	0.75	0.75	0.5	0.5	0.5
P3	0.5	0.6	0.5	0.65	0.65	0.6	0.5	0.5
P1	0.6	0.5	0.5	0.85	0.85	0.6	0.5	0.5

Table 4. Comparative results with the Bok and Park's model.

K[1]	Bok and Park, 1998 [5]			This model, 2007		
	Binary,cont.,rows	Iterations	MK, h	Binary,cont.,rows	Iterations	MK, h
2	207, 340, 442	6375	8.20	66, 43, 174	903	8.28
UN[2]	207, 340, 441	100751	5.66	66, 43, 162	244	5.54

[1](K) Number of moveable vessels

[2](UN) Unconstrained case: number of moveable vessels ≥ number of batches

Looking at the model size, it is remarkable the significant saving of binary variables, continuous variables and constraints achieved by the model, directly translated into the number of solver's iterations in order to reach to optimality. Furthermore, if the number is equal or higher than the number of batches (unconstrained), handling both resources, processing units and moveable vessels, through different sets of constraints, allows discarding the moveable vessels constraints. Thus, reducing significantly the magnitude of the problem.

However, as it was expected, the model complexity increases rapidly with the number of batches to be scheduled in combination with the number of available moveable vessels. Table 5 shows how a direct application of this model to the same problem but with an increased demand of two batches of each product (six batches) needs an extremely high CPU time increase. In order to overcome this problem, this model allows a sequential treatment of the resources by decomposing the scheduling problem into two sub-problems in which every kind of resource is sequenced and allocated separately. The priority order for solving these scheduling sub-problems will be given by those resources considered as more critical. The underlying idea here is trying to provide efficient solutions (not necessarily optimal) when the problem size makes the solution unaffordable in a reasonable calculation time. Therefore, the unconstrained problem is solved first discarding the variables and constraints related to the moveable vessels. Then, the binary allocation and sequencing variables for the processing unit are fixed and the model is solved again just working only in the decision variables related to the moveable vessels. Table 5 shows the results obtained using this sequential approach and its CPU time. Although the optimal solution cannot be found, better solutions are encountered in very short computational times. This situation poses a trade-off between obtaining optimal solutions at the expense of huge calculation time and acceptable solutions in very small times.

Table 5. Comparative performance using the sequential approach for large-sized problems

K	Full problem using this model				Sequential approach	
	Binary,cont.,rows	Iterations	CPU, s	MK, h	Total CPU, s	MK, h
3	198, 85, 627	57624851	36635	9.75	27.7 + 0.1	10.07
4	204, 85, 654	12087918	9850	8.94	27.7 + 0.1	9.37
5	210, 85, 681	$7.3 \cdot 10^7$	46633	8.28	27.7 + 0.1	8.53
UN	192, 85, 546	135619	27.7	7.59	-	-

4. Conclusions

The proposed continuous-time MILP formulation based on the general precedence notion achieved an important saving of binary variables and computational effort by avoiding the use of time slots. The use of a sequential treatment approach to find effective solutions for large scheduling problems with modest computational time has been also illustrated.

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