

# An Iterative Solution Approach to Process Plant Layout using Mixed Integer Optimisation

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## Abstract

This paper presents an efficient solution approach to tackle large-scale single-floor process plant layout problems. Based on the mixed integer linear programming (MILP) model proposed by Papageorgiou and Rotstein [1], the final layout (i.e. coordinates and dimensions) is determined from an initial feasible solution by an iterative improvement procedure using mixed integer optimisation. The applicability of the solution algorithm is demonstrated through two illustrative examples.

**Keywords:** Process Plant Layout, Iterative Solution Approach, Mixed Integer Optimisation

## 1. Introduction

Plant layout is considered as one of the important parts in the design stage of a chemical plant. It deals with the spatial arrangement of equipment items and the required connections among them. The generation of a good layout needs great ingenuity and experience because of its significant impact on process design and operation. Engineering, economic, safety and management issues need to be considered simultaneously and a reasonable balance must be achieved between these criteria.

A number of methodologies have been proposed to tackle the process plant layout problem. Initial approaches were based on heuristic rules and graph partitioning techniques. Stochastic optimisation techniques [2] have been applied to obtain good quality solutions. Finally, mathematical programming models were presented to solve single and multiple floor process plant layout problems. A mixed integer nonlinear programming (MINLP) approach [3] integrated safety and economic considerations with layout issues. A discrete-domain MILP model was developed in [4]. A number of continuous-domain MILP formulations have been proposed to determine the land area, floor location and detailed layout of each process unit [1, 5-8].

It is widely accepted that the optimal solutions for large-scale process plant layout problems are very difficult to achieve using current computational resources. The development of efficient solution methods are of significant importance since it offers great opportunities to obtain near optimal solutions within modest computational times. Efficient solution approaches for single and multiple floor cases were proposed in [9,10]. The approach presented in this paper is an iterative one where the solution obtained from previous iteration is improved by releasing and reallocating a number of units in the flowsheet. This is tested on two illustrative examples and some comparative results are reported.

## 2. Problem Statement

The single-floor process plant layout problem can be stated as follows: *Given* (i) a set of equipment items and their dimensions, (ii) the connection costs among equipment items; *determine* the allocation of each equipment item (i.e., coordinates and orientations); *so as to* minimise the total connection cost. In this work, we adopt the continuous-domain MILP model (named as LAYOUT; Papageorgiou and Rotstein [1]) for the single-floor process plant layout problem where the optimal location of unit  $i$  is determined by continuous variables  $X_i$  and  $Y_i$ . Binary variables  $E1_{ij}$  and  $E2_{ij}$  are used to avoid overlapping between units  $i$  and  $j$ . Equipment items are simplified as rectangular shapes and the connections among them are calculated as rectilinear distances.

## 3. Iterative Solution Approach

In this section, we present an iterative approach to tackle the single-floor process plant layout problem efficiently. According to this approach, we start from the first integer solution obtained by solving the LAYOUT model. Several units are then selected and reallocated by solving the reduced MILP model. The items that are not released maintain their relative positions. Finally, the approach terminates when no improvement of the objective function value is observed after a prespecified number of successive iterations.

Next, the following sets are defined for the description of the iterative algorithm:

Sets

$I$	Set of plant equipment units considered
$\Delta$	Set of units released in the subproblems

The steps of the proposed approach are shown below:

- Step 1: Initialise  $\Delta = \phi$ . Solve LAYOUT for every  $i \in I$  to obtain the first integer solution.
- Step 2: Fix  $E1_{ij}$  and  $E2_{ij}$  for every  $(i, j) \in I$ .
- Step 3: Decide which units are released either randomly or by probabilistic rules (see Table 1). Update  $\Delta$ .
- Step 4: Release  $E1_{ij}$  and  $E2_{ij}$  if  $i$  and/or  $j \in \Delta$
- Step 5: Solve LAYOUT. If the objective function value over a prespecified number of successive iterations remains the same, STOP. Otherwise,  $\Delta = \phi$ , go to Step 2.

It is believed that the selection of released units is of significant importance to the final solution quality. Here, we propose random and probabilistic selection schemes as shown in Table 1.

Table 1. Unit selection probability

Approach	Selection probability
Random_M	Uniform distribution
Connect_M	$P_i = \frac{NC_i}{\sum_i NC_i}$
Cost_M	$P_i = \frac{\sum_j (C_{ij} + C_{ji})}{\sum_{ij} (C_{ij} + C_{ji})}$
Link_N	$PL_{ij} = \frac{C_{ij} \cdot D_{ij}}{\sum_{ij} C_{ij} \cdot D_{ij}}$

All algorithms used are named as Random\_M, Connect\_M, Cost\_M and Link\_N, where  $M$  and  $N$  represent the number of released units and links, respectively. Algorithm Random\_M indicates that  $M$  units are chosen randomly. Alternatively, equipment items can be selected based on different probability distributions. In algorithms Connect\_M and Cost\_M, selection probabilities of each item, defined as  $P_i$ , are associated with the number of connections,  $NC_i$ , and the unit connection costs of item  $i$ , respectively. Algorithm Link\_N attempts to release all pairs of nodes that are connected by the  $N$  chosen links. The selection probability of each pair,  $PL_{ij}$ , is related to the connection costs between  $i$  and  $j$ .

#### 4. Computational Results

Two illustrative examples are investigated to demonstrate the applicability of the proposed iterative approach. Tables 2 and 3 list all the input data for both examples (rmu stands for relative monetary units).

Table 2. Dimensions of equipment units for Examples 1 and 2

Example 1			Example 2		
Unit	$\alpha_i$ [m]	$\beta_i$ [m]	Unit	$\alpha_i$ [m]	$\beta_i$ [m]
1	5.22	5.22	1	5.00	4.00
2	11.42	11.42	2	5.00	4.00
3	7.68	7.68	3	5.00	6.00
4	8.48	8.48	4	5.00	4.00
5	7.68	7.68	5	5.00	6.00
6	2.60	2.60	6	5.00	8.00
7	2.40	2.40	7	5.00	8.00
			8	5.00	6.00
			9	5.00	4.00

Table 3. Connection costs for Examples 1 and 2

Example 1		Example 2	
Connection	Cost [rmu/m]	Connection	Cost [rmu/m]
(1,2)	346.0	(1,18)	200
(1,5)	416.3	(18,7)	240
(2,3)	118.0	(2,7)	230
(3,4)	111.0	(3,6)	400
(4,5)	85.3	(6,7)	230
(5,6)	86.3	(7,10)	270
(5,7)	82.8	(6,11)	280
(6,7)	6.5	(13,10)	170
		(10,6)	300
		(8,17)	250
		(9,12)	170

Examples 1 and 2 are solved using 8 iterative algorithms and model LAYOUT as shown in Table 4. All problems are implemented in GAMS [11] using CPLEX mixed integer optimisation solver with 0% margin of optimality. All runs are performed on an hp pavilion laptop with 10000 seconds CPU limit. The proposed approach terminates when the objective function can not be improved after 20 successive iterations. Each algorithm is repeated 10 times and the best and median objective function values are reported together with the median computational times.

Example 1 considers a 7-unit ethylene oxide plant introduced by Penteado and Ciric [3]. The optimal solution is 9948.03 rmu achieved by model LAYOUT

within 2.86 seconds. When applying iterative algorithms with different values of  $M$  and  $N$ , all algorithms end up with the optimal solution thus illustrating the robustness of the proposed approach.

Example 2 considers the layout design of an 18-unit industrial multi-purpose batch plant presented by Georgiadis *et al* [4]. Within the prespecified CPU limit (10000s), model LAYOUT can not solve this example to optimality resulting in an integer feasible solution with an objective function value of 32550 rmu. The best result achieved through the iterative approach is 31640 rmu from Link\_1 and Link\_2, which is 2.80% better than model LAYOUT. Also, note that the best median results has been obtained by Link\_2 (31810 rmu), which constitutes a 2.27% improvement over the LAYOUT model.

Table 4. Computational results for Examples 1 and 2

Approach	Example 1			Example 2		
	Best	Median	CPU	Best	Median	CPU
Random_2	9948.03	9948.03	4.49	31775	33067.5	30.50
Random_3	9948.03	9948.03	6.76	31645	32492.5	92.15
Connect_2	9948.03	9948.03	4.91	31710	32672	30.21
Connect_3	9948.03	9948.03	6.23	31677.5	32331.25	87.25
Cost_2	9948.03	9948.03	3.78	31715	32691.25	34.99
Cost_3	9948.03	9948.03	8.33	31765	32813.75	75.75
Link_1	9948.03	9948.03	3.92	31640	31972.5	39.90
Link_2	9948.03	9948.03	16.08	31640	31810	498.34
LAYOUT	9948.03	9948.03	2.86	32550	32550	10000*

\*Maximum CPU limit (10000s)

The layouts for both examples associated with the best objective function values obtained from the iterative approach are shown in Figure 1.

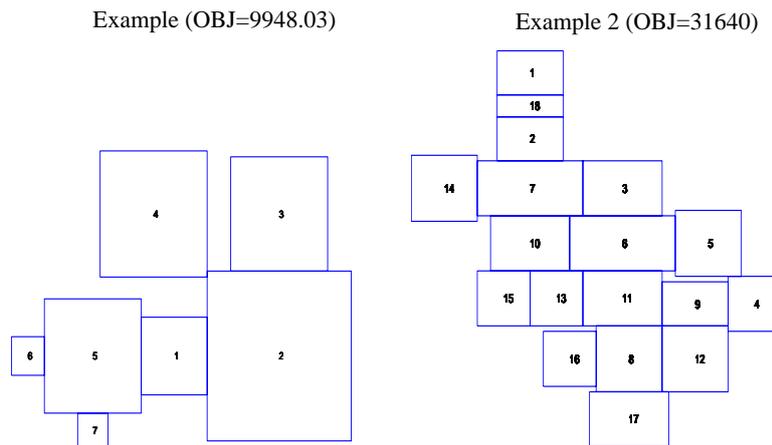


Figure 1. Best layout obtained for Examples 1 and 2

## 5. Conclusions

In this work, an iterative solution approach has been proposed to solve large-scale process plant layout problems. According to the MILP formulation [1], the solution quality has been improved from an initial feasible layout through an iterative process using releasing and reallocation schemes. During each iteration, process units are selected either randomly or based on specific probabilistic rules. Finally, the applicability of the proposed approach has been demonstrated by two illustrative examples. The results show that the iterative solution approach has great potential to obtain good quality solutions for process plant layout problems with large sizes using modest computational requirements.

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